Lesson 5: Relativity and Black Holes

Reading Assignment

- Chapter 22.5: Black Holes
- Chapter 22.4: Gamma-Ray Bursts
- Chapter 22.6: Einstein’s Theories of Relativity
  - Discovery 22-1: Special Relativity
- Chapter 22.7: Space Travel Near Black Holes
- Chapter 22.8: Observational Evidence for Black Holes
  - More Precisely 22-1: Tests of General Relativity
  - Discovery 22-2: Gravity Waves: A New Window on the Universe
- Chapter 23.7: The Galactic Center

Postulates of Special Relativity

- Read Chapter 22.6 and Discovery 22-1.
- Special Principle of Relativity
  - The laws of physics are the same the same in every inertial (non-accelerated) frame of reference.
  - The laws of physics are such that no experiment performed in an inertial frame of reference can determine whether that frame is at absolute rest or in motion with respect to an absolute frame of rest.
  - Any inertial frame of reference can be treated as if at absolute rest.
- Special Principle of Constancy of Speed of Light
  - The speed of light is the same is the same in every inertial frame of reference.

Postulates of General Relativity

- Read Chapter 22.6.
- Principle of Relativity
  - The laws of physics are the same in every frame of reference.
  - The laws of physics are such that no experiment performed in any frame of reference can determine whether that frame is inertial or accelerated.
- Principle of Constancy of Speed of Light
  - The speed of light is the same is the same in every frame of reference.
- Principle of Equivalence
  - The laws of physics are such that no experiment performed in any frame of reference can determine whether accelerations are gravitational or non-gravitational.
Summary of General Relativity

- Read Chapter 22.6.
- No accelerations are gravitational.
- Rather, mass and energy travel on the shortest path possible through curved spacetime.
- Spacetime is curved by mass and energy.

Summary of Consequences of Special Relativity

- Read Chapter 22.6 and Discovery 22-1.
- Time dilation
- Length contraction
- Relativistic mass

Summary of Consequences of General Relativity

- Read Chapter 22.6, More Precisely 22-1, Discovery 22-2, and Chapter 22.7.
- Deflection of light
- Non-Keplerian orbits
- Gravity waves
- Gravitational redshift
- Gravitational time dilation

Math Notes

- Schwarzschild Radius
  - Read Chapter 22.5.
  - \( v_{\text{esc}} = \) escape speed from surface of object
  - \( M = \) mass of object
  - \( R = \) radius of object
  - \( R_s = \) Schwarzschild radius of object
  - \( G = \) Newton’s gravitational constant
  - \( c = \) speed of light
  - Recall the escape speed equation from Lesson 2:
    - \( v_{\text{esc}} = (2GM / R)^{1/2} \)
  - Solving for \( R \) yields:
    - \( R = 2GM / v_{\text{esc}}^2 \)
  - Setting \( v_{\text{esc}} = c \) yields the Schwazschild radius of the object:
    - \( R_s = 2GM / c^2 \)
  - If an object of mass \( M \) is compressed to the point that its radius \( R \) is less than its Schwarzschild radius \( R_s \), then it would require a speed greater than the speed of light to escape its surface. Since nothing travels faster than light, nothing, not even light, would be able to escape its surface, making a “black hole”. 
For stars, M is usually measured in solar masses (M_{\text{sun}}). Hence, you might find this, equivalent, form easier to use:

\[ R_s = 3 \text{ km} \left( \frac{M}{M_{\text{sun}}} \right) \]

<table>
<thead>
<tr>
<th>Object</th>
<th>M</th>
<th>R</th>
<th>R_s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth</td>
<td>$3 \times 10^6 M_{\text{sun}}$</td>
<td>6,400 km</td>
<td>0.9 cm</td>
</tr>
<tr>
<td>Sun</td>
<td>$1 M_{\text{sun}}$</td>
<td>$7 \times 10^5$ km</td>
<td>3 km</td>
</tr>
<tr>
<td>Neutron star</td>
<td>$1.4 M_{\text{sun}}$</td>
<td>10 km</td>
<td>4.2 km</td>
</tr>
<tr>
<td>Neutron star</td>
<td>$3 M_{\text{sun}}$</td>
<td>9 km</td>
<td>9 km</td>
</tr>
</tbody>
</table>

Hence, Earth would have to be compressed to a radius of 0.9 cm, the sun to a radius of 3 km, and a 1.4 solar-mass neutron star to a radius 4 km. However, >3 solar-mass neutron stars would be smaller than their Schwarzschild radii and consequently collapse to form black holes.

Addition of Velocities

- Read Chapter 22.6 and Discovery 22-1.
- \( v_1 \) = speed of object 1 with respect to observer
- \( v_2 \) = speed of object 2 with respect to object 1
- \( v \) = speed of object 2 with respect to observer
- Newtonian addition of velocities:
  - \( v = v_1 + v_2 \)
  - But what if object 2 is light? Then \( v > c \), which would contradict the principle of the constancy of the speed of light.
- Relativistic addition of velocities:
  - \( v = \frac{(v_1 + v_2)}{1 + \frac{v_1 v_2}{c^2}} \)
  - When \( v_1 \) or \( v_2 \) is c, \( v = c \).
  - When \( v_1 \) or \( v_2 \) is much less than c, \( v \approx v_1 + v_2 \).

Time Dilation

- Read Chapter 22.6 and Discovery 22-1.
- \( t_0 \) = rest time = time that passes in a moving object’s frame of reference
- \( t \) = time that passes in a stationary observer’s frame of reference
- \( v \) = speed of moving object with respect to stationary observer
- Newtonian time:
  - \( t = t_0 \)
  - Time passes at the same rate in both frames of reference.
- Relativistic time dilation:
  - \( t = \frac{t_0}{\sqrt{1 - (v/c)^2}} \)
  - When \( v > 0 \), \( t > t_0 \).
  - When \( v > 0 \), time passes slower in the moving object’s frame of reference.
  - As \( v \rightarrow c \), \( t \rightarrow \infty \).
  - As \( v \rightarrow c \), time stops in the moving object’s frame of reference.
• When v is much less than c, \( t \approx t_0 \).

• Length Contraction
  • Read Chapter 22.6 and Discovery 22-1.
  • \( l_0 = \) rest length = length of an object when at rest
  • \( l = \) length of object when moving, in the direction of motion
  • \( v = \) speed of moving object
  • Newtonian length:
    • \( l = l_0 \)
    • Length does not change with speed.
  • Relativistic length contraction:
    • \( l = l_0 \sqrt{1 - (v/c)^2} \)
    • When \( v > 0 \), \( 1 < l_0 \).
    • When \( v > 0 \), length contracts in the direction of motion.
    • As \( v \to c \), \( l \to 0 \).
    • When \( v \) is much less than \( c \), \( l \approx l_0 \).

• Relativistic Mass
  • \( m_0 = \) rest length = mass of an object when at rest
  • \( m = \) mass of object when moving
  • \( v = \) speed of moving object
  • Newtonian time:
    • \( m = m_0 \)
    • Mass does not change with speed.
  • Relativistic mass:
    • \( m = m_0 \sqrt{1 - (v/c)^2} \)
    • When \( v > 0 \), \( m > m_0 \).
    • When \( v > 0 \), mass increases.
    • As \( v \to c \), \( m \to \infty \).
    • Since \( E = mc^2 \), it would take an infinite amount of energy to accelerate an object to the speed of light. Since an infinite amount of energy is not available, massive objects cannot be accelerated to the speed of light.
    • When \( v \) is much less than \( c \), \( m \approx m_0 \).

Homework 5

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