Overview of and Future of BSM Calculations, $0\nu\beta\beta$, WIMPs

J. Engel

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Can $\beta$ Decay Measurements Inform Other BSM Experiments?

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Can $\beta$-Decay Measurements Inform Other BSM Experiments?

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New Physics

Some Basic Questions

What underlies the Standard Model?
Why is the $\theta$ parameter in QCD so small?
Why is there more matter than antimatter in our universe?
Are neutrinos Majorana?
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What underlies the Standard Model?
Why is the $\theta$ parameter in QCD so small?
Why is there more matter than antimatter in our universe?
Are neutrinos Majorana?

The observation (or continued non-observation) of electric dipole moments and $0\nu\beta\beta$ decay will help us address these questions.

Atomic EDMs yield some of the best experimental limits. Heavy diamagnetic atoms are particularly sensitive to new physics within the nucleus.

Many $\beta\beta$ decay candidates are also heavy.
EDMs: How Atoms Can Get Them

EDMs require $CP$ violation
and
an undiscovered source of $CP$ violation is required to explain why there is so much more matter than antimatter.

The source can work its way into nuclei through $CP$-violating $\pi NN$ vertices (in chiral EFT)…

leading, e.g. to a neutron EDM…
EDMs: How Atoms Can Get Them (cont.)

...and to a nuclear EDM from the nucleon EDM or a $T$-violating $NN$ interaction:

\[ V_{PT} \propto \bar{g} \left( \hat{\sigma}_1 \pm \hat{\sigma}_2 \right) \cdot \left( \vec{\nabla}_1 - \vec{\nabla}_2 \right) \frac{\exp \left( -m_\pi |\vec{r}_1 - \vec{r}_2| \right)}{m_\pi |\vec{r}_1 - \vec{r}_2|} + \text{contact term} \]

The $\bar{g}$'s (isoscalar, isovector and isotensor) depend on source of CP violation.
EDMs: How Atoms Can Get Them (cont.)

...and to a nuclear EDM from the nucleon EDM or a $T$-violating $NN$ interaction:

Note: $\mathcal{CP} = \mathcal{T}$

$$V_{PT} \propto \bar{g} \left( \vec{\sigma}_1 \pm \vec{\sigma}_2 \right) \cdot \left( \vec{\nabla}_1 - \vec{\nabla}_2 \right) \frac{\exp \left( -m_\pi |\vec{r}_1 - \vec{r}_2| \right)}{m_\pi |\vec{r}_1 - \vec{r}_2|} + \text{contact term}$$

The $\bar{g}$'s (isoscalar, isovector and isotensor) depend on source of CP violation.

Atoms gets an EDM from nuclei. But electronic shielding replaces nuclear dipole operator with “Schiff operator,”

$$S \approx \sum_p r_p^2 z_p + \ldots ,$$

making relevant nuclear quantity the Schiff moment:

$$\langle S \rangle = \sum_m \frac{\langle O | S | m \rangle \langle m | V_{PT} | O \rangle}{E_0 - E_m} + \text{c.c.}$$
EDMs: How Atoms Can Get Them (cont.)

...and to a nuclear EDM from the nucleon EDM or a $T$-violating $NN$ interaction:

Note: $CP = T$

Job of nuclear-structure theory: compute dependence of $\langle S \rangle$ on the $\bar{g}$'s (and on the contact term and nucleon EDM).

It's up to QCD to compute the dependence of the $\bar{g}$ vertices on fundamental sources of $CP$ violation.

Nuclear dipole operator with Schiff operator:

$$S \approx \sum_p r_p^2 z_p + \ldots ,$$

making relevant nuclear quantity the Schiff moment:

$$\langle S \rangle = \sum_m \frac{\langle 0 | S | m \rangle \langle m | V_{PT} | 0 \rangle}{E_0 - E_m} + c.c.$$
What to Compute

More precisely, because the $\tilde{g}_i$ are so small,

$$\langle S \rangle = \sum_i a_i g \tilde{g}_i,$$

and we have to calculate the three $a_i$. These reflect action of both the $S$ and $V_{PT}$ operators.

Most heavy nuclei must be treated in something like DFT for now, leading to uncertainty in the $a_i$ that is large and difficult to estimate.

But other observables can help.
Unlike in other nuclei, these two states are the whole story.
Correlation of $\langle S \rangle_{\text{intr.}}$ with Octupole Defm. in $^{224}\text{Ra}$

Gaffney et al., 2013
Correlation of $\langle S \rangle_{\text{intr.}}$ with Octupole Defm. in $^{224}\text{Ra}$

\[ \text{Intrinsic } S_0 \ (\text{e fm}^3) \]

\[ ^{224}\text{Ra Intrinsic } Q_0^3 \ (1000 \text{ e fm}^3) \]

J. Dobaczewski, JE, M. Kortelainen, P. Becker

Correlation with octupole moment of $^{225}\text{Ra}$ even better.

Will be determined at ANL.
Light Actinides More Generally

The error bars represent statistical uncertainty only, but systematic variation is not large.
Looks good, but situation is more complicated when we include octupole moments in other nuclei. The resulting $a_i$ for $^{225}$Ra:

<table>
<thead>
<tr>
<th>Type</th>
<th>$a_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>isoscalar</td>
<td>$-0.4 - 0.8$</td>
</tr>
<tr>
<td>isovector</td>
<td>$-2 - -8$</td>
</tr>
<tr>
<td>isotensor</td>
<td>$2 - 5$</td>
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Range doesn’t include systematic uncertainty.
Reducing Uncertainty of Lab Moments

The problem is that we don’t have information about $\langle V_{PT} \rangle$.

Can $\beta$ decay constrain its matrix element?

$V_{PT}$ has same space-spin form as two-body axial-charge operator:

$$A_{2b}^0 \propto \tilde{\tau}_1 \times \tilde{\tau}_2 (\tilde{\sigma}_1 + \tilde{\sigma}_2) \cdot (\tilde{\nabla}_1 - \tilde{\nabla}_2) \frac{e^{-m_\pi |\tilde{r}_1 - \tilde{r}_2|}}{m_\pi |\tilde{r}_1 - \tilde{r}_1|}$$

Because the one-body part,

$$A_{2b}^0 \propto \frac{1}{M} \tilde{\sigma} \cdot \tilde{\nabla}$$

is suppressed by $q/M$, the pion-exchange contribution is significant. Also, the effective one-body form of $V_{PT}$:

$$V_{PT}^{\text{eff}} \propto \tilde{\sigma} \cdot \tilde{\nabla} \rho$$

has a similar form.
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Also, the effective one-body form of $V_{PT}$:

$$V_{PT}^{\text{eff}} \propto \vec{\sigma} \cdot \vec{\nabla} \rho$$

has a similar form.

Can we measure

1. charge-changing transition strength to analog of $|1/2^-\rangle$ in $^{225}$Fr?
2. axial-charge $\beta$ decays in other nuclei?
Review of $0\nu\beta\beta$ Decay

Standard operator

Review of $0\nu\beta\beta$ Decay

Standard operator

I’ll focus on this one.

Forbidden in Standard Model.
New physics inside blobs.
Nuclear Matrix Element (Simplified)

\[ M^{0\nu} = g_A M_{GT}^{0\nu} - g_V M_{F}^{0\nu} + \ldots \]

with

\[ M_{GT}^{0\nu} = \langle f | \sum_{a,b} H_{GT}(r_{ab}) \bar{\sigma}_a \cdot \bar{\sigma}_b \tau_a^{-} \tau_b^{-} | i \rangle \]

\[ M_{F}^{0\nu} = \langle f | \sum_{a,b} H_{F}(r_{ab}) \tau_a^{+} \tau_b^{+} | i \rangle \]

\[ H_{GT}(r) \approx H_{F}(r) \approx \frac{R_{\text{nucl.}}}{r} \]
Nuclear Matrix Element (Simplified)

\[ M^{0\nu} = g_A^2 M^{0\nu}_{GT} - g_V^2 M^{0\nu}_F + \ldots \]

with

\[ M^{0\nu}_{GT} = \langle f | \sum_{a,b} H_{GT}(r_{ab}) \vec{\sigma}_a \cdot \vec{\sigma}_b \tau_a^+ \tau_b^+ | i \rangle \]

\[ M^{0\nu}_F = \langle f | \sum_{a,b} H_{F}(r_{ab}) \tau_a^+ \tau_b^+ | i \rangle \]

\[ H_{GT}(r) \approx H_{F}(r) \approx \frac{R_{\text{nucl.}}}{r} \]

Also:

\[ M_{2\nu} = g_A^2 \sum_m \frac{\langle f | \sum_a \vec{\sigma}_a \tau_a^+ | m \rangle \cdot \langle m | \sum_b \vec{\sigma}_b \tau_b^+ | i \rangle}{E_m - \frac{E_f + E_i}{2}} \]
**Ab Initio Methods for (Fairly) Heavy Nuclei**

**Partition of Full Hilbert Space**

<table>
<thead>
<tr>
<th></th>
<th>(P)</th>
<th>(Q)</th>
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<tr>
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\(P\) = states we care about

\(Q\) = the rest

**Task:** Find unitary transformation to make \(H\) block-diagonal in \(P\) and \(Q\), with \(H_{\text{eff}}\) in \(P\) reproducing most important eigenvalues.

For transition operator \(\hat{M}\), must apply same transformation to get \(\hat{M}_{\text{eff}}\).

As difficult as solving full problem. But \(N\)-body effective operators with \(N > 2\) or 3 can be treated approximately.
Ab Initio Methods for (Fairly) Heavy Nuclei

Partition of Full Hilbert Space

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$P$ = states we care about
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Task: Find unitary transformation to make $H$ block-diagonal in $P$ and $Q$, with $H_{\text{eff}}$ in $P$ reproducing most important eigenvalues.

Now includes more.
Ab Initio Methods for (Fairly) Heavy Nuclei

Partition of Full Hilbert Space

$P$ $Q$

$P$ $H_{\text{eff}}$

$Q$ $H_{\text{eff-Q}}$

$P = \text{states we care about}$

$Q = \text{the rest}$

**Task:** Find unitary transformation to make $H$ block-diagonal in $P$ and $Q$, with $H_{\text{eff}}$ in $P$ reproducing most important eigenvalues.

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*P = states we care about*

*Q = the rest*

**Task:** Find unitary transformation to make *H* block-diagonal in *P* and *Q*, with *H*_{eff} in *P* reproducing most important eigenvalues.

For transition operator *M*, must apply same transformation to get *M*_{eff}.

As difficult as solving full problem. But *N*-body effective operators with *N* > 2 or 3 can be treated approximately.
Wave function ansatz:

\[ |\Psi\rangle = e^{\hat{T}} |\text{Slater det.}\rangle = \exp\left( t_{ij}^1 a_i^\dagger a_j + t_{ijkl}^2 a_i^\dagger a_j^\dagger a_k a_l + \ldots \right) |\text{Slater det.}\rangle \]

Then using a similarity transform:

\[ \hat{H} \longrightarrow e^{-\hat{T}} \hat{H} e^{\hat{T}}, \]

means that you work with a Slater determinant rather than the fully correlated state when building excitations.
Flow equation for effective Hamiltonian. Gradually decouples shell-model space.

Trick is to keep all 1- and 2-body terms in $H$ at each step after normal ordering (approximate treatment of 3-, 4- ... terms).

If model space contains just a single state, approach yields ground-state energy. If it is larger, result is effective interaction and operators.
ββ Decay in $^{48}$Ca with Coupled Clusters

Convergence with respect to model-space sizes of $^{48}$Ti (top) and $^{48}$Sc (bottom), and comparison with data.

Neutrinoless decay of $^{48}$Ca
Small spread of results for different interactions
Small corrections when going from CCSD to more precise CCSDT-3
Ab initio results close to shell-model (SM) results

Coupled cluster method S. Novario et al., in prep.

A little larger than shell-model result.

From G. Hagen
Small Fly in the Ointment

Usual light neutrino exchange:

must be supplemented, at same order in chiral EFT, by short-range operator (representing high-energy $\nu$ exchange):

Coefficient of this term is unknown.
Two-Body Axial Current and Connection with $\beta$ Decay

$\beta$ Decay (simplified) with electron lines omitted

Leading order in $\chi$ EFT:

Usual $\beta$-decay current.

Finite-momentum corrections at next order.

plus a contact
Two-Body Axial Current and Connection with $\beta$ Decay

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Higher order:

$\approx$

plus a contact
Two-Body Axial Current and Connection with $\beta$ Decay

$\beta$ Decay (simplified) with electron lines omitted

Leading order in $\chi$EFT:

Usual $\beta$-decay current.
Finite-momentum corrections at next order.

Higher order:

Coefficients same as in three-body interaction:

plus a contact
Product of Currents

In first quantization, let

\[
\sum_i \hat{O}_i^{1b} = \text{1-body operator in } J^+
\]

\[
\sum_{ij} \hat{O}_{ij}^{2b} = \text{2-body operator in } J^+
\]

\[
J^+(\vec{q})J^+(\vec{-q}) = \sum_{ij} \hat{O}_i^{1b}\hat{O}_j^{1b} + \sum_{ijk} \left( \hat{O}_{ij}^{2b}\hat{O}_k^{1b} + \hat{O}_i^{1b}\hat{O}_{jk}^{2b} \right) + \text{4-body op.}
\]

\[
+ \sum_{ij} \left( \hat{O}_{ij}^{2b}[\hat{O}_i^{1b} + \hat{O}_j^{1b}] + [\hat{O}_i^{1b} + \hat{O}_j^{1b}]\hat{O}_{ij}^{2b} \right)
\]

\[
\text{2-body op.}
\]
Inclusion of Two-Body Currents

Diagrams for these contributions:

Three-body

Two-body
Inclusion of Two-Body Currents

Diagrams for these contributions:

Three-body

Two-body

Uh oh... divergent loops
Normal ordered two-body current, to get effective one-body current. Corresponds to:
Prior Work on Effects in Heavy Systems

Javier, Doron, Achim: Symmetric Nuclear Matter

Normal ordered two-body current, to get effective one-body current. Corresponds to:

\[
\begin{array}{c}
\nu \\
n/p \\
\end{array}
\]

In nuclear matter:

\[
g_A \rightarrow g_A - g_A \frac{\rho}{F_{\pi}^2} \left[ \frac{c_d}{g_A \Lambda} + \frac{2c_3}{3} \frac{q^2}{q^2 + 4m_{\pi}^2} + I(\rho, P) \left( \frac{2c_4 - 33}{3} + \frac{1}{6m} \right) \right]
\]

\[I(\rho, P) \approx \frac{2}{3}\] at nuclear density, with weak dependence on \(P\).

\(0\nu\beta\beta\) decay quenched by about 30%, somewhat less than \(2\nu\beta\beta\) decay because of \(q\) dependence of effective \(g_A\).
More Complete Nuclear Matter Calculation

With Simplest Operator: $g_A$ at one-body vertex, $c_D$ at two-body vertex

Goldstone (Time-Ordered) Diagrams

![Goldstone Diagrams](image)

Need counter-term to renormalize these

$$\sum_{i<F} \langle F | p_d^\dagger n_i^\dagger n_a n_b p_c^\dagger n_i | I \rangle$$

Three-body operators contribute (a) and (b) plus twice (c) and (d) $\approx 0$.

$$(c) + (d) \approx -\frac{1}{2} [(a) + (b)]$$

$$(e) + (f) \approx (\Lambda/k_F - 1) [(a) + (b)]$$
Approximate $^{76}$Ge wave function in \textit{fp} shell, inert core underneath.
Approximate $^{76}$Ge wave function in fp shell, inert core underneath.

Takeaway: Effects of three-body operators are small.
Two-Body Operators
With Nucleon Form Factors

Right side includes usual modifications.

Almost entire contribution from $c_D$ and short-range parts of $c_3, c_4$. 
Two-Body Operators
With Nucleon Form Factors

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Need counter term, just like in leading order. Help!
Two-Body Operators
With Nucleon Form Factors

Right side includes usual modifications.

Takeaway: Effects of two-body current look moderate... But we don’t know for sure.

Almost entire contribution from $c_D$ and short-range parts of $c_3, c_4$.

Need counter term, just like in leading order. Help!
So, to Sum Up…

1. Schiff moments, for now, must be calculated in DFT, which makes drastic and uncontrolled approximations. Other observables can help constrain calculations.

Can $\beta$-decay rates do that?
So, to Sum Up…

1. Schiff moments, for now, must be calculated in DFT, which makes drastic and uncontrolled approximations. Other observables can help constrain calculations.

   Can $\beta$-decay rates do that?

2. Application of chiral EFT to $0\nu\beta\beta$ decay implies short-range contribution to neutrino exchange with unknown coefficient. A similar issue hampers our ability to fully examine effects of the two-body current in $0\nu\beta\beta$ decay.

   The part for which we do know coefficients seems to quench very little, however.
Finally…

\begin{Acknowledgments}

Thanks!

\end{Acknowledgments}

\end{Talk}