Chern-Simons Gravity:

its effects on bodies orbiting the Earth

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arXiv: 0708.0001, to appear in PRD

TASC: November 1, 2007

A Brief Outline

I. Introducing Chern-Simons Gravity
What is it and why do we care?

II. Hunting for Chern-Simons Gravity
Where do we look for modifications?

III. Chern-Simons Gravitomagnetism
Gravito-what??

IV. Constraining Chern-Simons Gravity
Do we know anything?

Defining Chern-Simons Gravity

Start with **the action**

Jackiw, Pi 2003

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{16\pi G} R + \frac{\ell}{12} \theta R \tilde{R} \right)$$

Action *Volume* *Standard GR* *Chern-Simons*

$$R\tilde{R} \equiv R^{\beta}{}_{\alpha}{}^{\gamma\delta} \tilde{R}^{\alpha}{}_{\beta\gamma\delta} = R^{\beta}{}_{\alpha}{}^{\gamma\delta} \left(\frac{1}{2} \epsilon_{\sigma\tau\gamma\delta} R^{\alpha}{}_{\beta}{}^{\sigma\tau} \right)$$

Dual of Riemann tensor

$\theta = v_\mu x^\mu$ is an **external field**

Why Chern-Simons?

I. A simple way to add parity violation to gravity

- $R\tilde{R}$ is a pseudoscalar $(R\tilde{R} \to -R\tilde{R})$
- use CS gravity to explore and constrain parity-violating gravitational effects

II. Connected to string theory

- The effective 4D action for heterotic and type II string theory yields the same field eqns. as CS gravity
  Campbell, Duncan, Kaloper, Olive 1990, 1991

III. Gives a mechanism for leptogenesis

Alexander, Peskin, Sheikh-Jabbari 2006
Alexander, Gates 2006

The CS Field Equations

We treat θ as a dynamical scalar field

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{16\pi G}R + \frac{\ell}{12}\theta R\tilde{R} - \frac{1}{2}(\partial\theta)^2 - V(\theta) + \mathcal{L}_{\rm mat}\right]$$

Action — Volume — Standard GR — Chern-Simons — Scalar Lagrangian

Vary this action with respect to $g_{\mu\nu}$

$$G_{\mu\nu} - \frac{16}{3}\ell\pi G C_{\mu\nu} = -8\pi G T_{\mu\nu},$$

Einstein Curvature — Cotton Curvature — Stress-Energy

$$C^{\mu\nu} = \frac{1}{2}\left[(\partial_\sigma\theta)\left(\epsilon^{\sigma\mu\alpha\beta}\nabla_\alpha R^\nu_\beta + \epsilon^{\sigma\nu\alpha\beta}\nabla_\alpha R^\mu_\beta\right) + \nabla_\tau(\partial_\sigma\theta)\left(\tilde{R}^{\tau\mu\sigma\nu} + \tilde{R}^{\tau\nu\sigma\mu}\right)\right]$$

Derivatives of Ricci tensor — Dual of Riemann Tensor

Hunting for CS Gravity

$$G_{\mu\nu} - \frac{16}{3}\ell\pi G C_{\mu\nu} = -8\pi G T_{\mu\nu},$$

Einstein Curvature *Cotton Curvature* *Stress-Energy*

The Cotton tensor vanishes in spherically symmetric spacetimes.

Campbell, Duncan, Kaloper, Olive 1991

- The Schwarzschild spacetime is still the solution for a non-rotating star. Nearly all Solar System tests of gravity cannot distinguish between CS gravity and General Relativity.

- The Friedmann-Robertson-Walker spacetime is still the solution for a homogeneous and isotropic expanding universe. Probes of cosmic evolution cannot distinguish between CS gravity and General Relativity.

Hunting for CS Gravity

We need to **break spherical symmetry** if we want to differentiate between CS gravity and General Relativity!

I. **Gravitational waves** are produced differently

- Binary systems produce circularly polarized gravitational waves.
  Jackiw, Pi 2003
- The gravitational wave background from inflation could also be circularly polarized. *Lue, Wang, Kamionkowski 1999*
  Satoh, Kanno, Soda 2007

II. What about **spinning masses in the Solar System?**

- Spinning masses source a gravitomagnetic field.
- Current and future satellites probe the Earth's gravitomagnetic field.
- Can we use measurements of the Earth's gravitomagnetic field to constrain CS gravity? ***YES!***

Introducing Gravitomagnetism

Start with a **perturbed metric:** $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ (*Flat*: $\eta_{\mu\nu}$; *Small perturbation*: $h_{\mu\nu}$)

Define the **vector potential:** $$A_\mu \equiv -\frac{1}{4}\left(h_{0\mu} - \frac{1}{2}\eta_{0\mu}h\right)$$

$$\partial_\mu A^\mu = 0$$ *Lorenz gauge*

Define the **mass current density:** $$J_\mu \equiv -T_{\mu 0} = (-\rho, \vec{J}\,)$$

density (ρ), *density x velocity* ($\vec{J}$)

The **Einstein and geodesic equations** have familiar forms:

$$\Box A_\mu = -4\pi G J_\mu$$

$$\vec{a} = -\vec{E} - 4\vec{v} \times \vec{B}$$

acceleration ($\vec{a}$), *velocity* ($\vec{v}$)

$$\vec{E} = \vec{\nabla} A_0 - \partial_t \vec{A}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

Chern-Simons Gravitomagnetism

Restrict to a homogeneous scalar field $\theta(t)$.

The Cotton tensor does not vanish: $C^{\text{linear}}_{0i} = -\dot{\theta}\ \partial^{\alpha}\partial_{\alpha}B_i$

The New "Maxwell" Equations

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \vec{\nabla}\cdot\vec{E} = 4\pi G(\rho+\rho_\theta)$$

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$$\vec{\nabla}\times\vec{B} - \frac{\partial\vec{E}}{\partial t} - \frac{1}{m_{cs}}\Box\vec{B} = 4\pi G\vec{J}$$

Standard GR

Mass Current Density

New parity-violating term!

$$m_{cs} \equiv \frac{-3}{8\pi G\ell\dot{\theta}}$$

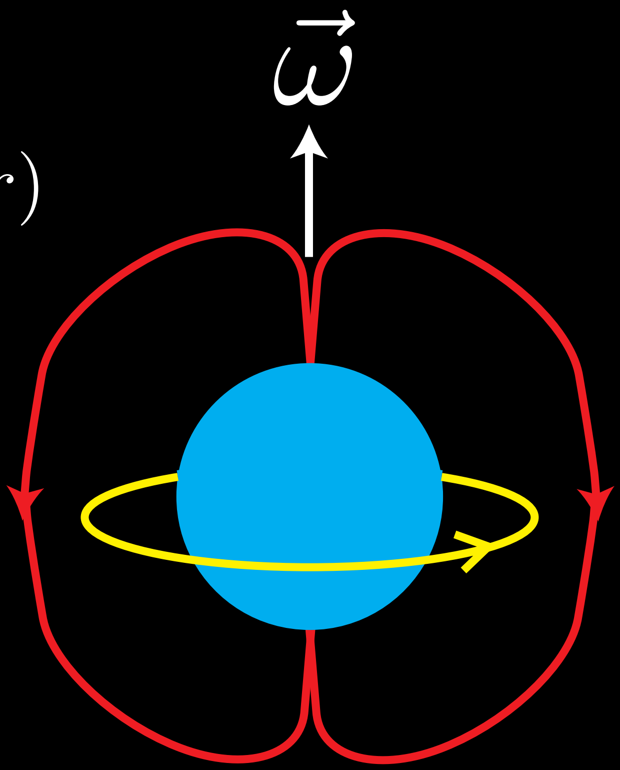
A Spinning Sphere

A homogeneous spinning sphere: $\vec{J} = \rho(\vec{\omega} \times \vec{r})\Theta(R - r)$

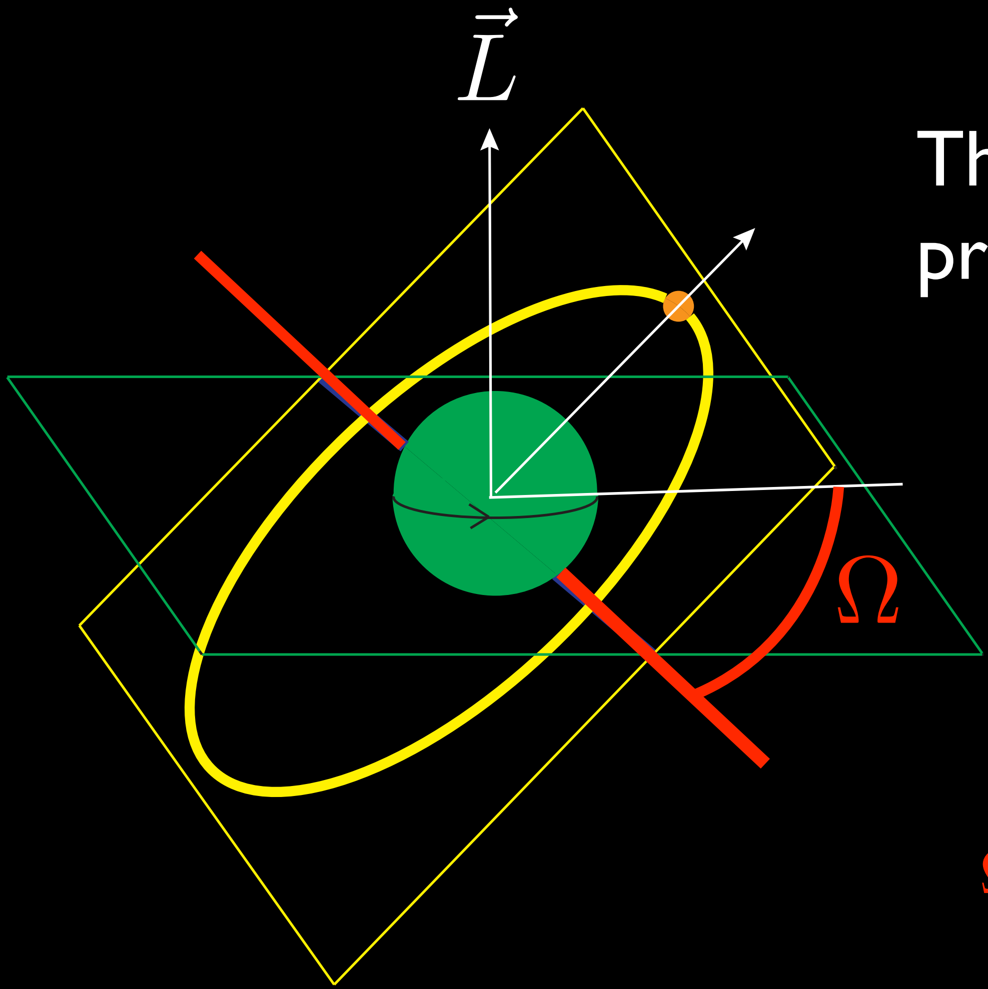
Solving the modified Ampère's Law and imposing continuity of the vector potential yields

$$\vec{B} = \vec{B}_{GR} + \vec{B}_{CS}$$

- $\vec{B}_{CS}$ is ocsillatory: $B_{CS} \propto y_{1,2}(m_{cs} r)$
- While $\vec{B}_{GR}$ is purely poloidal, $\vec{B}_{CS}$ has poloidal and toroidal components. Toroidal fields violate parity.



Constraints from Orbital Precession



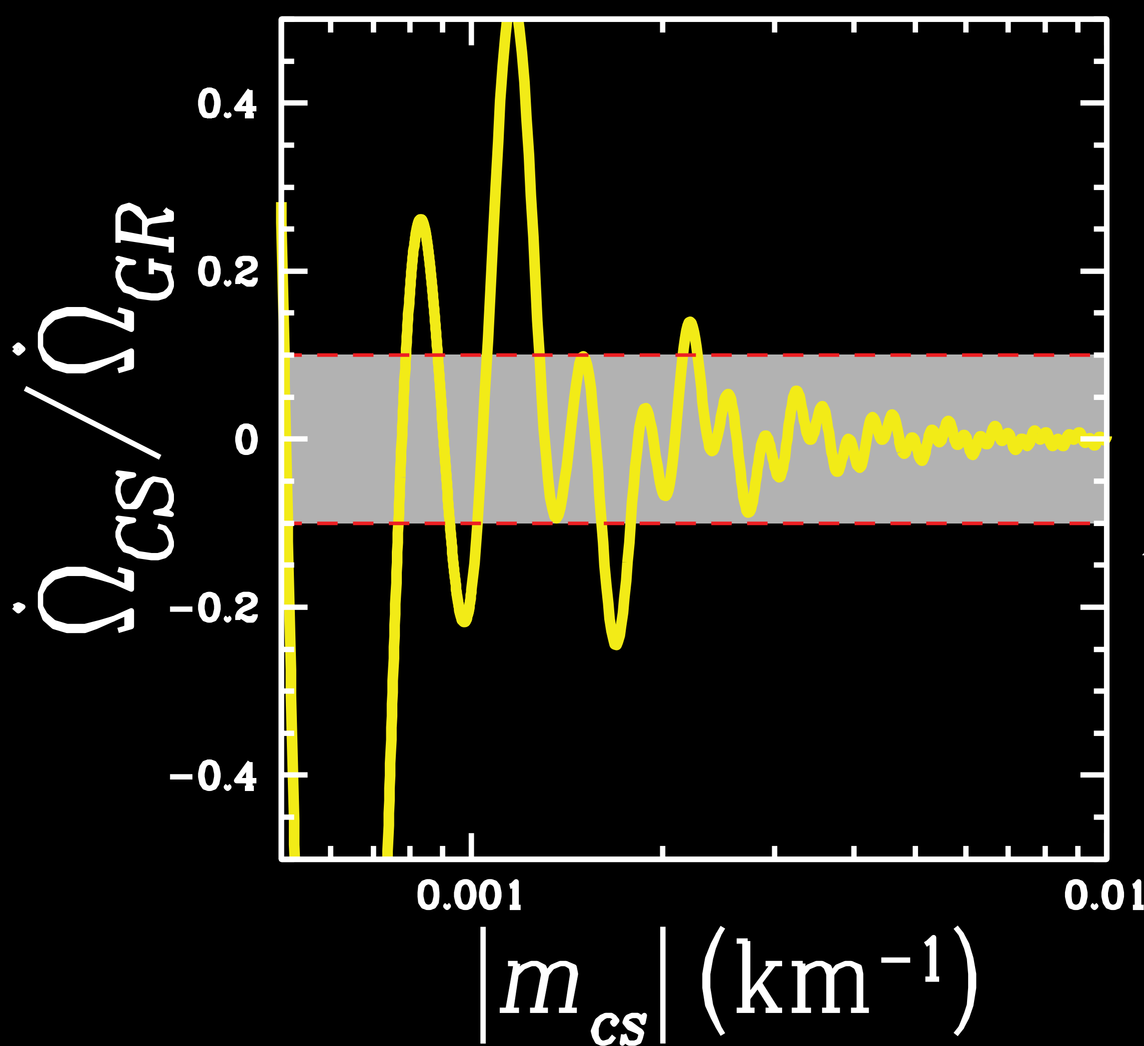
The gravitomagnetic field causes a precession of the line of nodes.

$$\dot{\Omega}_{GR} = \frac{2GL}{a^3(1-e^2)^{3/2}}$$

$\dot{\Omega} \simeq 31 \text{ mas yr}^{-1}$ for $a = 12,200\text{km}$

$$\left(\vec{B} = \vec{B}_{GR} + \vec{B}_{CS}\right) \Rightarrow \left(\dot{\Omega} = \dot{\Omega}_{GR} + \dot{\Omega}_{CS}\right)$$

Constraints from Orbital Precession



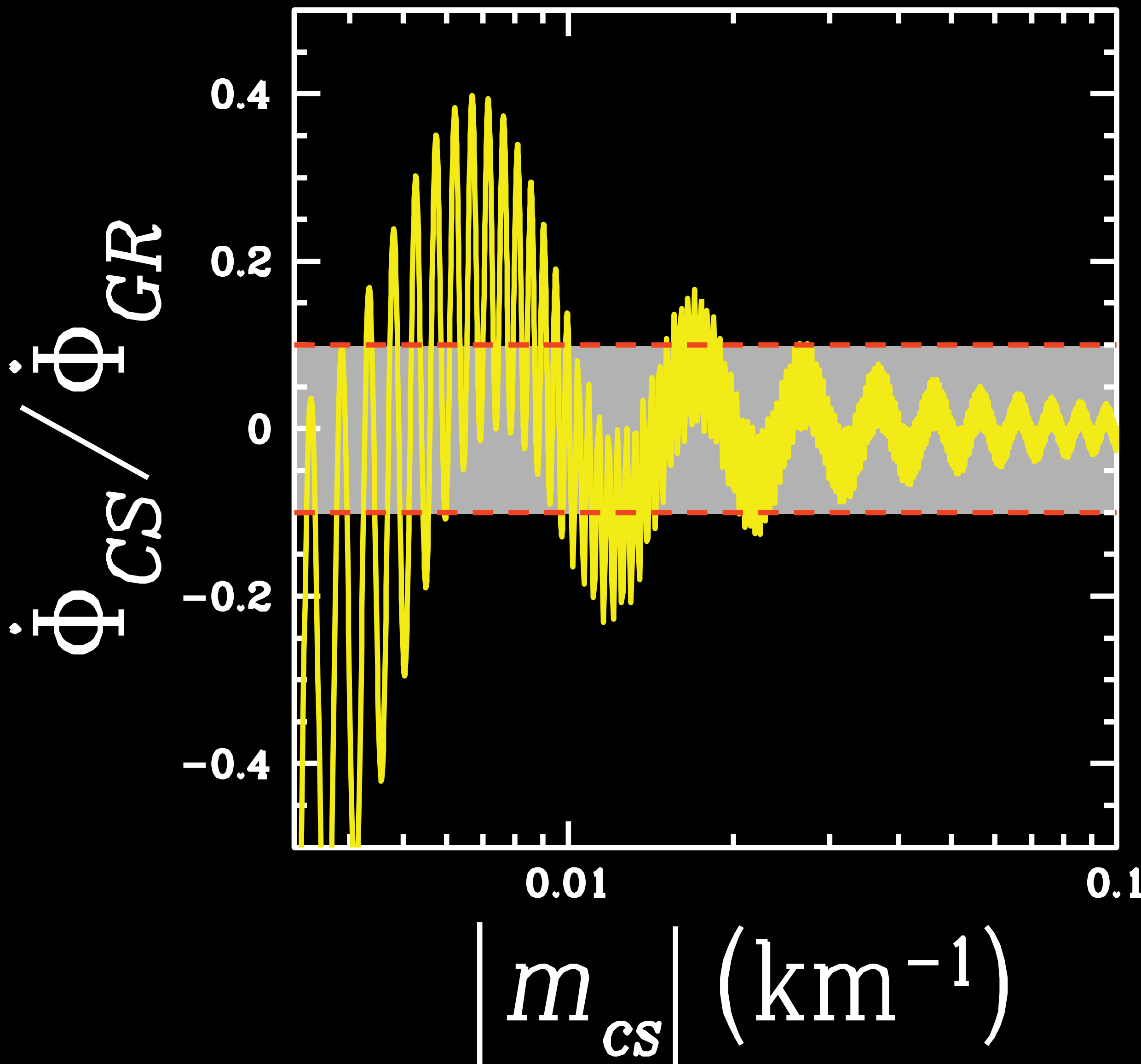
$$\frac{\dot{\Omega}_{CS}}{\dot{\Omega}_{GR}} = 15\frac{a^2}{R^2} j_2(m_{cs}R)\, y_1(m_{cs}a)$$

Fractional Deviation from GR

LAGEOS satellites measured $\dot{\Omega}$ and found it to agree with GR to within 10 %. *Ciufolini, Pavlis 2004*

$$|m_{cs}| \gtrsim 0.001\ \mathrm{km}^{-1}$$

Constraints from Gyroscopic Precession



Parallel transport of spin:

$$\dot{\vec{S}} = 2\vec{B} \times \vec{S}$$

$$\dot{\Phi} \equiv |\dot{\vec{S}}|/|\vec{S}|$$

$\dot{\Phi}_{GR} \simeq 42$ mas yr^{-1} for GPB

Gravity Probe B will measure this precession to unknown precision.

Even a 10% uncertainty will improve bound:

$$|m_{cs}| \gtrsim 0.01\ \mathrm{km}^{-1}$$

Summary

★ In Chern-Simons gravity, a spinning mass produces a parity-violating gravitomagnetic field.

★ This gravitomagnetic field affects the orbits of satellites and the spin of freely-falling gyroscopes.

★ Using LAGEOS measurements of the precession of the satellites' line of nodes, we are able to constrain a combination of parameters of Chern Simons gravity:

$$\left| \frac{3}{8\pi G \ell \dot{\theta}} \right| \gtrsim 2 \times 10^{-22} \text{ GeV}$$

★ Gravity Probe B will probably improve this bound by a factor of ten.