Double-Beta Decay and Nuclear Structure

J. Engel

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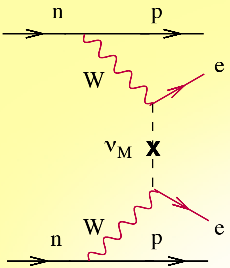
University of North Carolina

October 1, 2013



Neutrinos: What We Know

Come in three "flavors", none of which have definite mass.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} & & \\ & U_\nu & \\ & & \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

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$m_i \lesssim 1$ eV

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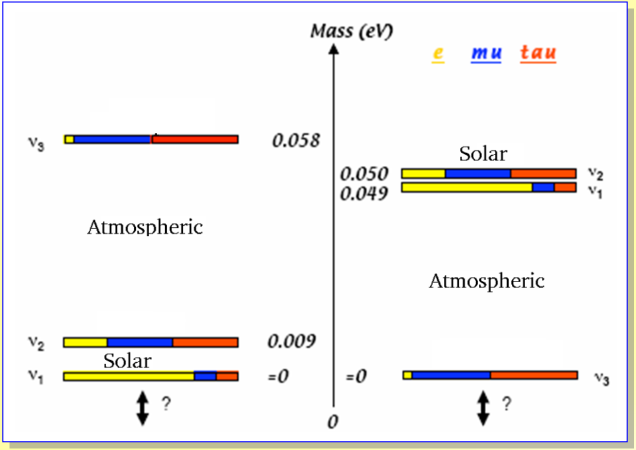
What We Still Don't Know



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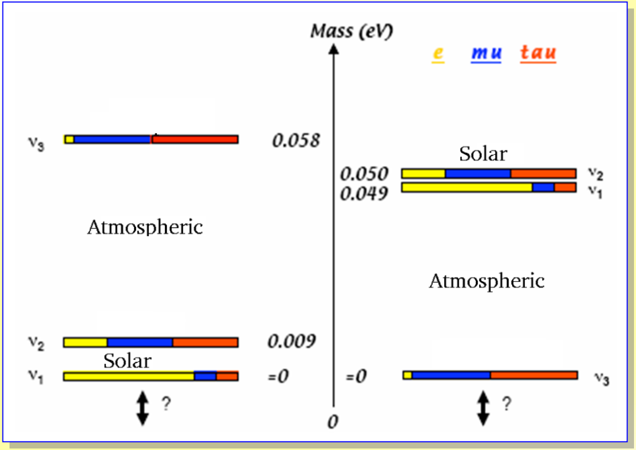

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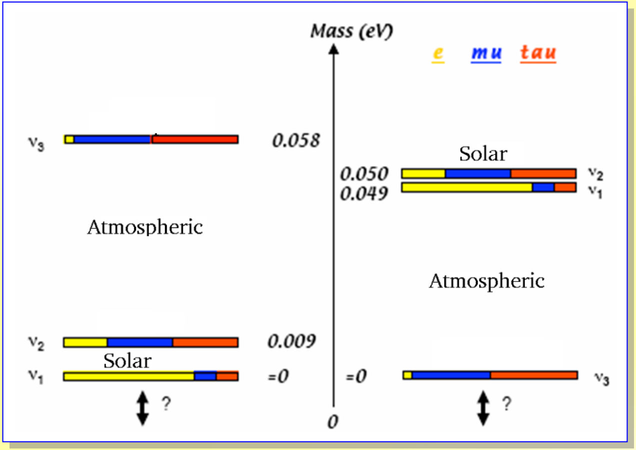

- "Hierarchy": normal or inverted?
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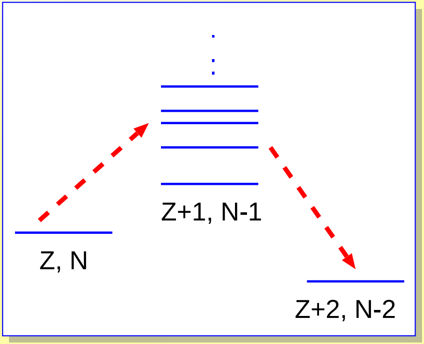

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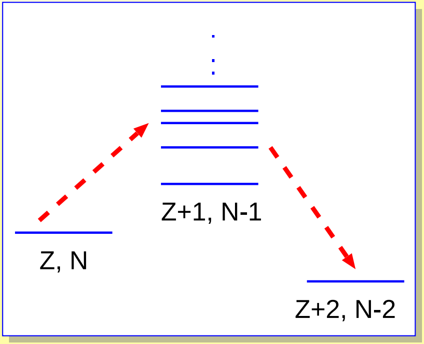

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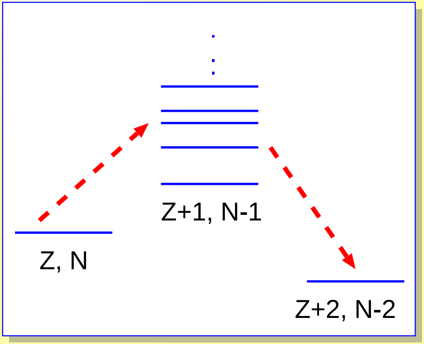

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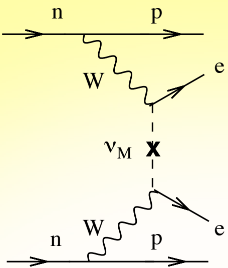
can observe two neutrons turning into protons, emitting two electrons and nothing else.



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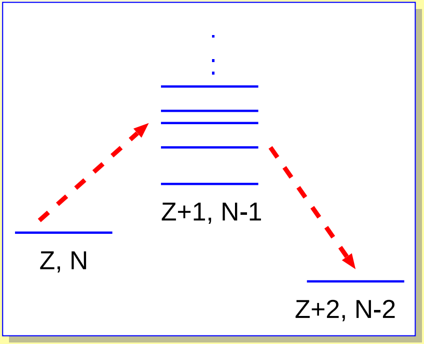

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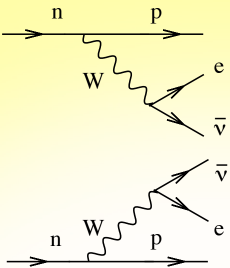
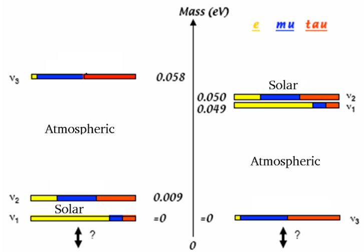
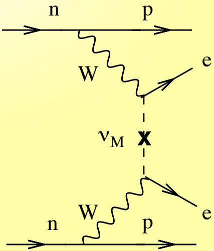


Different from already observed two-neutrino process.

Usefulness of Double-Beta Decay





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$$m_{\text{eff}} \equiv \sum_i m_i U_{ei}^2$$

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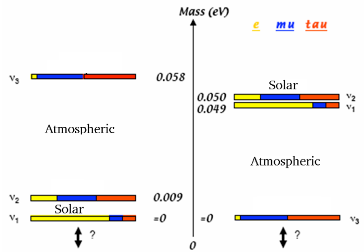

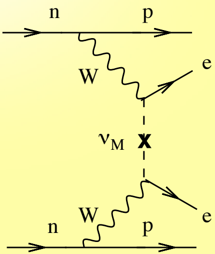

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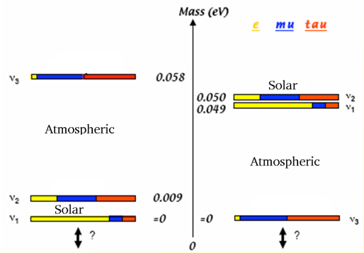

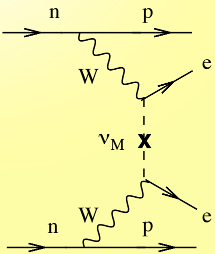

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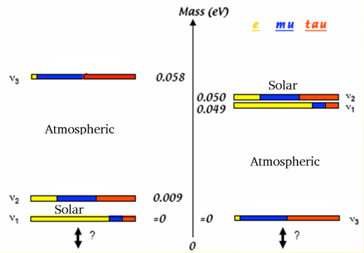

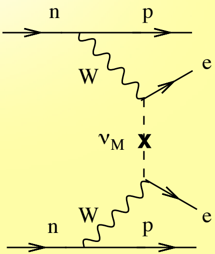

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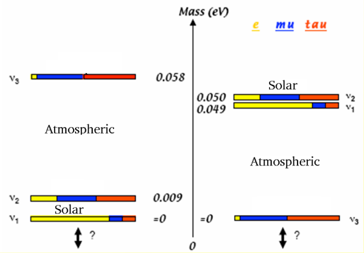

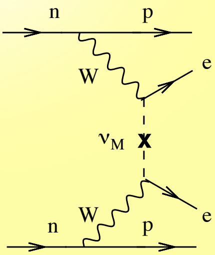

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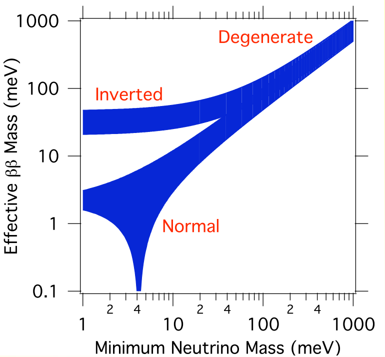

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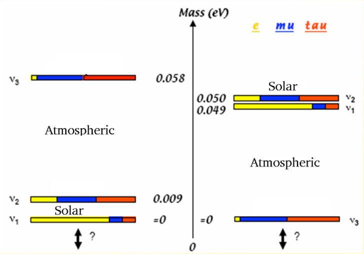

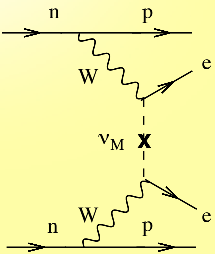

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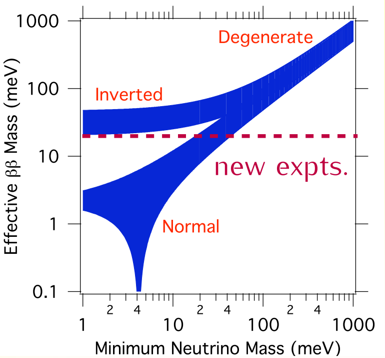

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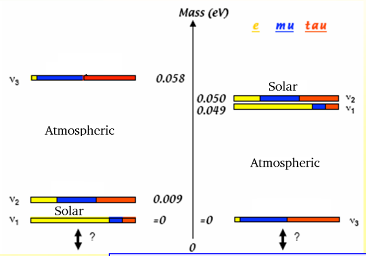

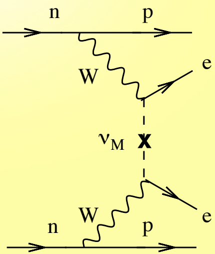

But rate also depends on a nuclear matrix element!

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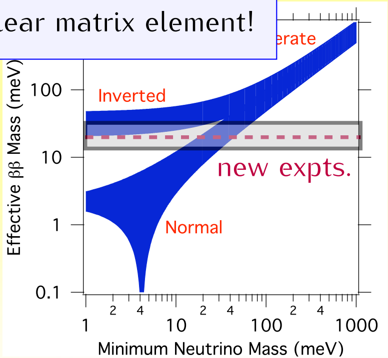

How Effective Mass Gets into Rate

$$[T^{0\nu}_{1/2}]^{-1} = \sum_{\text{spins}} \int |Z_{0\nu}|^2 \delta(E_{e1} + E_{e2} - Q_{\beta\beta}) \frac{d^3p_1}{2\pi^3} \frac{d^3p_2}{2\pi^3}$$

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The $q^\rho\gamma_\rho$ part vanishes in trace, leaving a factor

$$\boxed{m_{\text{eff}} \equiv \sum_k m_k U_{ek}^2.}$$

What About Hadronic Part?

Integral over times produces a factor

$$\sum_n \frac{\langle f|J_L^\mu(\vec{x})|n\rangle\langle n|J_L^\nu(\vec{y})|i\rangle}{q^0(E_n + q^0 + E_{e2} - E_i)} \; + (\vec{x},\mu \leftrightarrow \vec{y},\nu)\,,$$

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$$\langle p|J^\mu(x)|p'\rangle = e^{iqx}\overline{u}(p)\bigg(g_V(q^2)\gamma^\mu - g_A(q^2)\gamma_5\gamma^\mu - ig_M(q^2)\frac{\sigma^{\mu\nu}}{2m_p}q_\nu + g_P(q^2)\gamma_5 q^\mu\bigg)u(p')\ .$$

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q^0 typically of order inverse nucleon distance, 100 MeV, so denominator can be taken constant and sum done in closure.

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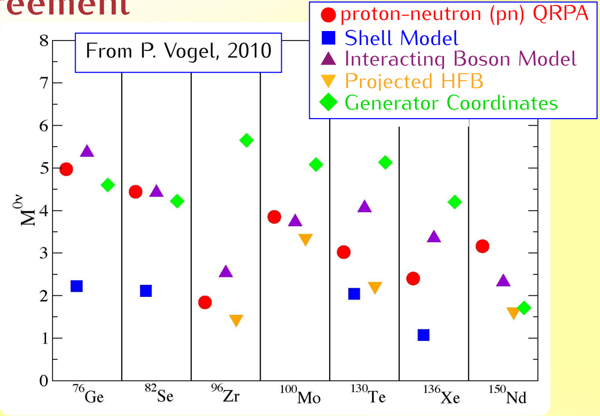
Corrections ("forbidden" terms, weak form factors) $\lesssim 30\%$.

Calculations of Matrix Elements

Nuclear-structure theory in heavy nuclei still an art, but becoming a science.

Recent Level of Agreement

Same level of agreement in 2013.



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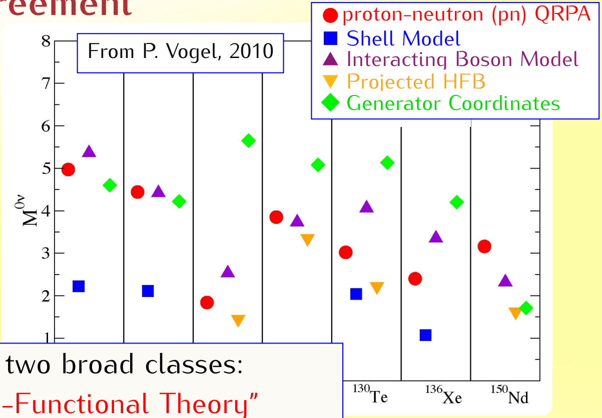

Calculations fall into two broad classes:

I. “Energy-Density-Functional Theory”
- Generator Coordinates
- QRPA
- Projected HFB

II. Shell Model and derivatives
- Shell Model (Duh!)
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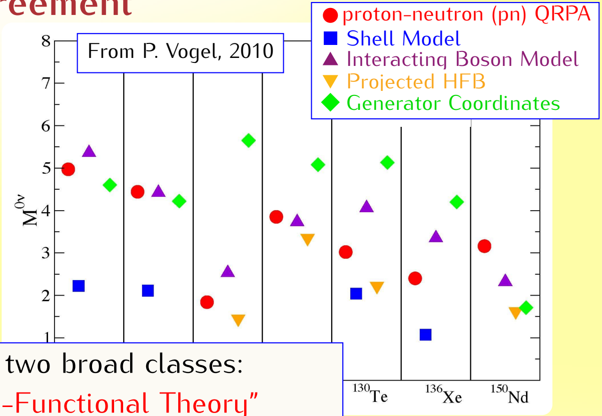

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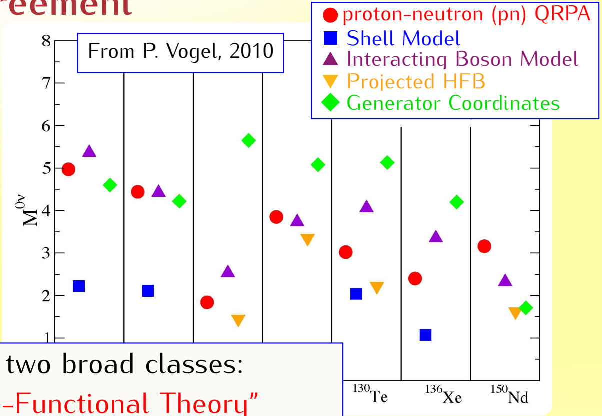

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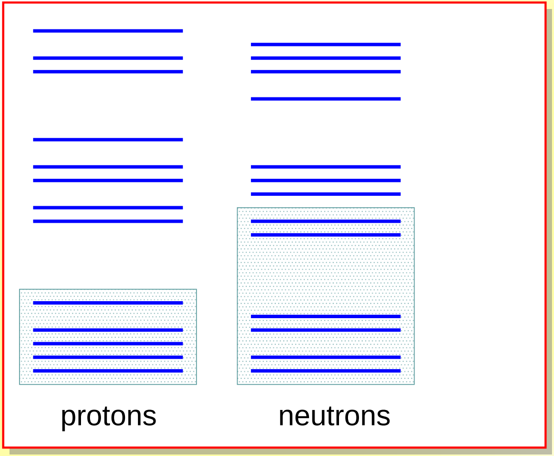
Contrasting the Various Approaches



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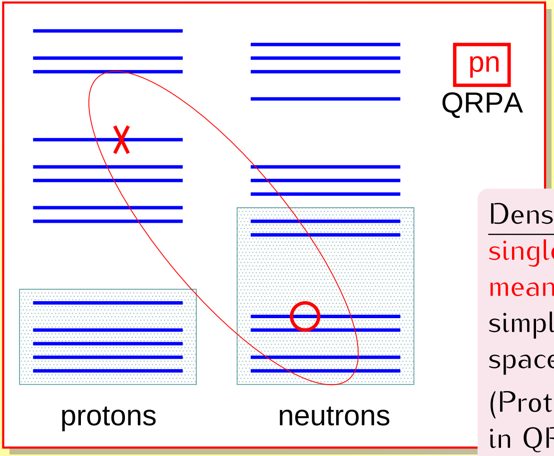

Density-functional theory: Large single-particle spaces in arbitrary mean field or set of mean fields; simple correlations within the spaces.

(Proton-neutron correlations here in QRPA, which is based on single mean field).

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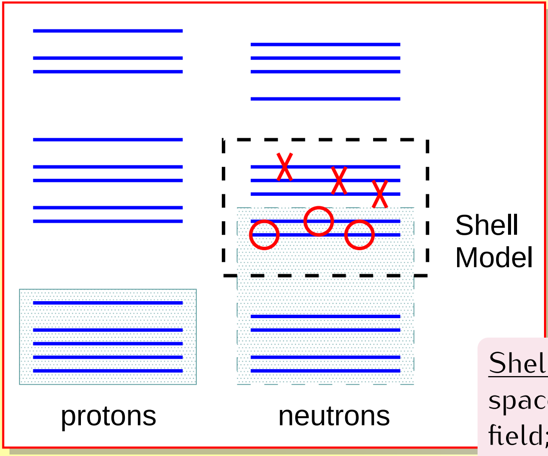

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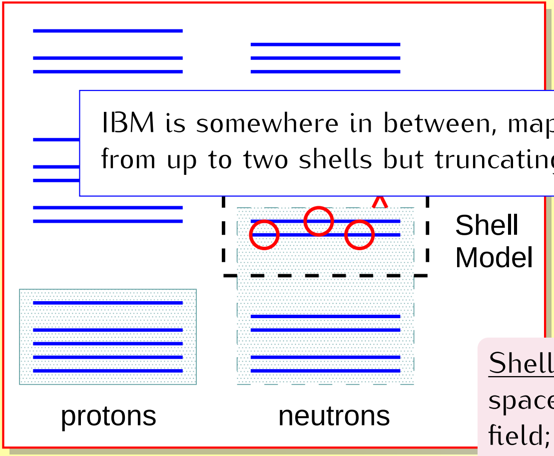

IBM is somewhere in between, mapping matrix elements from up to two shells but truncating to collective pairs.

Shell Model: Small single-particle space in simple spherical mean field; arbitrarily complex correlations within the space.

First Large-Scale Deformed QRPA

QRPA inserts complete set of states in intermediate nucleus, provides single-beta matrix elements from ground states of initial and final nuclei to this complete set.

Used modern Skyrme functional SkM*, consumed $\approx$ 7M CPU hours.

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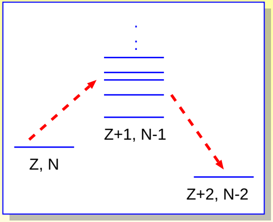
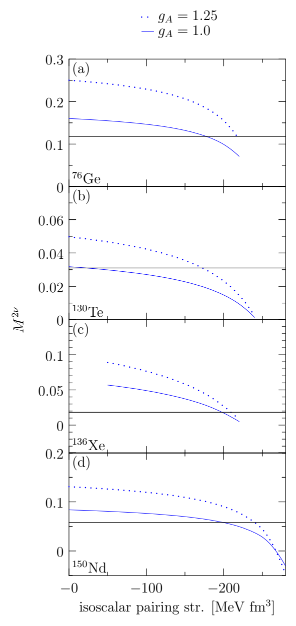
Worth noting:

QRPA is linear response of mean field; gives two sets of intermediate-nucleus energies and strengths (for transitions involving initial/final nuclei) but not corresponding wave functions. Doesn't tell you how the two sets of states are related.

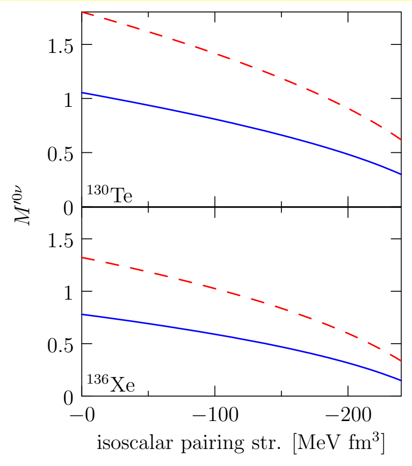


Must finesse the problem (i.e. cheat).

Sensitivity to Proton-Neutron Pairing



Have to tune isoscalar pairing to get 2ν decay right. Process can cover up of virtues as well as sins.



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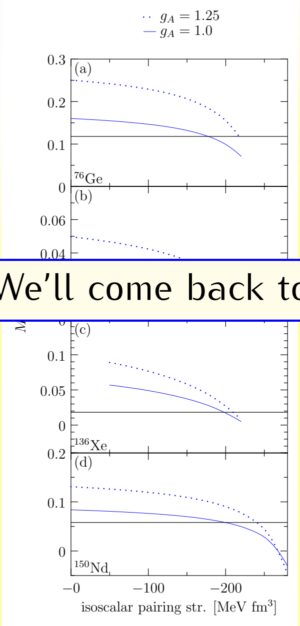

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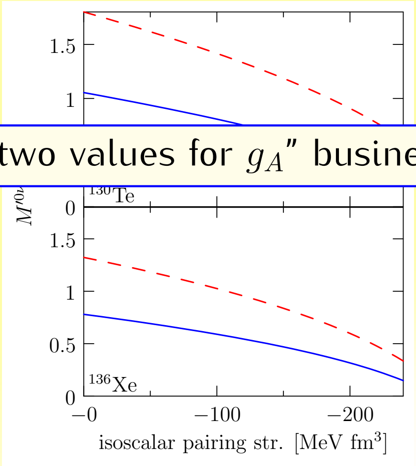
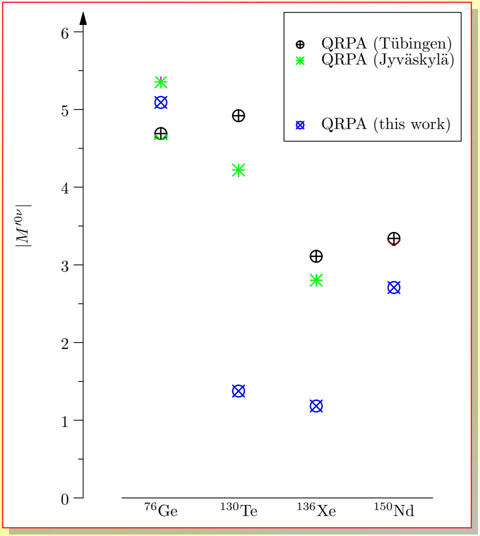


We'll come back to this "two values for g_A" business.

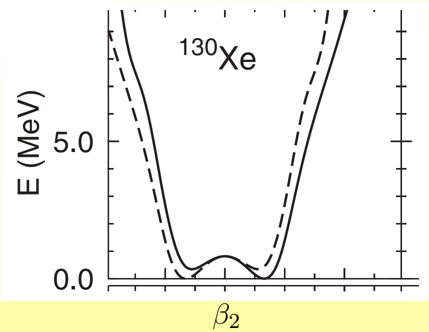
Results

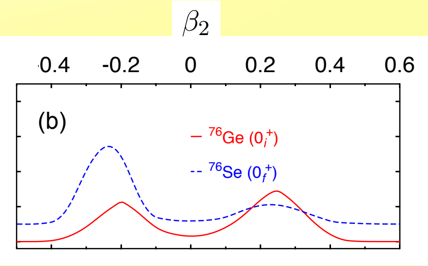


Results different from other QRPAs in some nuclei, but this actually points to problems with method.

The QRPA Has Issues Beyond Ambiguity

Some of the nuclei in these decays don't have well defined shape, can't be represented by single mean field.





Robledo et al.: Energy minima at $\beta_2 \approx \pm .15$

Solid line is actual result; dashed line a symmetric potential for comparison

Rodríguez and Martinez-Pinedo: Wave functions peaked at $\beta_2 \approx \pm .2$

Beyond QRPA

Want to avoid the problems:

1. Overlap of intermediate states not well defined.
2. No mixing of mean fields with different shapes, pairing...
3. Correlations too simple.
4. **Response to proton-neutron pairing unrealisticaly strong** (as phase transition to pn pairing is approached)?

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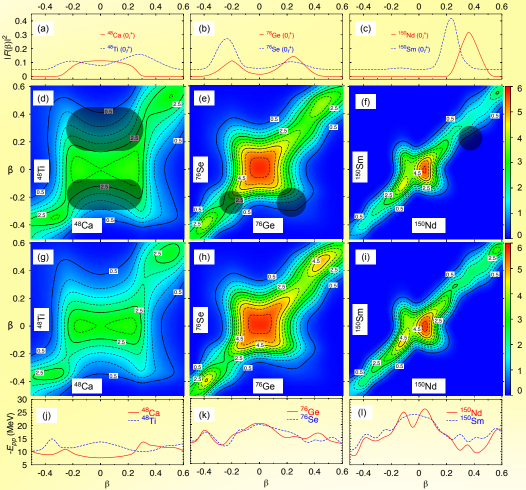
We're generalizing the method to include proton-neutron pairing and spin correlations, deal with problem 4.

Rodríguez et al Generator-Coordinate Calculation

Basic idea: Construct set of mean fields by constraining coordinate(s), e.g. quadrupole moment $\langle Q_0 \rangle \equiv \langle \sum_i r^2(i) Y_0^2(i) \rangle$. Minimize

$$\langle H' \rangle = \langle H \rangle - \lambda \langle Q_0 \rangle$$

Then use $\langle Q_0 \rangle$ as a collective coordinate; diagonalize H in space of number- and angular-momentum-projected quasiparticle vacua with different values of $\langle Q_0 \rangle$.



Rodríguez and Martinez-Pinedo

Adding Proton-Neutron Correlations to GCM

GCM results missing physics that affects QRPA calculations.

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Adding Proton-Neutron Correlations to GCM

GCM results missing physics that affects QRPA calculations.

So we generalize the approach:

1. Pairing currently treated as mean field, but not pn pairing. So we construct quasiparticles that mix not only particles and holes, but also protons and neutrons.
2. Constrain proton-neutron pairing and particle-hole condensation as well as deformation, i.e. minimize

$$H' = H - \lambda_Q \langle Q_0 \rangle - \lambda_P \langle P_0^\dagger \rangle - \lambda_{\sigma\tau} \langle O_{\sigma\tau} \rangle$$

with

$$P_0^\dagger = \sum_l \left[a_l^\dagger a_l^\dagger \right]^{L=0,S=1,T=0}_{M_S=0}, \quad O_{\sigma\tau} = \sum_i \sigma_z(i) \left(\tau^+(i) + \tau^-(i) \right)$$

The pn operators have zero expectation value at HFB minimum, but we add HFB states constrained to have non-zero values.

Calculation in $fp + sdg$ Shells

Pairing Operators

Usual spin-singlet pair operators

pn (spin-triplet) pair operators

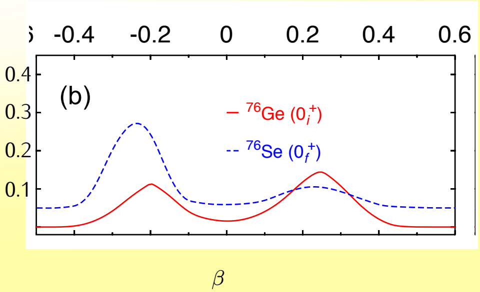
$$S^\dagger_\nu = \sum_l \left[a^\dagger_l \tilde{a}_l\right]^{L=-0,S=0,T=1}_{M_T=\nu} \qquad P^\dagger_\mu = \sum_l \left[a^\dagger_l \tilde{a}_l\right]^{S=1,T=0}_{M_S=\mu}$$

Interaction is

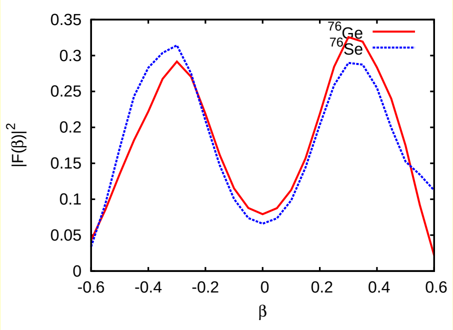
$$V = -g_{\text{pair}} \sum_\nu S^\dagger_\nu S_\nu - g_{pp} \sum_\mu P^\dagger_\mu P_\mu + g_{ph} \sum_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\tau}_i \vec{\tau}_j - g_Q \sum_\mu (-1)^\mu Q_\mu Q_{-\mu}$$

Competition between ordinary pairing and spin-triplet pairing.

Deformation Distributions for $A = 76$

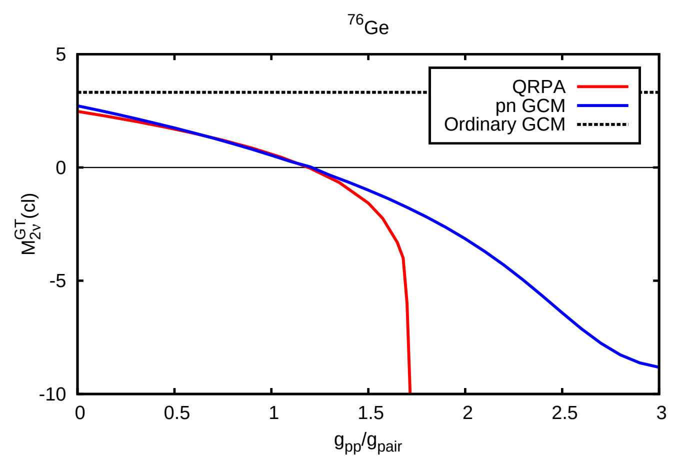


Rodrígez and Martinez-Pinedo



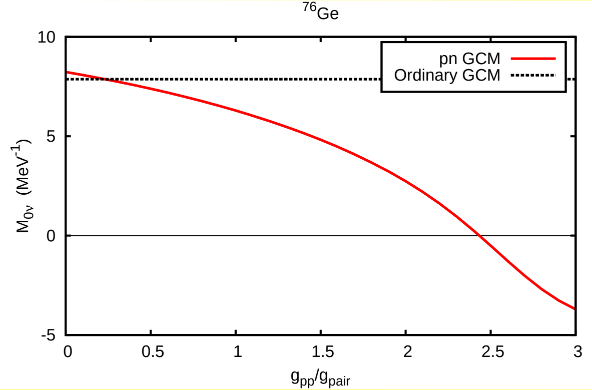
Hinohara

2ν (Closure) Matrix Element in ^{76}Ge



Realistic g_{pp}/g_{pair} is perhaps 1.5 or 1.6.

0ν Matrix Element in ^{76}Ge



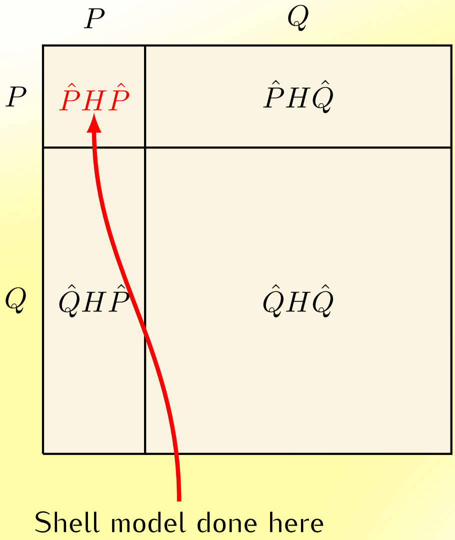
(Haven't done QRPA calculation yet.)

Next steps:

1. Fully include deformation. Initial results show little change.
2. Add proton-neutron physics to Gogny- or Skyrme-based GCM!

Corrected Shell Model

Partition of Full Hilbert Space



P = valence space
Q = the rest

Task: Find unitary transformation to make H block-diagonal in P and Q, with H_{eff} in P reproducing d most important eigenvalues.

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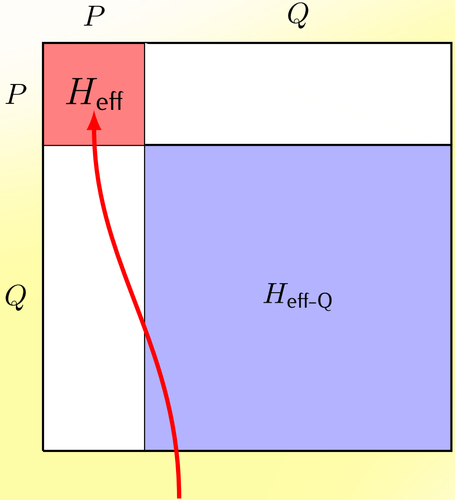

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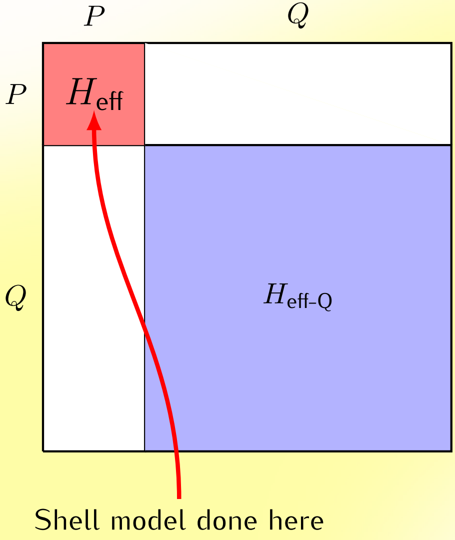

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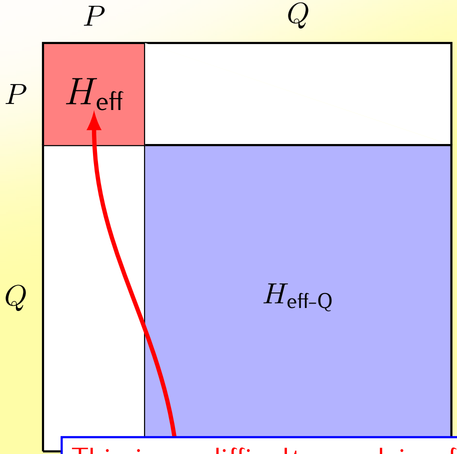

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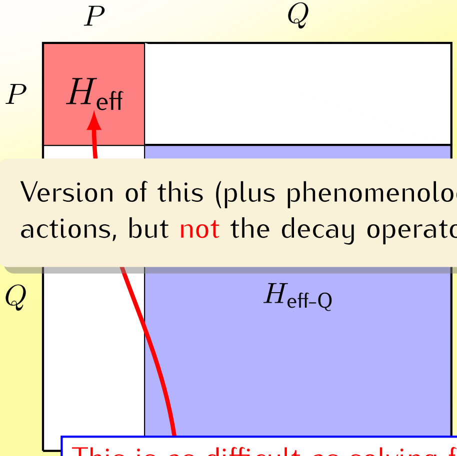

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Version of this (plus phenomenology) used to get shell-model interactions, but not the decay operator. Bare operator generally used.

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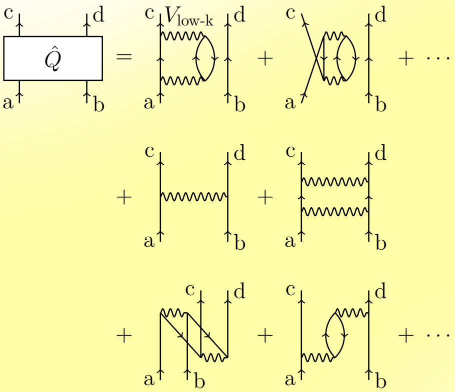
Shell model done here

Peturbation-Theory Approach

Q-Box



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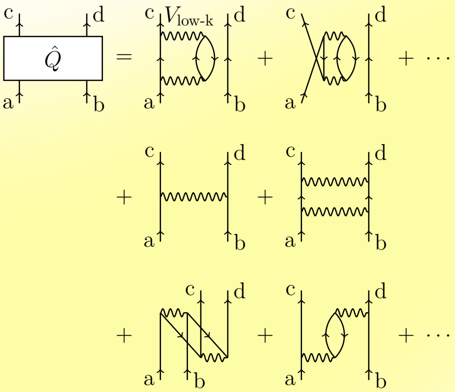
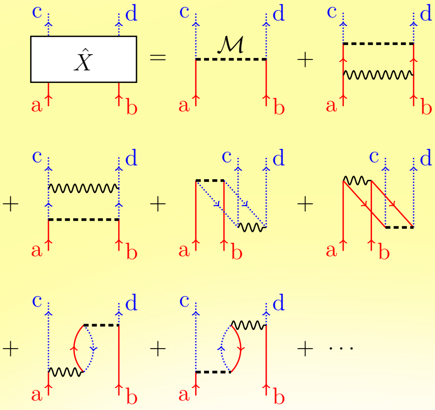


X-Box



Equation for Effective Transition Operator

$$\langle cd|\,\mathcal{M}_{\text{eff}}\,|ab\rangle =$$

$$\left(\left[1 + \frac{1}{2}\frac{d\hat{Q}(\varepsilon)}{d\varepsilon} + \frac{1}{2}\frac{d^2\hat{Q}(\varepsilon)}{d^2\varepsilon}\hat{Q}(\varepsilon) + \frac{3}{8}\left(\frac{d\hat{Q}(\varepsilon)}{d\varepsilon}\right)^2 \ldots \right] \right.$$

$$\times \left[\hat{X}(\varepsilon) + \hat{Q}(\varepsilon)\frac{\partial\hat{X}(\varepsilon_f,\varepsilon)}{\partial\varepsilon_f}\bigg|_{\varepsilon_f=\varepsilon} + \frac{\partial\hat{X}(\varepsilon,\varepsilon_i)}{\partial\varepsilon_i}\bigg|_{\varepsilon_i=\varepsilon}\hat{Q}(\varepsilon)\ldots \right]$$

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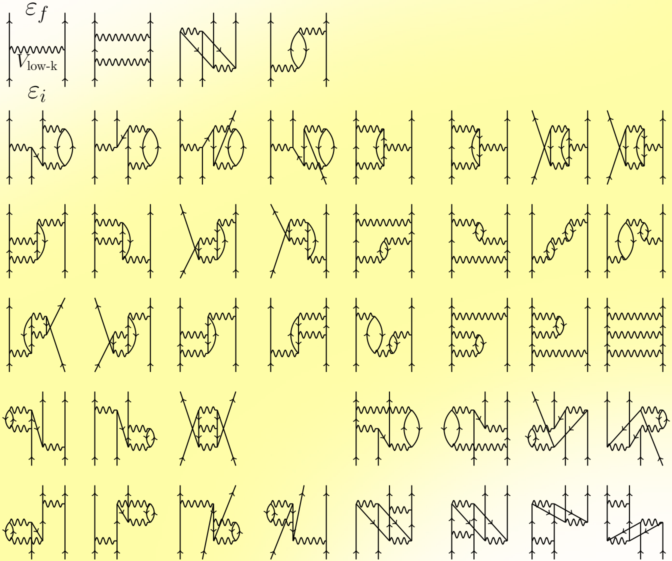
Perturbative Effective Decay Operator

Evaluated $\beta\beta$ version of these (which are for effective interaction).



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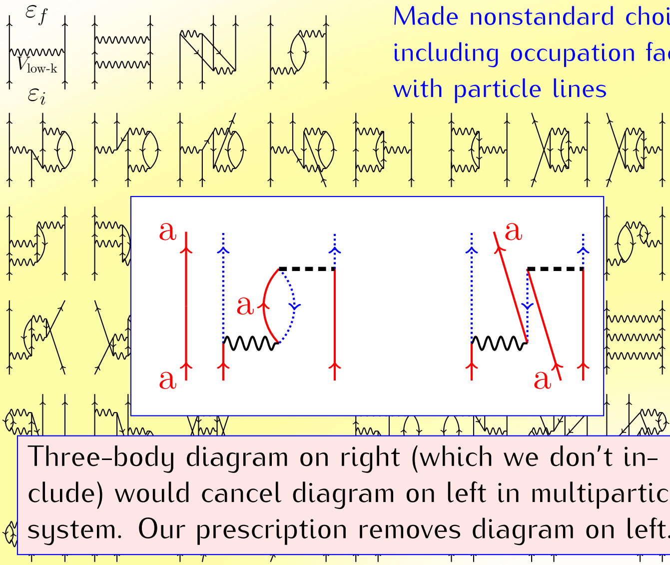

Made nonstandard choice of including occupation factors with particle lines

Three-body diagram on right (which we don't include) would cancel diagram on left in multiparticle system. Our prescription removes diagram on left.

Results

	Bare	Effective
^{82}Se	2.73	3.62
^{76}Ge	3.12	3.77

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Can we really believe the results? Convergence is an issue, but a deeper one may be effect of many-body induced operators.

Nonperturbative Test

Perturbation theory still may not be perfect so we also try to do without it.

So far, have just tested in p shell:

- Do pseudo-exact (6 or 8 $\hbar\omega$) no-core calculations for ^{5}He, ^{5}Li, get p-shell single-particle energies

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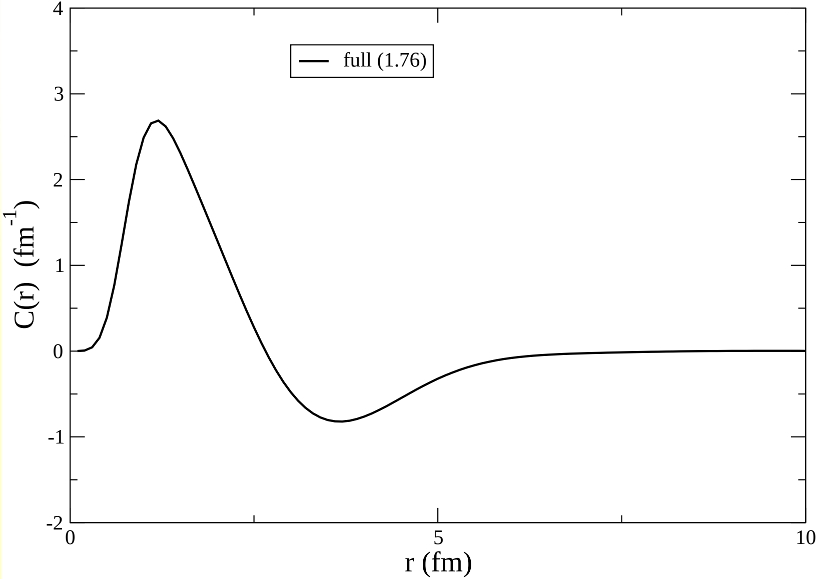
- Do the same for ^{6}He, ^{6}Be, get effective p-shell two-body interaction, effective two-body $\beta\beta$ operator.
- Use those operators to calculate ^{7,8,10}He $\longrightarrow$ ^{7,8,10}Be. Test adequacy of two-body operator. Can do the same for 3-body Hamiltonian and decay operator.

$^7He \longrightarrow {}^7Be$



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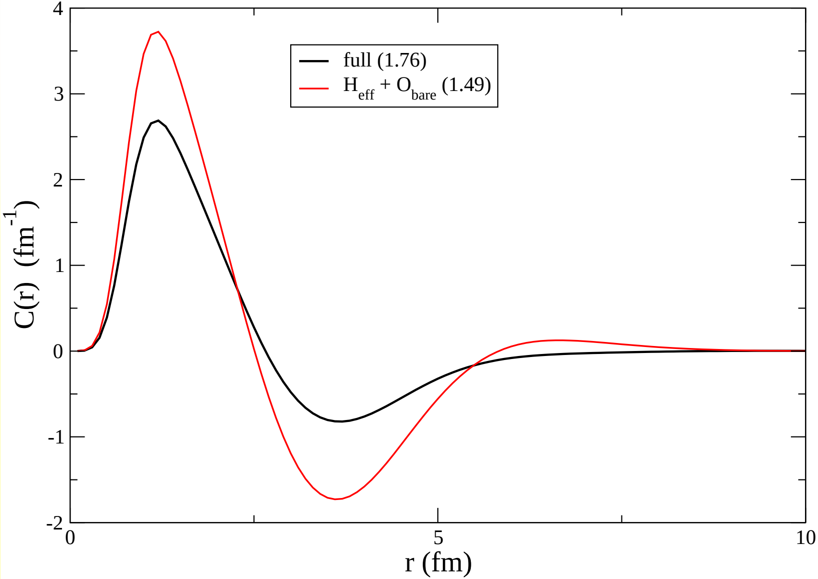

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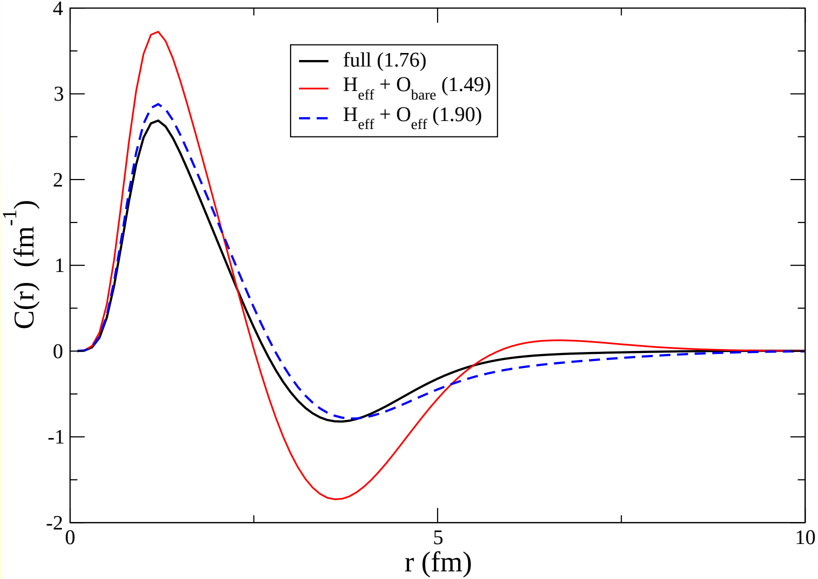

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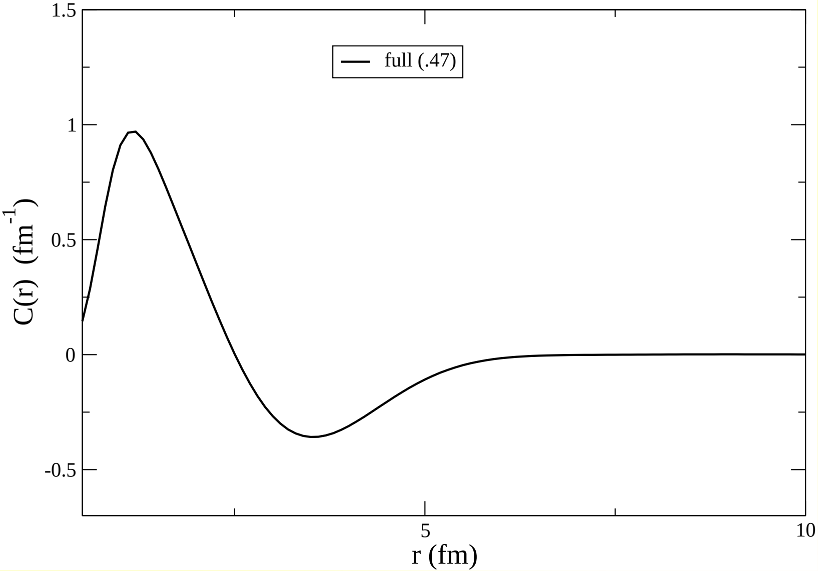

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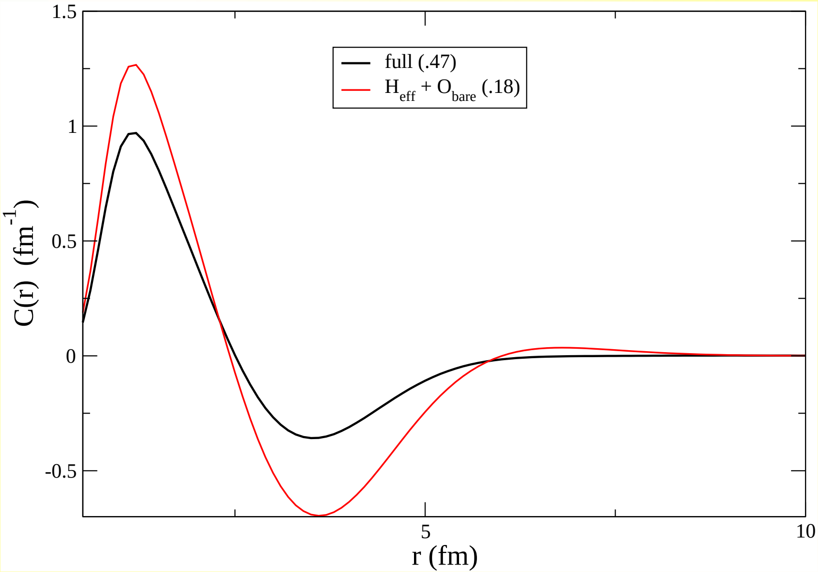

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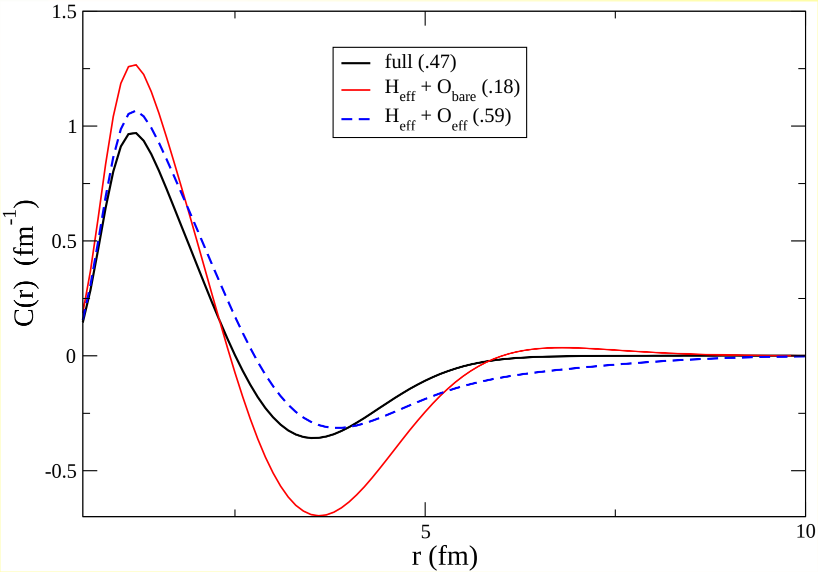
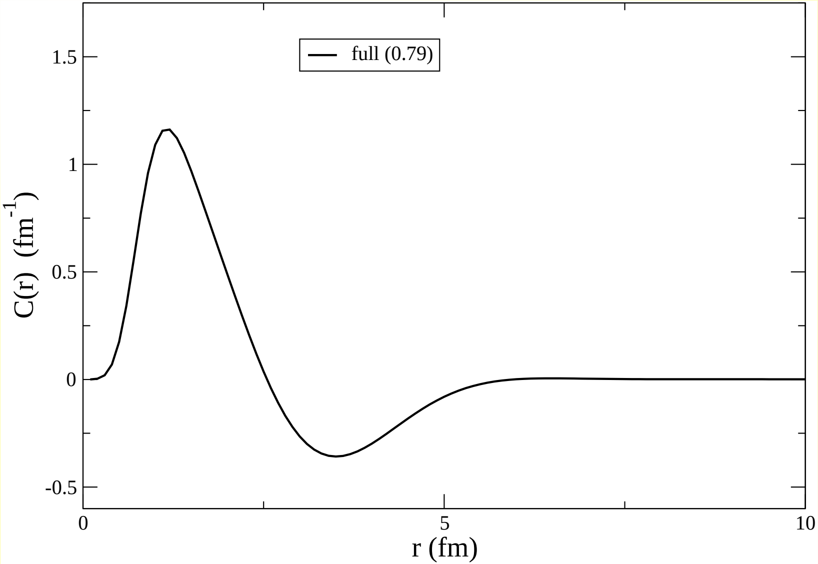


^{10}He$\longrightarrow$^{10}Be



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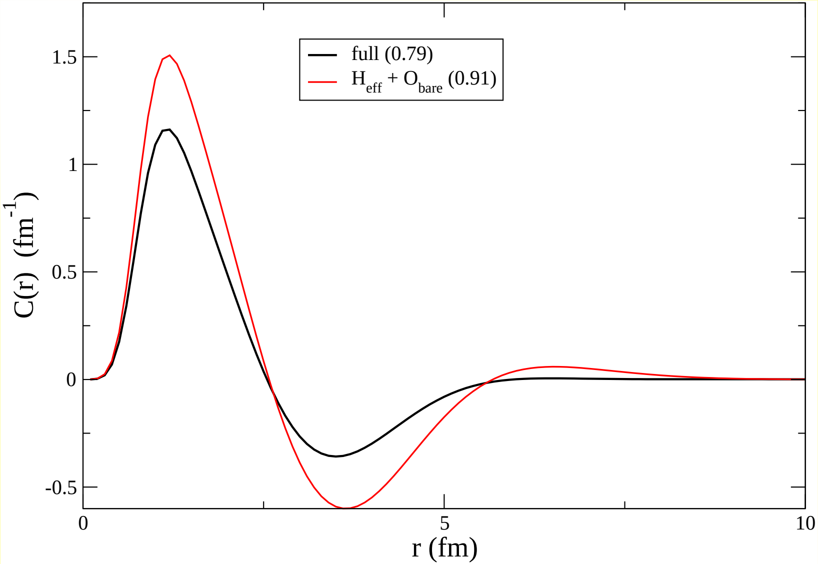

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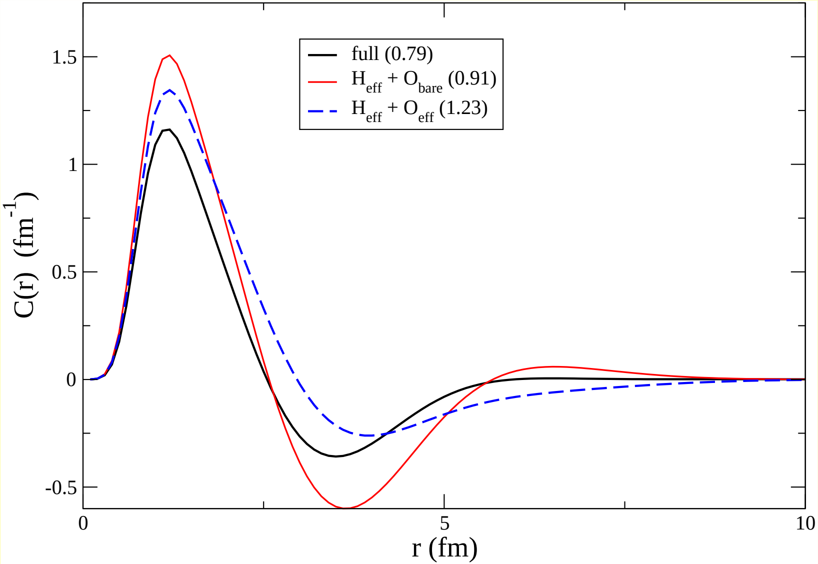


Want to test improvement from three-body operators.

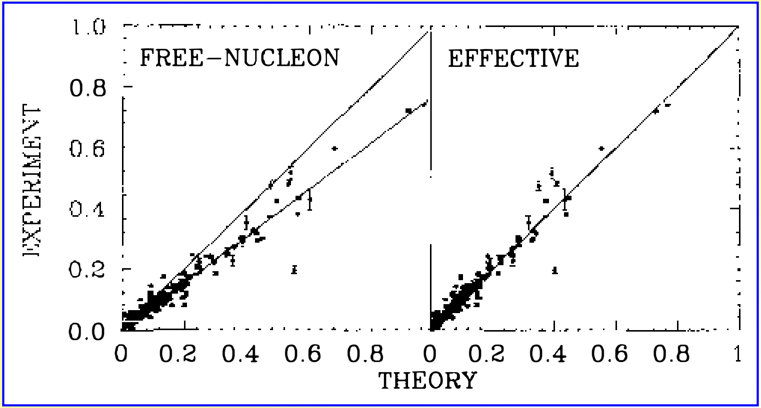
Nonperturbative Future

- **Coupled Clusters:** Solve the two-particle attached problem (closed shell + 2) on top of e.g., ^{56}Ni and three-particle-attached in some approximation, do Lee-Suzuki mapping of lowest eigenstates onto $f_{5/2}pg_{9/2}$, determine effective Hamiltonian and decay operator (up to three-body), calculate matrix element for ^{76}Ge. Already working with Jannsen and Hagen on this.
- **In-Medium SRG:** Hergert, Bogner, et al have published preliminary results for effective interaction in sd shell. Should be able to extend procedure to decay operator and $f_{5/2}pg_{9/2}$ shell.

Issue Facing All Models: "g_A"

Forty(?)-year old problem: Single-beta rates, 2ν double-beta rates, related observables overpredicted in heavy nuclei.

Brown & Wildenthall: Beta-decay strengths in sd shell



Solution: Not Yet Clear

Typical practice: "Renormalize" g_A to get correct results. But if g_A is renormalized by same amount in 0ν decay as in 2ν decay (a lot in shell model), experiments will fail; rates go as $(g_A)^4$.

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Better practice: Understand reasons for over-prediction. In modern language, must be due to

1. Many-body weak currents, either modeled explicitly as π, ρ exchange, etc., or from effective-field-theory fits. Who's right? The many old-school practitioners who say meson-exchange effects are small, or the chiral effective field theory folk, who say they can be large?

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2. Truncation of model space, to be fixed in shell model as already discussed. Can treat "bare many-body" operators as well.

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People are attacking both sides of this problem.

2. ... s already discussed. Can treat "bare many-body" operators as well.

So. . .

We should be able to improve nearly all methods for treating double-beta decay.

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Future is ~~dazzling~~ pretty bright.

So...

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That's all.