Physics 174 19 March, 2006

Exercise 10—Due 26 March

Name: ______________________

Planetary Temperatures

Table: _______

The principle of equilibrium means that the forces and energies in a system are balanced. Astronomers use this principle in many settings. We will apply it to determine the temperatures of the planets in the Solar System. The key assumption is that the Sun and planets can be approximated as blackbodies. The rate at which a blackbody radiates energy is its luminosity:

$$L = 4\pi r^2 \sigma T^4. \tag{1}$$

The surface of each planet is heated by the Sun. To stay in equilibrium, each planet must radiate energy into space at the same rate it absorbs energy from the Sun:

$$L_{in} = L_{out},$$

where L_{out} is just the luminosity in equation (1) above. If we can figure out L_{in}, then we can use equation (1) to solve for the temperature of the planet. L_{in} is just the fraction of the Sun's luminosity absorbed by the planet:

$$L_{in} = f\, L_{Sun},$$

where the fraction f depends on the distance from a planet to the Sun, because the intensity of sunlight falls as the distance squared. The fraction also depends on the size of the planet, because a larger planet would absorb more sunlight. In other words:

$$f = A_p \,/\, A_{sunlight},$$

where A_p is the absorbing area of the planet and $A_{sunlight}$ is the area over which the sunlight is spread. The Sun radiates in all directions. At a distance D, the sunlight is spread over a sphere of area

$$A_{sunlight} = 4\pi D^2.$$

What about the absorbing area of the planet? It can't be the total surface area ($4\pi r_p^{\,2}$) because half the planet is in shadow at any one time. And it's not half this area ($2\pi r_p^{\,2}$) either, because the sunlight doesn't fall directly onto most of the surface. If we were right behind the planet, what would the area of its shadow be? That area is its absorbing area, and it's just the area of a circle of radius r_p:

$$A_p = \pi r_p^{\,2}.$$

Putting this all together:

$$f = (A_p/A_{sunlight}) = (\pi r_p^{\,2}/4\pi D^2) = (r_p^{\,2}/4D^2),$$

and

$$L_{in} = f\, L_{Sun} = (r_p^{\,2}/4D^2)\; 4\pi R_{sun}^{\;2}\sigma T_{sun}^{\;4}.$$

So,

$$L_{in} = (r_p^{\,2}/D^2)\; \pi R_{sun}^{\;2}\sigma T_{sun}^{\;4}. \tag{2}$$

That what the planet absorbs. It emits (from equation 1):

$$L_{out} = 4\pi r_p^{\,2}\sigma T_p^{\;4}. \tag{3}$$

To be in equilibrium, L_{in} must equal L_{out}, so we can set equations (2) and (3) equal to each other and solve for T_p.

$$\pi R_{sun}^{2}\sigma T_{sun}^{4}\ (r_p^{2}/D^2) = 4\pi r_p^{2}\sigma T_p^{4},$$

$$R_{sun}^{2}T_{sun}^{4} \,/\, D^2 = 4T_p^{4}.$$

To solve for T_p, we must divide by four and take both sides to the 1/4 power:

$$T_p = T_{sun}\ (R_{sun}/2D)_{1/2} \qquad (4)$$

Notice that the radius of the planet and the constants π and σ have canceled out.

1. Does it make sense that the size of the planet canceled out? If we set a big marble and a small marble on a sidewalk in the sun, would they have the same or different temperatures?

In equation (4), we know R_{sun} = 696,000 km and T_{sun} = 5780 K, and for a given distance D, we can solve for T_p. Converting R_{sun} to AU (0.00465 AU) and substituting it and T_{sun} into equation (4) gives:

$$T_p\ (\mathrm{K}) = 279/D^{1/2}, \text{ where } D \text{ is in AU.} \qquad (5)$$

2. Using equation (5), fill out the following table:

Planet	D (AU)	T_p (K)	Planet	D (AU)	T_p (K)
Mercury	0.39	________	Jupiter	5.20	________
Venus	0.72	________	Saturn	9.54	________
Earth	1.00	________	Uranus	19.2	________
Mars	1.52	________	Neptune	30.1	________
Ceres	2.77	________	Pluto	39.5	________
			Eris	67.7	________

3. The mean surface temperature of Venus is 737 K. Why is this value different from your answer?

4. The mean surface temperature of Earth is 287 K. How does this compare to your answer? Explain any differences.