Physics 174 7 March, 2007

Exercise 9—Due 21 March

Name: ______________________

Radioactive Dating

Table: _______

In this assignment, you will determine the ages of two moon rocks, using the isochrone dating method discussed in class. The rocks are from the Apollo 11 and Apollo 12 missions, and both have multiple minerals with measurable amounts of the parent isotope ^{87}Rb, its daughter ^{87}Sr, and the comparison isotope ^{86}Sr.

Background

Radioactive decay is a statistical process. In a given time, a certain fraction of an unstable isotope will decay to something else. This process is random. While we know what the odds are that a given atom will decay in some interval, we have no way of knowing in advance which atoms will actually decay. Typical samples contain extremely large numbers of atoms, which effectively guarantees that, for example, if the odds are 50:50 that an atom will decay in some time interval, half the atoms will actually decay in that interval.

The time interval for half of a sample to decay is referred to as its half-life, and it is a convenient way to quantify how quickly a given isotope decays. The half-lives of many radioactive isotopes are well known from laboratory measurements.

As explained in the textbook and in your lecture notes, if a parent isotope (P) decays into a daughter element (D), we can determine its age using the Age Equation:

$$t = \tau \ln (1 + D/P), \qquad (1)$$

where τ is the mean lifetime and is related to the half-life t_{hl} by

$$\tau = t_{hl} / \ln 2. \qquad (2)$$

Here, we are measuring ages based on the decay of ^{87}Rb to ^{87}Sr. The half-life (t_{hl}) for this decay process is 48.8 Gyr (4.88×10^{10} yr), and the mean lifetime (τ) is 70.4 Gyr (7.04×10^{10} yr).

The catch to this method is that a sample will almost certainly contain some of the daughter isotope when it cools and forms into rock, so we need a way of determining how much of the daughter the rock started with. That method is called **isochrone dating**.

When a rock forms, it will usually form with multiple minerals, each of which has a different chemical composition. As a result, the rock will consist of minerals with different ratios of the parent to the daughter. To calibrate how much of the daughter the rock started with, we also measure a different isotope of the daughter (D_i). Chemistry determines how much of each isotope goes into each mineral, and since all isotopes of the same element are chemically identical, the ratio of any two isotopes of the same element will not vary from one mineral to the next in a given rock. In other words, the ratio of a daughter to its comparison isotope (D/D_i) will be the same in each mineral in a given rock when it formed.

By measuring the ratios of the parent to the comparison isotope (P/D_i) and daughter to the comparison isotope (D/D_i) and plotting D/D_i as a function of P/D_i, we can determine the initial ratio of daughter to comparison isotope and correct for it. This plot is our **isochrone plot**. When the rock has just formed, all of the ratios D/D_i will be the same, so the isochrone plot would be a horizontal line. As time passes, more and more of the parent will decay into the daughter, moving each point to the left (less P) and up (more D). How far to the left and up a point moves is proportional to how much of the parent one started with, so the horizontal line will remain a line, but over time it will tilt up with a steadily increasing slope. Each of these lines is an isochrone. If a mineral in the rock contained none of the parent, then none of the daughter would ever form. Thus, the y-intercept of the isochrone, no matter its slope, will never change. It always corresponds to the initial value of the ratio D/D_i, which we will now call D_o/D_i.

All we have to do for a given isochrone is find the y-intercept, which is equal to D_o/D_i, and correct for this when using the Age Equation. The total measured amount of the daughter D consists of two parts,

$$D = D_o + D_R, \tag{3}$$

where D_R is the quantity we are after. It is the amount of the daughter produced by decay of the parent. In terms of ratios to the comparison isotope,

$$(D_R/D_i) = (D/D_i) - (D_o/D_i). \tag{4}$$

We need to correct the Age Equation by replacing D in it with the corrected value D_R, giving

$$t = \tau \ln (1 + D_R/P), \tag{5}$$

where we will determine (D_R/P) from the ratio

$$(D_R/P) = (D_R/D_i) / (P/D_i). \tag{6}$$

Note that the isochrone method only determines the time that has passed since the rock cooled. If a rock is remelted and reformed, that resets the clock, since all of the minerals in the rock would again have the same ratio D/D_i, making the isochrone horizontal once more.

Name: ________________________

1. The Apollo 11 Rock

Table: ________

Mineral	(1) $^{87}Rb/^{86}Sr$ P/D_i	(2) $^{87}Sr/^{86}Sr$ D/D_i	(d) $^{87}Sr_R/^{86}Sr$ D_R/D_i	(e) $\ln(1+D_R/P)$	(f) t (Gyr)
1	2.3	70.04	________	________	________
2	3.9	70.14	________	________	________
3	7.4	70.31	________	________	________
4	10.3	70.46	________	________	________
5	14.5	70.68	________	________	________

a. Plot columns (1) and (2) on the isocrhone plot on the last page of this assignment.

b. Carefully draw a **straight line** through **all of the data**. Do not connect the data points with multiple lines—just draw one line which comes closest to all of them. (Alternatively, you may make a least-squares linear fit and draw this on the plot.)

c. Carefully read off the y-intercept from the plot (or the least-squares fit). The y-intercept is the point where the line you just drew crosses the left-hand side of the plot (where $P/D_i = 0$).

y_intercept = D_o/D_i = ____________

d. Subtract this amount from each measurement of D/D_i to get D_R/D_i, and enter the results in column (d) in the table above. Carry your calculations to the hundredths place.

e. Compute the next column in the table by using the formula at the top of column (e). Remember that the ratio D_R/P is the ratio of column (d) over column (1).

f. Compute the ages in the last column by multiplying column (e) by the mean lifetime, 70.4 Gyr.

g. Find the average of column (f). That is your estimated age of the rock.

Average age = ____________ Gyr

h. Estimate the uncertainty in the age of the rock by how far the minimum and maximum in column (f) deviate from the average.

Uncertainty = ____________ Gyr

2. The Apollo 12 Rock

Repeat step 1 for the Apollo 12 rock. You should put these data on the same plot as used in step 1, but using a different color of lead or ink.

Mineral	(1) $^{87}Rb/^{86}Sr$ P/D_i	(2) $^{87}Sr/^{86}Sr$ D/D_i	(d) $^{87}Sr_R/^{86}Sr$ D_R/D_i	(e) $\ln(1+D_R/P)$	(f) t (Gyr)
1	1.51	70.02	________	________	________
2	2.19	70.07	________	________	________
3	8.79	70.36	________	________	________
4	15.40	70.67	________	________	________

c. y_intercept = D_o/D_i = ____________

g. Average age = ____________ Gyr

h. Uncertainty = ____________ Gyr

3. Questions

a. Which of the two ages you determined is more reliable? Why?

b. Most highland rocks range in age from 3.8 to 4.2 Gyr, with some as young as 3.5 Gyr. Most maria rocks range in age from 3.2 to 3.9 Gyr. Which areas on the Moon do you think the two rocks came from? How certain are you for each rock?

Name: ______________________

The isochrone plot

Table: ________