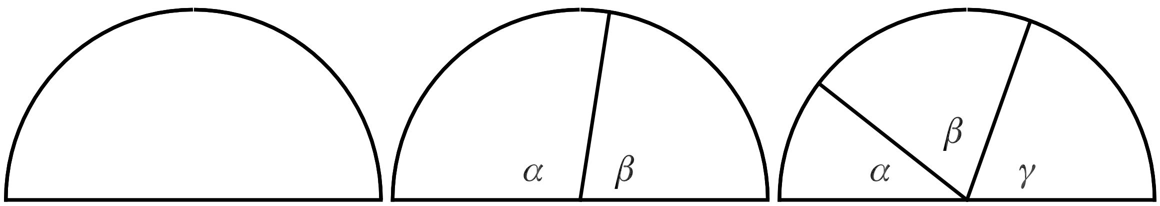
Physics 174 2 February, 2007

An Introduction to Angles in the Sky

The celestial sphere model explains how the appearance of the skies depends on an observer's location on Earth. This handout will help you understand this relationship. We will begin with some basic concepts from your high school geometry.

If you will recall, supplementary angles are defined to be angles whose sum is 180°. Now, 180° is just the angle of a straight line, so supplementary angles always form a line.



180° $\alpha + \beta = 180°$ $\alpha + \beta + \gamma = 180°$

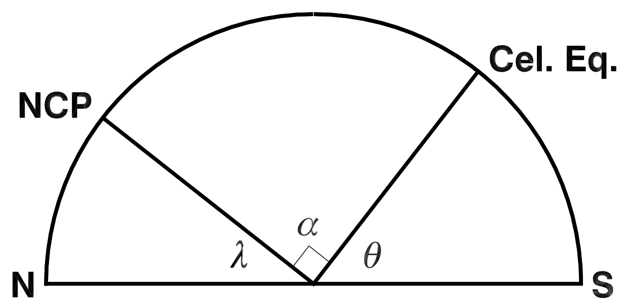
This can be very useful when studying the celestial sphere. Suppose we wish to know how high above the southern horizon the celestial equator gets at a certain position on Earth. This altitude will just be the altitude of the celestial equator on the meridian, labelled θ in the figure below. In the figure, the angle α is just a right angle, 90°. The angle between the NCP and the northern horizon is simply the latitude λ. Since the angles θ, α, and λ form a straight line, we have,

$$\theta + \alpha + \lambda = 180°,$$

$$\theta + 90° + \lambda = 180°,$$

$$\theta + \lambda = 90°,$$

$$\theta = 90° - \lambda.$$



As an example, let's pick Athens, Greece, which has a latitude λ = 38° N. Thus, the NCP is 38° above the northern horizon. We can also find the angle θ:

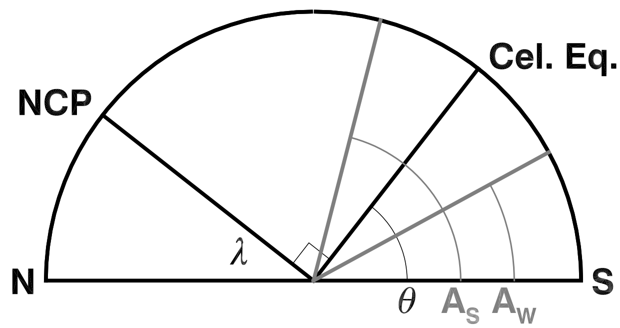
$$\theta = 90° - 38° = 52°.$$

That is, the celestial equator is 52° above the southern horizon in Athens.

The next question to consider is how high the Sun gets in the sky. The Sun reaches its maximum altitude at noon, when it transits (i.e. crosses the meridian). At that point, it is directly over the southern horizon.

Because of the tilt of the Earth's axis, the Sun can have a declination anywhere between $\delta = 23.5°$ N = +23.5° or $\delta = 23.5°$ S = −23.5°. When does the Sun reach these positions?

Event	Approximate Date	δ (°)
Vernal equinox	22 March	0
Summer solstice	22 June	+23.5
Autumnal equinox	22 September	0
Winter solstice	22 December	−23.5



For our example of Athens, Greece, the figure above shows the range of angles to the Sun when it crosses the meridian (in grey). A_S and A_W are the altitude of the Sun at the summer and winter solstices respectively. On the equinoxes, the Sun is on the celestial equator, so its altitude is just θ, the altitude of the celestial equator over the southern horizon. In general:

$$A = \theta + \delta.$$

In Athens, θ = 52°. Using the values for δ in the table above, we can solve for A_S and A_W:

$$A = \theta + \delta,$$

$$A_S = 52° + 23.5° = 75.5°.$$

$$A_W = 52° + -23.5° = 52° - 23.5° = 28.5°.$$

To complete our example, the maximum altitude of the Sun in Athens, Greece can range from only 28.5° in the winter to 75.5° in the summer.

With these tools, we can now determine the maximum altitudes of the Sun from any place on the Earth on any date, provided we know the Sun's declination (δ) on that date.