Astronomy 1102/1104 19 January, 2009

Powers of Ten

Astronomy deals with a wide range of numbers, as large as the size of the Universe and as small as the mass of an electron. As a consequence, we will need to express numbers using **scientific notation**, a shorthand way of representing very large or very small numbers. For example, one hundred can be expressed as

$$100 = 10 \times 10 = 10^2;$$

this is ten **squared** or ten to the **second** power (2 is the exponent of 10). Similarly, one million is

$$1{,}000{,}000 = 10^6.$$

Generally, numbers involve more than factors of ten. Using scientific notation, a number like 362,900 could be written as

$$3.629 \times 100{,}000 = 3.629 \times 10^5.$$

The right-hand side of the above equation is in correct scientific notation. Numbers smaller than one can also be expressed with scientific notation. For example,

$$0.000595 = 5.95 \times 1/10000 = 5.95 \times 10^{-4}.$$

Now we can discuss a new concept, the **logarithm**. People often think that logarithms are far more complicated than they really are. Essentially, the log of a number is the power to which you must raise 10 to get that number. For example,

$$10^2 = 100.$$

Therefore, the log of 100 is 2. Similarly, the log of one million is 6. This is written as

$$\log 100 = 2$$

or

$$\log 1{,}000{,}000 = 6.$$

More formally, if

$$10^a = x,$$

then

$$\log_{10} x = a.$$

Note that 10 is the **base** of the logarithm. If ten is the base, you are working with **common** logs and the subscript 10 is usually dropped (as was done in some of the equations above). You can have other bases; mathematicians usually work with **natural** logs, which have the base e = 2.718.

One useful aspect of exponents and logarithms is that they are additive. For example, we know that

$$\log 100{,}000 = 5$$

since

$$100{,}000 = 10^5.$$

But notice that we could write

$$100{,}000 = 100 \times 1000 = 10^2 \times 10^3 = 10^{2+3}.$$

Similarly,

$$\log 100{,}000 = \log 100 + \log 1{,}000.$$

or

$$5 = 2 + 3.$$

To express this more generally,

$$\log (ab) = \log a + \log b,$$

and

$$\log (a/b) = \log a - \log b.$$

Usually, the logarithm of a number is not an integer. For example,

$$\log 3.162 = 0.5$$

What then would be the logarithm of 316.2? Well,

$$\begin{aligned} \log 316.2 = \log (3.162 \times 10^2) &= \log 3.162 + \log 10^2 \\ &= 0.5 + 2 \\ &= 2.5. \end{aligned}$$

When solving problems, keep in mind that you can exponentiate or take the log of both sides of an equation and it will still be an equation. For example, if we exponentiate the equation

$$x = 6y,$$

we get

$$10^x = 10^{6y}.$$

This is still an equation. If you don't believe it take $x=6$ and $y=1$. That certainly solves the first equation. It also solves the second, since both sides are 10^6, which is just a million.

Fun, huh? Understanding and using scientific notation is very important in this course. While the rest of this handout covers less essential material, you may find some of these handy rules useful later on.