

Exercise 13—Due in section from 13 April

Name: _____

Orbital Resonances

Section: _____

1. The Galilean Satellites

Io, Europa, and Ganymede all show evidence of geologic activity (in the case of Io, it is pretty obvious). This activity has been tied to orbital resonances between the three moons. The question to answer is what those resonances are.

a. In the table below, divide the period of the moon by the period of Io and record the answer in the column for period ratio.

Moon	a (Gm)	p (d)	period ratio	resonance
Io	0.422	1.77	1.00	1:1
Europa	0.671	3.55	_____	_____
Ganymede	1.070	7.15	_____	_____
Callisto	1.880	16.7	_____	_____

b. In the table above, round the number to the nearest tenths and write the result as a ratio in the last column (e.g. 3:1).

c. Which of the moons is farthest from a resonance? _____

d. Which of the moons is least geologically active? _____

e. Explain your answers to (c) and (d) above.

(More on the back.)

2. The Moons and Rings of Saturn

The table below lists the semi-major axes and orbital periods of the larger moons of Saturn.

Moon	a (Gm)	p (d)	Moon	a (Gm)	p (d)
Mimas	0.186	0.94	Titan	1.220	16.0
Enceladus	0.238	1.37	Iapetus	3.560	79.3
Tethys	0.295	1.89			
Dione	0.377	2.74			
Rhea	0.527	4.52			

The Cassini Division lies between the B and A rings, between 117,500 and 122,300 km (0.118-0.122 Gm). Its inner edge, which is the outer edge of the B ring, is maintained by an orbital resonance with one Saturn's moons. To determine which moon is responsible, we must first determine the orbital period at 0.118 Gm. Saturn has a mass of $2.86 \times 10^{-4} M_{\text{Sun}}$. Substituting this into Kepler's Third Law gives:

$$p^2 = K a^3,$$

where

$$K = 139.4 \text{ d}^2 \text{ Gm}^{-3}. \text{ (Remember that "d}^2 \text{ Gm}^{-3}\text{" gives the units of } K\text{.)}$$

- Solve this equation for p as a function of a . $p = \underline{\hspace{2cm}}$.
- What is the period of the outer edge of the B ring? $p = \underline{\hspace{2cm}}$ days.
- The outer edge of the B ring has a $\underline{\hspace{1cm}} : \underline{\hspace{1cm}}$ resonance with $\underline{\hspace{2cm}}$.
- The Cassini mission has confirmed that Enceladus is geologically active. Does it have a resonance with any other moon? If so, identify the resonance.
- There are other resonances among Saturn's moons.
 - Mimas has a $\underline{\hspace{1cm}} : \underline{\hspace{1cm}}$ resonance with $\underline{\hspace{2cm}}$.
 - Titan is in a $\underline{\hspace{1cm}} : \underline{\hspace{1cm}}$ resonance with Iapetus.