Physics 174 24 January, 2007

Exercise 2 — Due 29 January, 2007

Name: ________________________

The Meaning of Measure

Table: ________

In this exercise, we will explore what a measurement really is.

Step 1. Measure your textbook (Seed, Fifth Ed.).

Lay your ruler down on your textbook and measure its length (along its longest axis). Be sure to:

- Ensure that the zero point of your ruler is exactly at one end of your textbook. You will have to sight the ruler from directly above to avoid any parallax effects.
- Ensure that the ruler is aligned parallel to the long axis of the book. Don't let it go diagonal!
- Ensure that when you measure the length, your eye is directly above the right end of the ruler. Again, this is to avoid any parallax effects.
- Estimate the length to the nearest tenth of a millimeter. You *can* do it.

Length $\equiv L$ = ____________ mm.

Enter this number using your clicker when prompted.

Step 2. Make some predictions.

Using the entire class, we have a sample of about 85 measurements of the same quantity. The spread in these measurements can give us some idea of how precise our measurement really is. Before proceeding, we need to define the **mean length** ($\langle L \rangle$) and **standard deviation** (σ) of the sample. The mean is easy; it's just the average of everyone's measurements. The standard deviation is a measure of the spread of the individual measurements around the mean, defined such that roughly two thirds of the measurements will be between the mean minus one standard deviation and the mean plus one standard deviation.

What is your estimate of the standard deviation of the sample? $\sigma \sim$ ________ mm.

Taking the mean of the class's measurements gives a value that is more precise than any individual measurement. We can use the standard deviation to estimate the uncertainty in an individual measurement, but the uncertainty in the class mean is smaller.

What do you think the uncertainty in the mean (u) will be? $u \sim$ ________ mm.

Step 3. Examine the measurements made by the entire class.

Your instructor will collect the measurements made in step 2, calculate the mean and standard deviation, and report these to you. Record all of the digits your instructor gives to you. Not all of them are meaningful, but we'll get to that in Step 5 below.

$<L>$ = ________________ mm. σ = ____________ mm.

How does the standard deviation of the sample compare to your estimate in Step 2?

The uncertainty in the mean (u) is the standard deviation (σ) divided by the square root of the number of measurements (N).

N = ________. $u = \sigma/N^{1/2}$ = ____________ mm.

How does u, the uncertainty in the mean, compare to your estimate in Step 2?

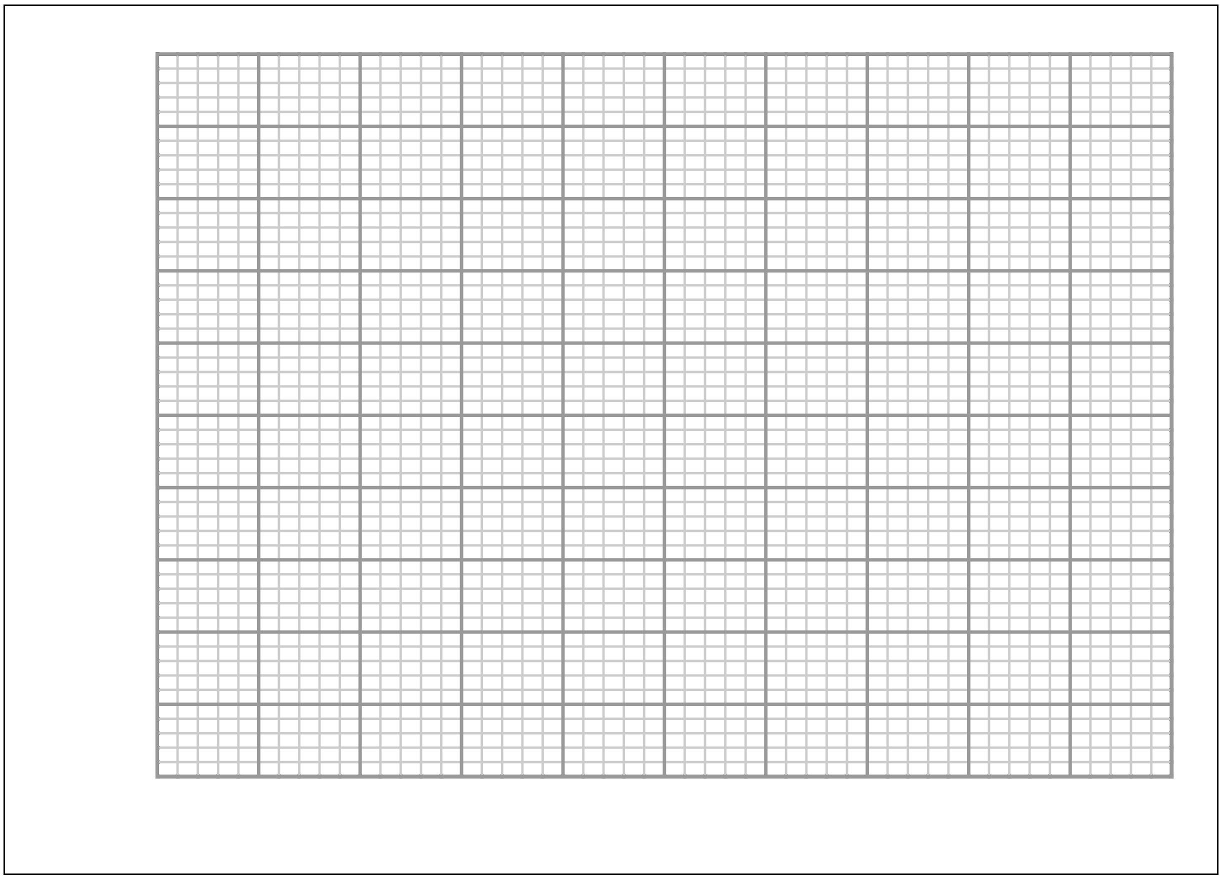
Your instructor will give you the distribution of the measurements made by all of the students in the class. Record them below:

L (mm)	n	L (mm)	n	L (mm)	n
________	______	________	______	________	______
________	______	________	______	________	______
________	______	________	______	________	______
________	______	________	______	________	______
________	______	________	______	________	______
________	______	________	______	________	______
________	______	________	______	________	______
________	______	________	______	________	______
________	______	________	______	________	______

 Name: ________________________

Step 4. Plot the results.

Use the graph below and the data table on the previous page to make a histogram of the measurements. Label the vertical axis "*n* (number of measurements)" and the horizontal axis "length (mm)". Choose a range of values for the horizontal and vertical axes in order to use as much of the plotting area as possible, and label some of values along the axes.



On the histogram, draw a vertical line from top to bottom indicating the mean length.

What is the range of lengths within one standard deviation of the mean? I.e., calculate the two values below:

$\langle L\rangle - \sigma =$ ________ mm.

$\langle L\rangle + \sigma =$ ________ mm.

Draw these on the histogram as dashed vertical lines.

Estimate the number of measurements between these lines: ________

The number of measurements within a standard deviation of the mean should be about two thirds of the total sample.

How many measurements are there? (i.e. What is N?) ________

What is two thirds of this number? ________

How does this compare to your answer at the bottom of the previous page?

Step 5. The meaning of a measurement.

The point to the previous few questions is that even a simple concept like the length of your textbook is not a definite quantity. We only know the length within the uncertainty of the measurement. And what is this uncertainty? Even though the sample of measurements is spread over a range $\langle L \rangle \pm \sigma$, the more measurements we make, the more precisely we know the mean of the sample. That is why we defined the uncertainty in the mean as $u = \sigma / N^{1/2}$, and that is the true uncertainty.

Scientists will state a measurement with an indication of how precisely they know what they measured, and there are two ways they do this. First, they will limit how many digits they write down. If we only know something to a hundredth of a millimeter, then the thousandths place has little meaning. The digits after that are completely meaningless. In this case, the common approach would be to include the thousandths place when writing the measurement. Second, they will quote the mean, along with the uncertainty. Applying both of these techniques, and using the values for $\langle L \rangle$ and u in Step 3, fill in the following statement:

The length of the textbook is: ________________ ± ____________ mm.

For the remainder of this course, we will usually not quote uncertainties explicitly, but we will imply them by the number of significant figures we write down. This takes some getting used to. A general rule is that if you measure a quantity to X significant digits, we should carry all calculations with one extra digit, but when we write the final answer, we should record only X digits. The rest are meaningless, and writing them down gives an impression that our answer is more precise than it really is. With that in mind, fill in the blanks below:

6/9 = ____________. 24/36 = ___________.

222/333 = ____________. 804/1206= ____________.