PHY492: Final Report

Chaotic Advection in a Blinking Vortex Flow

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Fluid Dynamics

- **Advection** is the transport of substance by a fluid due to its motion.
  - It differs from convection in that no diffusion takes place.
- Tracers are any fluid property that can be used to track the flow of a fluid.
  - They can either be naturally occurring in the fluid, or be artificially introduced.
Lyapunov exponent

- Chaotic mixing: simple tracer fields in a mixture develop into complex fractals under the action of a fluid flow.
- Chaotic flows: small differences in initial position will lead to exponential diverging paths i.e. the flows are highly sensitive to the initial conditions.
- The Lyapunov exponent ($\lambda$) can characterize this difference in path length over time, and give a measure of this chaotic nature.

$$\Delta r(t) = \Delta r_0 e^{\lambda t}$$ (1)

Here $\Delta r_0$ gives the initial separation between two tracer particles and $\Delta r(t)$ gives their separation at time $t$. 
Fluid mixing

- *Turbulent mixing*: random structures are produced by fluid instability at high Reynolds number (Re).
- *Chaotic mixing*: laminar flows at modest Reynolds number produce complex distributions of the material.
- In both cases, stretching and folding of fluid elements causes nearby points to separate from each other irreversibly.
- A *laminar flow* occurs where fluid flows in parallel layers without disruption between layers.
Reynolds number

- *Reynolds number*: a dimensionless number that gives the measure of the ratio of inertial forces to viscous forces in a fluid.

\[
\text{Re} = \frac{\rho v L}{\mu}
\]  \hspace{1cm} (2)

\(\mu\) is dynamic viscosity, \(\rho\) is density, \(v\) is velocity and \(L\) is characteristic dimension (radius).

- In recent years it has been shown that a high Reynolds number (i.e. turbulent flow) is not necessary for complex particle trajectories in a fluid [Cartwright, 1999].

- Hassan Aref in 1984 showed that a simple fluid setup – a blinking vortex flow – could be used to create laminar fluid flow that becomes chaotic.
Applications

- Mixing of fluids is highly important process for many chemical and industrial processes.
  - In many cases however, mixing occurs in a confined space where there is not much room for disorderly flow.
- Systems such as Aref’s blinking vortex flow efficiently mix two or more liquids while maintaining a laminar fluid flow.
- The blinking vortex flow has been use in blending of polymers, creation of nanoscale structures and more.
- Chaotic flows can observed in many natural processes and system; in environmental flows such as the atmosphere and the ocean, where fluid flow are traced out by advected impurities or by biological species.
System

- Two vortices, equally spaced from the center of a cylindrical container.
  - A vortex is a region in a fluid where the fluid spins around an imaginary axis.
- Blinking: One vortex is turned on, while the other is off, and then they are reversed.
  - Cycle is repeated with a period $T$
A 2D blinking vortex flow can be described by the following

\[
\dot{x} = -\frac{Ay}{x_s^2 + y^2} \quad \dot{y} = \frac{Ax}{x_s^2 + y^2}
\]  

(3)

Where \( x_s \) is given by

\[
x_s = \begin{cases} 
  x + b & \text{if } 0 < t < T/2 \\
  x - b & \text{if } T/2 < t < T 
\end{cases}
\]  

(4)

- \( \dot{x} \) and \( \dot{y} \) are components of the velocity
- Two vortices centered at \((\pm b, 0)\)
- Describes circular motion
Theory contd.

For a single vortex, these equations reduce to

\[
\dot{x} = -\frac{Ay}{x^2 + y^2} \quad \dot{y} = \frac{Ax}{x^2 + y^2}
\]  \hspace{1cm} (5)

Squaring and adding together the two equations, and converting to polar coordinates with \(x = r \cos \theta, y = r \sin \theta\), and \(\dot{x} = -r \sin(\theta) \dot{\theta}, \dot{y} = r \cos(\theta) \dot{\theta}\)

\[
\dot{x}^2 + \dot{y}^2 = r^2 \dot{\theta}^2 = \frac{A^2}{r^2}
\]  \hspace{1cm} (6)

So, with \(v = \omega r\), and \(\omega = \dot{\theta}\),

\[
v = \frac{A}{r}
\]  \hspace{1cm} (7)
Setup

- Two glass rods as the vortices.
- Attached to two high speed DC motors, connected to the variable DC power supply.
- Controlled by relay switches that are operated by script on the Arduino Mega 2560 board.
- Rods were immersed in the fluid placed in Pyrex cylinder.
Figure: Complete setup
(a) Glass rods supported by bearings.

(b) Electronics control: Arduino Board and relays

Figure: Details of the experimental setup
Measuring rotational frequency of motors

- Investigated different methods. Ex: ripple current, stroboscope, half-silvered rod.
- Used photo-transistor based sensor.
  - A bright LED is placed on one side of the rod and a photo-transistor is placed on the other side
  - A opaque flag is placed on the rod. As the rod rotates, the flag interrupts the light falling on the photo-transistor, causing a spike in the output voltage.
  - This signal was transmitted to an oscilloscope as well as an analog input port on the Arduino board.
Figure: Phototransistor based speed sensor
Processing the Signal

Figure: A periodic signal passes the threshold nine times in four periods (4T).

- Count the number of times that the signal passed an experimentally determined threshold.
- The frequency of the motors is given by

\[
f = \frac{1}{T} \text{Hz} = \frac{4}{(4T)} \text{Hz} = \frac{4 \times 60}{(4T)} \text{rpm}
\]  

where rpm is the revolutions per minute.
Simulation

- MATLAB program to numerically solve the coupled differential equations.
- Piecewise defined differential equations are solved in segments
- Inbuilt DE solver `ode45()` used
  - Variable step Runge-Kutta Method
- The simulations tracks a single particle over time.
Figure: Comparison between results of theoretical simulation and experimental data for a single particle.
Data Acquisition

- Camera positioned below the setup - video data.
- Motion of the particles was calibrated and traced with LoggerPro software.
  - Tracer particles: small plastic spheres (diameter: 2-5mm)
- The constant in equation $A$ was determined using the fit for the equation of $v$ vs $r$. (speed dependent)
- The frequency used between 1000 – 1200rpm. (Tested 500-6000 rpm)
- At higher frequencies the vibrations in the vortices build up and generated bubble in the liquid used.
- Period $T$ between 20s to 60s.
- Liquid used was glycerin (Tested corn syrup and water)
Figure: Verification of $1/r$ dependence of velocity for a single vortex rotating at $\sim 1200$ rpm.
Tests

- Advective nature: The system should return to its original state upon reversal.
- Experimentally: run the system in the reverse direction.
- Reversing switch shown flips polarity of the DC motors, and thus the whole system would run backwards.
- Mechanical limitations: not completely reproducible

![Schematic for a reversing switch](image)

**Figure:** Schematic for a reversing switch.
Figure: The position of a particle upon reversal after 1.5 periods in a blinking vortex flow at $\sim$1000 rpm
Figure: Exponential separation of two particles with similar initial conditions in a blinking vortex flow with vortices spinning at \( \sim 1000 \text{ rpm} \).
Results

- System is advective in nature
- Verified the equations that we assumed to be true, both using our numerical simulation, as well as with experimental data.
- Position of the particles deviates exponentially
- For glycerin at \( \sim 1000 \text{rpm} \), the Lyapunov exponent is 1.0354.
- This occurs at low Reynolds number 0.4277 (Glycerin, Density: 1.26g/cm\(^3\), \( L = 0.0509 \text{m} \), \( v = 0.0063 \text{ m/s} \), Viscosity = 0.950 Pa·s)
Conclusion

- The blinking vortex system is thus a successful system with which to accomplish chaotic mixing.
- It is an advective system, where particles with small initial separation will diverge exponentially.
- It has a slew of applications, mainly in industry, for the mixing of materials both large and small.
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