Astronomy 1102/1104 6 April, 2009
Exercise 12—Due in section from 13 April

Name: ________________________

The Mass of Jupiter

Section: ________

In this assignment, we will explore how to use Kepler's Third Law to measure the mass of a planet. Kepler's Third Law relates the mass of an object to the period and semi-major axis of any moon orbiting it. If we know two of these quantities, we can solve for the third. Jupiter has an extensive system of satellites. We can measure the orbital period and distance for any one of them and from this determine the mass of Jupiter. In general, Kepler's Third Law takes the form:

$$p^2 = (4\pi^2/GM)\, a^3, \qquad (1)$$

where p is the period, a is the semi-major axis, M is the mass of the central body, and G is the gravitational constant (6.67×10^{-11} N m^2 kg^{-1}). We can simplify this equation a little with a proper choice of units. For the orbits of moons around Jupiter, we can express the period p in days, the semi-major axis a in units of one million km (i.e. a billion meters, or a gigameter, Gm!), and the mass M in solar masses. Then, Kepler's Third Law becomes:

$$p^2 = (C/M)\, a^3, \qquad (2)$$

where

$$C = 0.0398\ \mathrm{M_{Sun}\ d^2\ Gm^{-3}}.$$

Note that C is just a number. The text "$\mathrm{M_{Sun}\ d^2\ Gm^{-3}}$" just gives the units for the number "0.0398" (solar masses times days squared over gigameters cubed).

a. Solve equation (2) for M. What is the formula? M = ____________________.

b. For each of the Galilean satellites, you can make an independent measurement of the mass of Jupiter. Fill in the following table using the equation above.

Moon	a (Gm)	p (d)	M_J ($\mathrm{M_{Sun}}$)
Io	0.422	1.77	__________
Europa	0.671	3.55	__________
Ganymede	1.070	7.15	__________
Callisto	1.880	16.7	__________

c. What is the mass of Jupiter as a percentage of the mass of the Sun? __________