When a magnetic field passes through a thin conducting sheet carrying a current perpendicular to the direction of the magnetic field, charge carrying particles are deflected by the field causing a pile-up on one edge of the sheet. This asymmetric distribution of charge sets up an electric field across the sheet (perpendicular to both the magnetic field and the current) which can be measured as a potential difference between the sides. Known as the Hall Effect, it was first discovered in 1879 by Edwin Hall, who clearly showed that electrical currents through conductors were caused by the movement of negatively, not positively, charged particles. In this paper, we use measurements of the Hall voltage in a given magnetic field to calculate the Hall constant \( R_H \) for bismuth and silver. We find this constant to be \(-6.12 \times 10^{-8} \, \text{m}^3/\text{C} \) and \(-2.00 \times 10^{-9} \, \text{m}^3/\text{C} \) for bismuth and silver respectively.

Theoretical calculations pertaining to current flow through a transverse magnetic field predict a linear relationship between the Hall voltage and \( \beta \), where

\[
\beta = \frac{IB}{d} \tag{1}
\]

In this equation, \( I \) is the current through the sheet, \( B \) is the magnetic field, and \( d \) is the thickness of the sheet. This experiment attempts to measure the constant of proportionality between these two variables for two different materials: bismuth and silver. Using the known charge of an electron and this measured Hall constant, it is possible to calculate the density of free electrons in each of these metals.

**Theoretical Background**

Current is the rate at which charge flows and can be described as \( Q/t \). As a current flows through a sheet with cross-sectional area \( A \), in any given length \( k \), the total charge is described by

\[
Q = Akqn \tag{2}
\]
where \( q \) is the charge of a single particle and \( n \) is the density of the charged particles in the material. Similarly, the amount of time it would take this amount of charge to move by a distance \( k \) is equal to \( k/v_d \) where \( v_d \) is the drift velocity of the charged particles. Substituting, the current can be rewritten as

\[
I = nqAv_d
\]  

(3)

The force on these charged particles due to the magnetic field is equal to the cross product of the magnetic field, \( B \), with the drift velocity, \( v_d \), multiplied by the charge. Substituting, this becomes

\[
F_B = \frac{qIB}{nqA}
\]  

(4)

This force results in the asymmetrical distribution of charge along the conducting sheet. An electric field forms to counter this \( F_B \). Upon equilibrium, \( F_B \) will be equal to \( F_E \), the force due to the electric field. This electric force is equal to \( Eq \) and, approximating this distribution of charge as a parallel plate capacitor, the electrical field is equal to \( V_{Hall}/w \) where \( w \) is the width of the sheet. Substituting for the electric force and setting it equal to the magnetic force yields

\[
F_B = F_E = \frac{V_{Hall}q}{w}
\]  

(5)

Finally, substituting for the magnetic force yields

\[
V_{Hall} = \frac{1}{nq} \frac{IB}{d}
\]  

(6)

where \( d \) is the thickness of the sheet. This equation shows that the Hall voltage, \( V_{Hall} \), is proportional to a parameter

\[
\beta = \frac{IB}{d}
\]  

(7)

with a constant of proportionality equal to the Hall constant

\[
R_H = \frac{1}{nq}
\]  

(8)

**Procedure**

Using a channel mask, thin films of bismuth and silver were evaporated onto glass slides inside a vacuum chamber at around \( 1 \times 10^{-6} \) torr. An additional layer of silver pads
was evaporated onto the slides: one on each end of the channel, two on one side of the channel, and one on the other side of the channel. The two pads on the same side of the channel were tied together with a potentiometer. Using a programmable current source, 1.0 mA was passed through the length of the channel. The voltage across the slide was then measured and minimized as a function of the potentiometer’s position. This minimized the magnetoresistance of the metal channel. The slides were then placed in a Varian electromagnet and subjected to a magnetic field. The current supplying the electromagnet was varied from 0.0 to 1.0 A, which, from a previous mapping of the magnetic field as a function of current, was equivalent to varying the magnetic field from 9.0 to 189.0 mT.

Due to equipment failure, a nano-Voltmeter was not used in the measurements of the Hall voltage. Instead, a less precise instrument was substituted and, therefore, less precise measurements were obtained as a function of the magnetic field. Thicknesses of each channel were measured using an angstrometer.

Data and Calculations

Table 1: Film Properties

Table 2: Data collected during the map of the magnetic field as a function of supplied current. The current was increased in 0.05 A increments up to a maximum of 1.0 A and then, to measure hysteresis effects, the current was lowered in 0.1 A increments back to 0.0 A at the center of the magnet. At a position halfway between the center and the outer edge of the magnet, the current was increased in 0.10 A increments up to a maximum of 1.0 A and then, to measure hysteresis effects, the current was lowered in 0.10 A increments back to 0.0 A. The corresponding magnetic field was measured with a Hall probe at each location. Error bars were estimated to be $\pm 0.1mT$ based on fluctuations in the readout of the Hall probe.
**Figure 1:** Plot of magnetic field as a function of supplied current at two different locations.

**Table 3:** Measurements of the Hall voltage with 1.0 mA running through the film as a function of magnetic field for both Bismuth and Silver. The current was increased in 0.05 A increments to a maximum of 1.0 A. Error bars were estimated to be $\pm 2.0\mu V$ based on fluctuations in the readout of the voltmeter.

**Figure 2:** Plot of Hall voltage as a function of supplied current for Bi and Ag.
<table>
<thead>
<tr>
<th>Metal</th>
<th>Channel Thickness (Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bismuth</td>
<td>13,252.5</td>
</tr>
<tr>
<td>Silver</td>
<td>589.0</td>
</tr>
</tbody>
</table>

*Table 1. Properties of the films*
<table>
<thead>
<tr>
<th>Current (A)</th>
<th>Magnetic field at center (± 0.1mT)</th>
<th>Magnetic field halfway out (± 0.1mT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>13.8</td>
<td>1.1</td>
</tr>
<tr>
<td>0.05</td>
<td>19.8</td>
<td>n/a</td>
</tr>
<tr>
<td>0.10</td>
<td>27.1</td>
<td>2.02</td>
</tr>
<tr>
<td>0.15</td>
<td>35.8</td>
<td>n/a</td>
</tr>
<tr>
<td>0.20</td>
<td>45.1</td>
<td>3.1</td>
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<tr>
<td>0.25</td>
<td>52.6</td>
<td>n/a</td>
</tr>
<tr>
<td>0.30</td>
<td>62.4</td>
<td>4.0</td>
</tr>
<tr>
<td>0.35</td>
<td>70.7</td>
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</tr>
<tr>
<td>0.40</td>
<td>80.6</td>
<td>5.2</td>
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<tr>
<td>0.45</td>
<td>88.8</td>
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</tr>
<tr>
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<td>98.1</td>
<td>6.4</td>
</tr>
<tr>
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<td>108.6</td>
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<tr>
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<td>116.2</td>
<td>7.5</td>
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<tr>
<td>0.65</td>
<td>126.3</td>
<td>n/a</td>
</tr>
<tr>
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<td>134.8</td>
<td>8.5</td>
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<tr>
<td>0.75</td>
<td>142.9</td>
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<tr>
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<td>152.7</td>
<td>9.7</td>
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<tr>
<td>0.85</td>
<td>162.2</td>
<td>n/a</td>
</tr>
<tr>
<td>0.90</td>
<td>171.1</td>
<td>10.9</td>
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<tr>
<td>0.95</td>
<td>182.0</td>
<td>n/a</td>
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<tr>
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<td>190.8</td>
<td>11.9</td>
</tr>
<tr>
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<td>10.9</td>
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</tr>
<tr>
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<td>30.3</td>
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</tr>
<tr>
<td>0.00</td>
<td>13.9</td>
<td>1.3</td>
</tr>
</tbody>
</table>

**Table 2.** Mapping of the Magnetic Field as a Function of Supplied Current at two Locations within the Magnet
Figure 1. Magnetic field as a function of supplied current.
### Table 3. Measurement of Hall Voltage as a Function of Current Supplied to Magnet

<table>
<thead>
<tr>
<th>Current (A)</th>
<th>Hall Voltage for Ag (± 2.0µV)</th>
<th>Hall Voltage for Bi (± 2.0µV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>-0.0</td>
<td>-0.0</td>
</tr>
<tr>
<td>0.05</td>
<td>-0.0</td>
<td>-0.8</td>
</tr>
<tr>
<td>0.10</td>
<td>-0.0</td>
<td>-1.4</td>
</tr>
<tr>
<td>0.15</td>
<td>-1.0</td>
<td>-1.6</td>
</tr>
<tr>
<td>0.20</td>
<td>-1.5</td>
<td>-1.9</td>
</tr>
<tr>
<td>0.25</td>
<td>-1.8</td>
<td>-2.9</td>
</tr>
<tr>
<td>0.30</td>
<td>-2.2</td>
<td>-4.0</td>
</tr>
<tr>
<td>0.35</td>
<td>-3.1</td>
<td>-4.5</td>
</tr>
<tr>
<td>0.40</td>
<td>-3.9</td>
<td>-3.8</td>
</tr>
<tr>
<td>0.45</td>
<td>-4.2</td>
<td>-3.8</td>
</tr>
<tr>
<td>0.50</td>
<td>-4.5</td>
<td>-3.5</td>
</tr>
<tr>
<td>0.55</td>
<td>-5.0</td>
<td>-5.0</td>
</tr>
<tr>
<td>0.60</td>
<td>-5.2</td>
<td>-5.8</td>
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<td>-5.9</td>
<td>-5.3</td>
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<tr>
<td>0.70</td>
<td>-6.2</td>
<td>-5.0</td>
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<tr>
<td>0.75</td>
<td>-6.3</td>
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<tr>
<td>0.80</td>
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<tr>
<td>0.85</td>
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<td>0.90</td>
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<td>0.95</td>
<td>-7.3</td>
<td>-5.9</td>
</tr>
<tr>
<td>1.00</td>
<td>-7.8</td>
<td>-9.0</td>
</tr>
</tbody>
</table>
Figure 2. Measurement of Hall Voltage as a Function of Current Supplied to Magnet
Discussion and Results

Using the data from the mapping of the magnetic field at the center of the magnet, a best-fit line was found using least squares fitting in Microsoft Excel. The best fit equation as well as the $R^2$ value are

$$B(I) = 179.51I + 9.4074$$

$$R^2 = 0.9993$$

where $B$ is measured in milli-Tesla and $I$ is measured in Amperes. This equation was then used to convert current supplied to magnet into mT. A plot was then constructed of $V_{Hall}$ vs. $\beta$ for both Bi and Ag. For the Bi sample, the current passing through the film was 1mA and thickness of the sample was found to be $13.252\,\text{Å}$. For the Ag sample, the current passing through the film was 1mA and the thickness of the sample was found to be $589.0\,\text{Å}$.

All units were converted to SI units. Another best-fit line was found for each sample using least squares fitting. The best-fit equations and corresponding $R^2$ values are

**Bismuth:**

$$V_{Hall} = -6.12 \times 10^{-8} \beta + 4.77 \times 10^{-7}$$

$$R^2 = 0.7949$$

**Silver:**

$$V_{Hall} = -2.00 \times 10^{-9} \beta - 7.04 \times 10^{-7}$$

$$R^2 = 0.9665$$

For bismuth and silver, $R_H$ is found to be $-6.12 \times 10^{-8} \pm 7.13 \times 10^{-9} \frac{m^3}{C}$ and $-2.00 \times 10^{-9} \pm 8.54 \times 10^{-11} \frac{m^3}{C}$ respectively (where the errors have been calculated using the $R^2$ value from each fit). From these numbers it can be stated that the charge carriers in Bi and Ag are negatively charged. This is known to be true since electrons are the charge carriers in metals. After spending many hours searching through libraries and online article databases, only two sources were found which quote experimental values for $R_H$ of Bi and Ag. In "Room temperature Hall coefficient and resistivity for selected chemical
elements” by Daniel W. Koon two values are quoted as the $R_H$ for Bi: $-1.0 \times 10^{-8} \text{ m}^3\text{ C}^{-1}$ and $-5.0 \times 10^{-7} \text{ m}^3\text{ C}^{-1}$. The percent error for each of these two values is 712 percent and 112 percent respectively. The only source found for Ag is quoted from “Hall coefficient of cubic metals” by Werner W. Schulz et al. They find $R_H$ to be $-8.81 \times 10^{-11} \text{ m}^3\text{ C}^{-1}$. Our percent error for this value is 2170 percent.

There are many possible explanations for such a high percent error. The most probable pertains to the voltmeter used in measuring the Hall voltage. Without the nano-voltmeter, we were forced to use a much less precise instrument. The change in the Hall voltage occurred in the last significant digit quoted by the voltmeter. This last digit was far from stable and quite frequently exhibited fluctuations on the order of 5.0 $\mu$V. With such great fluctuations, it was nearly impossible to obtain highly accurate results.

If more accurate results could be obtained, the Hall effect would not only serve as an experiment to determine the charge of the carriers, but additionally, it could also be used to determine the charge density of different metals.

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(1) Daniel W. Koon, ”Room temperature Hall coefficient and resistivity for selected chemical elements”, Centro de Investigaciones en Ciencia e Ingeniera de Materiales (CICIMA), (http://it.stlawu.edu/ koon/HallTable.html), Sept. 22, 2000.