Solar System Tests ***Do*** Rule Out $\frac{1}{R}$ Gravity

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Overview

- $\frac{1}{R}$ Gravity: its structure and motivation
- Scalar-tensor theory: *Ruled Out!*
- Vacuum solution: *NOT Ruled Out!*
- The Solar System solution: *Ruled Out!*

$\frac{1}{R}$ Gravity:

Cosmic Acceleration without Dark Energy

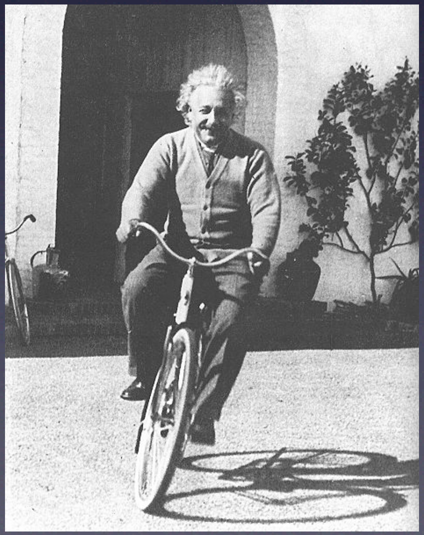
Carroll, Duvvuri, Trodden, Turner PRD 70, 043528 (2004)

The Theory:

Mass scale

$$S_{\mathrm{grav}} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R - \frac{\mu^4}{R}\right)$$

Action *Volume*



The Consequences:

- Constant curvature spacetime is **de Sitter**:

$$\Lambda_{\mathrm{eff}} \sim \mu^2 \sim H_0^2$$

- Late-time acceleration **without dark energy**

The Scalar-Tensor Twin Theory

Chiba Phys. Lett. B 575, 1 (2003)

Consider the **scalar-tensor** theory with the action

$$S_{\mathrm{grav}} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[\left(1+\frac{\mu^4}{\phi^2}\right)R - \frac{2\mu^4}{\phi}\right]$$

Action *Volume* *Curvature* *Scalar Field*

Varying with respect to $\phi \Longrightarrow \quad \phi = R$

and then this action is the same as $\frac{1}{R}$ gravity!

The Scalar-Tensor Twin Theory

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$$S_{\rm grav} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[\left(1+\frac{\mu^4}{\phi^2}\right)R - \frac{2\mu^4}{\phi}\right]$$

Action Volume Curvature Scalar Field

$$S_{\rm grav} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,[\varphi R - V(\varphi)]$$

Brans-Dicke theory with no scalar kinetic term: $\omega = 0$

Provided that the effective mass of the scalar is small, this theory is ***RULED OUT*** by Solar System tests.

Vacuum Solutions

Multamaki and Vilja PRD 74, 064022 (2006)

Field equation for $\frac{1}{R}$ gravity obtained by varying the metric:

$$8\pi G T_{\mu\nu} = \left(1 + \frac{\mu^4}{R^2}\right) R_{\mu\nu} - \frac{1}{2}\left(1 - \frac{\mu^4}{R^2}\right) R g_{\mu\nu} + \mu^4 \left(g_{\mu\nu}\nabla_\alpha\nabla^\alpha - \nabla_\mu\nabla_\nu\right) R^{-2}$$

Matter

GR-like terms

Nasty fourth-order derivatives of the metric!

Trace Equation: $$\frac{8\pi G T}{3} = \nabla_\alpha\nabla^\alpha \frac{\mu^4}{R^2} - \frac{R}{3} + \frac{\mu^4}{R}$$

Assume vacuum and constant curvature...

GENERAL RELATIVITY with a cosmological constant!

$$R = \sqrt{3}\mu^2 \qquad \Lambda_{\mathrm{eff}} = \frac{\sqrt{3}}{4}\mu^2$$

Vacuum Solutions

Multamaki and Vilja PRD 74, 064022 (2006)

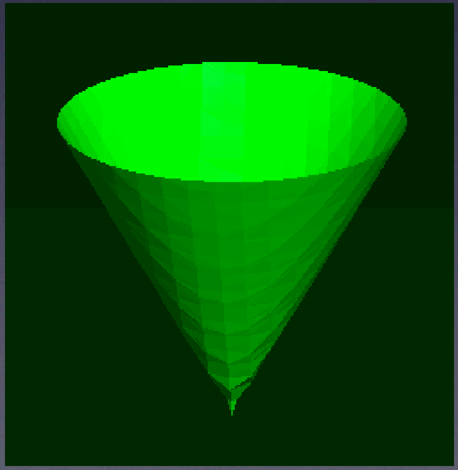
- The Schwarzschild-de Sitter metric is a solution to Einstein's equation with a cosmological constant.
- The Ricci scalar R for this metric is constant.
- The Schwarzschild-de Sitter metric is a solution to the field equation of $\frac{1}{R}$ gravity.
- The Schwarzschild-de Sitter metric passes all Solar System tests.
- Therefore, $\frac{1}{R}$ gravity is ***NOT RULED OUT!***

Beware!

This isn't General Relativity...

- In general relativity, the Schwarzschild-de Sitter metric is the only static spherically-symmetric solution to the vacuum equation. (Birkhoff's Theorem)
- Birkhoff's Theorem does not apply to $\frac{1}{R}$ gravity.
- Without Birkhoff's Theorem, we have no reason to believe that the Schwarzschild-de Sitter metric describes the spacetime in the Solar System.

Example: A conical spacetime solves Einstein's equations, but it can't be the spacetime around a pressureless string.



Finding the Solar System Solution I

In the Sun, $T \simeq -\rho$

Define a new function: $c(r) \equiv -\frac{1}{3} + \frac{\mu^4}{R^2(r)}$

Trace of the field eqn: $\nabla^2 c + \sqrt{3}\mu^2 c = -\frac{8\pi G}{3}\rho$

The solution:

$$R = \sqrt{3}\mu^2 \left(1 - \frac{GM}{r}\right)$$

Curvature Scalar

Background value

Leading order perturbation outside the Sun

The curvature in the Solar System is not constant!

Finding the Solar System Solution II

Begin with a perturbed de Sitter metric:

$$ds^2 = -\left[1 + a(r) - H^2 r^2\right] dt^2 + \left[1 + b(r) - H^2 r^2\right]^{-1} dr^2 + r^2 d\Omega^2$$

Line Element

Now that we have an expression for *R*, the field equations are second-order differential equations for *a(r)* and *b(r)*.

The Linearized Solution

$$ds^2 = -\left(1 - \frac{2GM}{r} - H^2 r^2\right) dt^2 + \left(1 + \frac{GM}{r} + H^2 r^2\right) dr^2 + r^2 d\Omega^2$$

Normal Newtonian Term = 2Φ

Half of Schwarzschild value:

$$\gamma = \frac{1}{2}$$

Conclusions

- Shapiro time delay measurements:

$$\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$$

Bertotti, Iess, Tortora
Nat. 425, 374 (2003)

- Relating to the equivalent scalar-tensor theory gives the correct PPN parameter for $\frac{1}{R}$ gravity: $\gamma = \frac{1}{2}$

- $\frac{1}{R}$ gravity is ***RULED OUT*** by Solar System tests.

- Analysis may be extended to other modifications of the Einstein-Hilbert action -- stay tuned!