HW1:

1. Prove the commutation relations between the creatiion-annihilation operators:

$$a_1^\dagger a_2^\dagger = \xi a_1^\dagger a_2^\dagger \quad a_1 a_2 = \xi a_1 a_2 \quad a_1 a_2^\dagger - \xi a_1^\dagger a_2 = < \phi_1 | \phi_2 >$$

Here $\xi = \pm 1$ for bosons/fermions, respectively.
2. Find eigenvectors (if any) and the corresponding eigenvalues of a and $a^\dagger$ in both cases of bosons and fermions.
3. Does a shift of an operator $a \to a + \alpha$ (where α is a complex number) constitute a canonical transformation? Find the corresponding unitary operator in both cases of bosons and fermions (if any)?
4. Compute the density correlation function

$$< n_\alpha(r) n_\beta(r') >$$

for both, free spinless bosons and free spin-1/2 fermions (in the latter case, for equal and opposite spins).

HW2
1. Find the result of a unitary transformation

$$A \to \exp(i\lambda a^\dagger a) A \exp(-i\lambda a^\dagger a)$$

for $A = a$ and $a^\dagger$.
2. Show that the number of particles N in a system described by the Hamiltonian with a generic quartic interaction is conserved.
3. Using the Green function method, calculate the scattering amplitude $f(\theta)$ in the presence of an arbitrary (rotationally-invariant) external potential $V(r)$ in dimensions $d = 3, 2, 1$.
4. Draw all the relevant Feynman diagrams for the second order self-energy correction $\Sigma^(2)$ and find the corresponding analytical expressions in both, coordinate and momentum, representations.

HW3

1. Calculate the lowest order term in the fermion polarization operator $\Pi(\omega, q)$ that yields a dispersion of the plasmon mode and find that dispersion ($\omega = \Omega_q$) in $d = 3, 2, 1$.
2. Compute the "on-shell" (that is, for $\epsilon = \xi_p$) value of the first order self-energy correction $\Sigma^{(1)}(\epsilon, p)$ in the theory of 3D fermions interacting via an "unscreened" (retarded and long-ranged) pairwise potential

$$U(\omega, q) = \frac{g}{q^2 + i\gamma|\omega|/q}$$

What is the Z-factor (wave function renormalization) associated with this self-energy?
3. Demonstrate that in the presence of a generic instantaneous and short-ranged (screened) pairwise interaction $U(\omega, q) = const$ the fermion decay rate (inverse lifetime) behaves as $Im\Sigma(\epsilon, \xi = \epsilon) \sim \epsilon^2$. What is the proportionality factor (by order of magnitude)?
4. Calculate the first order correction to the ground state energy of the 3D fermions/bosons interacting via a non-singular potential $U(\omega, q) = const$. What is a finite temperature counterpart of this expression?

HW4

1. Find the local current $J_\mu(x)$ associated (via the Noether's theorem) with the global $U(1)$ invariance ($\psi(x) \to \psi(x)e^{i\alpha}$) in the case of
i) a complex bosonic field with the Lagrangian density $L = (\partial\psi/\partial x_\mu)^2 - V(\psi)$;
ii) a Dirac fermion field with the Lagrangian density $L = (i\bar{\psi}\hat{\gamma}_\mu(\partial\psi/\partial x_\mu) - V(\psi)$.
2. Compute (in the momentum representation)
i) the propagator $< 0|T\psi(x)\psi^*(y)|0 >$ of free massive bosons ($V(\psi) = m^2\psi^*\psi$, see the previous problem);
ii) the propagator $< 0|T\psi(x)\bar{\psi}(y)|0 >$ of free massive fermions ($V(\psi) = m\bar{\psi}\psi$, see the previous problem).
3. Calculate the return probability amplitude $< 0, T|0, 0 >$ in the 1D quantum oscillator with friction: $S[q] = \int dt(m(dq(t)/dt)^2/2 - m\omega^2 q^2(t)/2) - \alpha \int dt_1 \int dt_2 (q(t_1) - q(t_2))^2/(t_1 - t_2)^2$ for large α.
4. Using path integral, compute the average $< \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) >$ in the theory of a real D-dimensional field governed by the Lagrangian density $L = (\partial\phi/\partial x_\mu)^2/2 - m^2\phi^2/2 - g\phi^4$ up to second order in the coupling g.