THE PSEUDO-L SCHEME IN STRONGLY DEFORMED BOSE–FERMI SYSTEMS AND ITS RELATION TO THE NILSSON MODEL

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The relation between the pseudo-L scheme and the Nilsson model is discussed within the context of SU(3) dynamical symmetry limits of boson–fermion models of odd-mass nuclei. It is shown that the pseudo-L approach incorporates certain non-adiabatic effects which modify the effective Coriolis matrix elements, thus offering a possible explanation for their observed deviation from experiment.

The idea of treating the single-particle states of the shell model in terms of pseudo-orbital and pseudo-spin angular momenta was introduced several years ago [1] and has since led to a number of interesting applications [2]. More recently this approach has emerged as a crucial element in the development of boson–fermion coupling schemes, via the recognition [3] of specific dynamical symmetries within the framework of the interacting boson fermion approximation (IBFA) model [4] of odd-mass nuclei. In certain limiting cases, a pseudo-L decomposition of the fermionic orbits allows the dynamical symmetry chains of the bosonic even–even core to be extended to encompass the odd fermion as well [5]. This is of particular interest in the description of strongly deformed odd-mass nuclei, where combined boson–fermion SU(3) dynamical symmetries may play an important role [5–7].

While dynamical symmetries contained in a nuclear model are certainly of interest in their own right, they become even more interesting if they have clear geometrical analogs. In the interacting boson approximation model of even–even nuclei, much of the beauty derives from the fact that the three dynamical symmetries are not only approximately

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realized but also have simple geometrical interpretations. The situation is less simple for boson–fermion symmetries. A natural possibility to consider is that dynamical SU(3) boson–fermion symmetries have their geometrical analog in the Nilsson or particle–rotor model. This idea was explored recently [6,7], and important progress in understanding the relation of Bose–Fermi SU(3) symmetries to the Nilsson model was achieved. In this work, we extend and generalize that understanding by considering a decomposition of the SU(3) Bose–Fermi symmetry hamiltonian into intrinsic and rotational degrees of freedom, in analogy with the Nilsson model approach. Essential to this decomposition is the use of the pseudo-L approach.

In the Nilsson model [8], an od-mass deformed system is treated as an even–even rotating core strongly coupled to an odd nucleon which moves in a deformed mean field produced by the core. Assuming that the rotation of the core is very slow compared to the motion of the odd particle, we can make an *adiabatic* separation of the total hamiltonian of the system into an intrinsic part and a collective part that describes the rotation of the inert core. For axially symmetric deformations,

$$H_{\text{collective}} = (1/2I)\, \boldsymbol{R}\cdot\boldsymbol{R}\,, \tag{1}$$

where $\boldsymbol{R}$ denotes the angular momentum of the core.

If we let $\boldsymbol{j}$ denote the angular momentum of the odd nucleon and $\boldsymbol{J}=\boldsymbol{R}+\boldsymbol{j}$ denote the total angular momentum of the system, then we can recast $H_{\text{collective}}$ in the form

$$H_{\text{collective}} = (1/2I)\, \boldsymbol{J}\cdot\boldsymbol{J} + (1/2I)\, \boldsymbol{j}\cdot\boldsymbol{j} - (1/I)\, J_3 j_3 - (1/2I)(J_+ j_- + J_- j_+)\,, \tag{2}$$

where the subscript 3 refers to the symmetry axis in the body-fixed system. The last term represents the well-known Coriolis interaction. Note that the same moment of inertia, that of the even–even core, enters in all terms in the collective hamiltonian. The many successes of this model in describing properties of odd-mass deformed nuclei are well documented in the literature [9].

We now consider a boson–fermion model of odd-mass nuclei, based on the algebraic group structure $\mathrm{U_B}(m_\mathrm{B})\times\mathrm{U_F}(m_\mathrm{F})$. Here m_B denotes the dimensionality of the boson space and m_F the dimensionality of the fermion space. Under appropriate conditions, it is possible to construct an SU(3) dynamical boson–fermion symmetry for such a model. The group chain that governs this symmetry has the generic form

$$\begin{aligned}&\mathrm{U_B}(m_\mathrm{B})\times\mathrm{U_F}(m_\mathrm{F})\\ &\quad\supset \mathrm{U_B}(m_\mathrm{B})\times\mathrm{U_F}(\tfrac{1}{2}m_\mathrm{F})\times\mathrm{SU_F}(2)\\ &\quad\supset \cdots\\ &\quad\supset \mathrm{SU_{B+F}}(3)\times\mathrm{SU_F}(2)\\ &\quad\supset \mathrm{SO_{B+F}}(3)\times\mathrm{SU_F}(2)\\ &\quad\supset \mathrm{Spin}(3)\,.\end{aligned} \tag{3}$$

The dots refer to the necessary intermediate subgroups in the chain, for which there is often more than one possibility.

For any such group chain, it is possible to decompose the total IBFA hamiltonian into

$$H = H_0 + H_{\text{rot}}\,. \tag{4a}$$

Here H_0 involves the Casimir operators up to and including that for $\mathrm{SU_{B+F}}(3)$ in the group chain and

$$H_{\text{rot}} = A_L C_{2\mathrm{SO_{B+F}}(3)} + A_J C_{2\mathrm{Spin}(3)}\,. \tag{4b}$$

Clearly H_{rot} governs collective rotations of the system, as is evident from the fact that

$$C_{2\mathrm{SO_{B+F}}(3)} = \boldsymbol{L}\cdot\boldsymbol{L},\quad C_{2\mathrm{Spin}(3)} = \boldsymbol{J}\cdot\boldsymbol{J}\,, \tag{5}$$

where $\boldsymbol{L}$ is the total pseudo-orbital angular momentum operator and $\boldsymbol{J}$ as before is the total angular momentum operator. More precisely, if $\boldsymbol{j}=\boldsymbol{l}+\boldsymbol{s}$ is the decomposition of the single-fermion angular momentum into its pseudo-orbital and pseudo-spin parts, then $\boldsymbol{L}=\boldsymbol{R}+\boldsymbol{l}$ and $\boldsymbol{J}=\boldsymbol{L}+\boldsymbol{s}$, where $\boldsymbol{R}$, as noted earlier, is the angular momentum of the (boson) core.

The pseudo-L decomposition appears explicitly in the reduction $\mathrm{U_F}(m_\mathrm{F}) \supset \mathrm{U_F}(\frac{1}{2}m_\mathrm{F})\times\mathrm{SU_F}(2)$. We will focus all subsequent discussions on a specific IBFA model, involving an s–d boson core and single-particle orbits with $j=\frac{1}{2}$, $\frac{3}{2}$ and $\frac{5}{2}$. Then $m_\mathrm{F}=12$ and $m_\mathrm{B}=6$ and the pseudo-L decomposition corresponds to treating the single-particle space as arising from the coupling of a pseudo-spin $s=\frac{1}{2}$ to pseudo-orbital $l=0$ and 2. The corresponding SU(3) spectrum consists of a series of rotational bands, as shown in fig. 1. The intrinsic excitation energies of the dif-

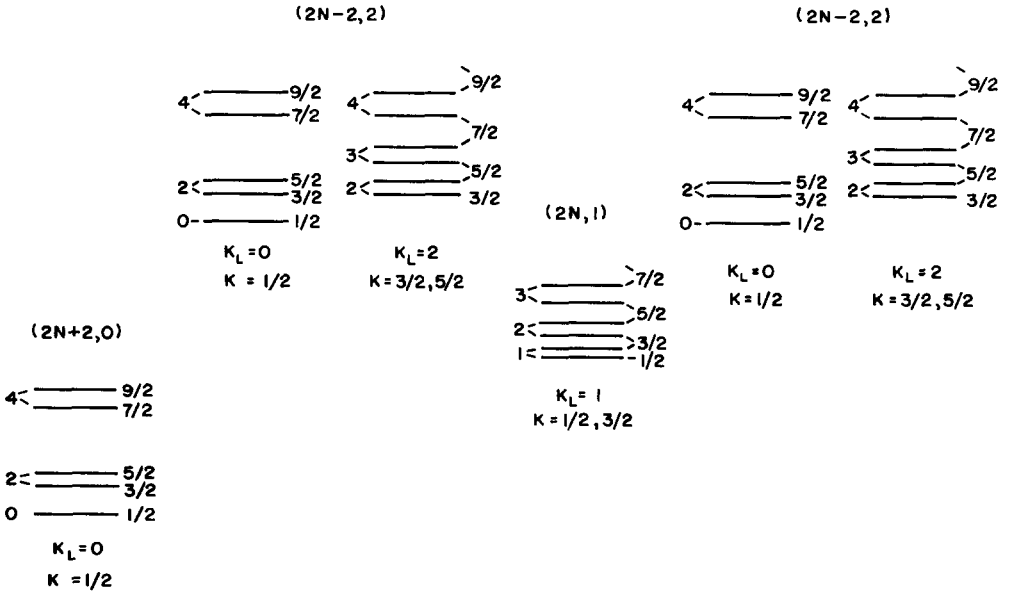

Fig. 1. Schematic representation of the low-lying band structure appropriate to a system of N bosons and one fermion described by the SU(3) limit of a $U_B(6)\times U_F(12)$ Bose–Fermi model. Shown above each intrinsic band are the (λ,μ) quantum numbers. States of each band are labelled on the left by the pseudo-L quantum number and on the right by the total angular momentum.

ferent bands are governed by the coefficients of the higher Casimir operators, which contribute to H_0, whereas the level spacings within bands are governed by $H_{\rm rot}$.

The intrinsic configurations defined by H_0 can be characterized by the SU(3) quantum numbers (λ,μ), by quantum numbers K, K_L and σ that give the projections of $\boldsymbol{J}$, $\boldsymbol{L}$ and $\boldsymbol{s}$, respectively, on the symmetry axis (note: $K=K_L+\sigma$) and by additional quantum numbers α that relate to earlier subgroups in the chain (1). We shall suppress these latter quantum numbers since they play no role in the rotational properties under discussion, and denote the intrinsic states by $|(\lambda,\mu)K\sigma\rangle$. Note that we choose a basis with good K quantum numbers [10], rather than the orthonormal Vergados basis [11] (the two bases become identical in the large N limit).

Each intrinsic state contains information about all the states in the corresponding intrinsic band. States with definite values of angular momentum can be obtained using standard angular momentum projection techniques. We denote them by $|(\lambda,\mu)K\sigma;JM\rangle$.

Each intrinsic state is doubly degenerate. For each state $|(\lambda,\mu)K\sigma\rangle$, there is another $|(\lambda,\mu)\ -K-\sigma\rangle$ with the same intrinsic energy. This double degeneracy results from the fact that the intrinsic hamiltonian is invariant with respect to rotations of 180° about an axis perpendicular to the symmetry axis. As a consequence, states projected from the two degenerate intrinsic states are not independent, but are related by a simple phase. Proper physical states are obtained from the symmetric linear combinations

$$|(\lambda,\mu)K\sigma;JM\rangle_s=(1/\sqrt{2})[\,|(\lambda,\mu)K\sigma;JM\rangle + (-1)^{J+K}|(\lambda,\mu)-K-\sigma;JM\rangle\,]\,. \qquad (6)$$

We now return to the rotational part of the symmetry hamiltonian. To make contact with the Nilsson description, it is useful to recast $H_{\rm rot}$ in a form similar to (2):

$$\begin{aligned} H_{\rm rot}&=A_L\boldsymbol{L}\cdot\boldsymbol{L}+A_J\boldsymbol{J}\cdot\boldsymbol{J}\\ &=A_L\boldsymbol{J}-\boldsymbol{s})(\boldsymbol{J}-\boldsymbol{s})+A_J\boldsymbol{J}\cdot\boldsymbol{J}\\ &=(A_L+A_J)\boldsymbol{J}\cdot\boldsymbol{J}+A_L\boldsymbol{s}\cdot\boldsymbol{s}-2A_LJ_3s_3\\ &\quad -A_L(J_+s_-+J_-s_+)\,. \end{aligned} \qquad (7)$$

The first three terms are diagonal for any intrinsic K band. The last term bears a striking similarity to the

Coriolis interaction of the Nilsson model, and is the term on which we will focus most of the discussion to follow. We will refer to it as the pseudo-Coriolis interaction and will denote it by $V_{\rm pC}$. Like the Coriolis interaction, it has diagonal matrix elements for $K=\frac{1}{2}$ bands and also has matrix elements mixing different K bands.

There are however two clear differences between the pseudo-Coriolis interaction and the usual Coriolis interaction of the Nilsson model. First, the pseudo-Coriolis interaction involves the pseudo-spin operator $s_{+(-)}$ rather than the odd-particle angular momentum operators $j_{+(-)}$. Second, the multiplicative constant in front of the pseudo-Coriolis interaction is different from the one in front of the rotational kinetic energy term. In contrast, in the Nilsson approach the same constant premultiplies both terms. This latter difference has potentially interesting consequences, and we will return to it later.

We now consider in more detail the matrix elements of the pseudo-Coriolis interaction. As noted earlier, it has diagonal matrix elements for $K=\frac{1}{2}^+$ bands, which follow from the presence of the two terms in the physical states (6). They are given by

$$
\begin{aligned}
&{}_{\rm s}\langle(\lambda,\mu)\,\tfrac{1}{2}\sigma;JM|\,V_{\rm pC}\,|(\lambda,\mu)\,\tfrac{1}{2}\sigma;JM\rangle_{\rm s}\\
&\quad=-A_L(-1)^{J+1/2}(J+\tfrac{1}{2})\langle s=\tfrac{1}{2},\sigma|\,s_+\,|s=\tfrac{1}{2},-\sigma\rangle\\
&\quad=-A_L(-1)^{J+1/2}(J+\tfrac{1}{2})\\
&\qquad\text{for } \sigma=+\tfrac{1}{2} \text{ and } K_L=0\,,\\
&\quad=0\quad\text{for } \sigma=-\tfrac{1}{2} \text{ and } K_L=1\,. \qquad (8)
\end{aligned}
$$

Thus, for $K=\frac{1}{2}$, $K_L=0$ bands the rotational contribution to the energy is given by

$$
\begin{aligned}
E^J_{\rm rot}&=(A_L+A_J)J(J+1)\\
&\quad+A_L[(-1)^{J+1/2}(J+\tfrac{1}{2})+\tfrac{1}{4}]\,, \qquad (9a)
\end{aligned}
$$

which is very similar in structure to the analogous Nilsson-model expression

$$
\begin{aligned}
E^J_{\rm rot}({\rm Nilsson})&=(1/2I)[J(J+1)\\
&\quad-\tfrac{1}{4}+a(-1)^{J+1/2}(J+\tfrac{1}{2})]\,, \qquad (9b)
\end{aligned}
$$

where a is the decoupling parameter.

Next, we discuss pseudo-Coriolos mixing between different intrinsic bands. From the form of the pseudo-Coriolis interaction, it is clear that such band mixing is limited to cases in which $\Delta K=1$, $\Delta K_L=0$ and $\Delta\sigma=1$. The relevant non-zero pseudo-Coriolis mixing matrix elements are

$$
\begin{aligned}
&{}_{\rm s}\langle(\lambda',\mu')K+1,\tfrac{1}{2};JM|\,V_{\rm pC}\,|(\lambda,\mu)K,-\tfrac{1}{2};JM\rangle_{\rm s}\\
&\quad=-A_L\sqrt{(J-K)(J+K+1)}\times\delta_{\lambda\lambda'}\delta_{\mu\mu'}\,. \qquad (10)
\end{aligned}
$$

As a specific example, we consider the bands with $K_L=1$ and 2 that arise within the $\mathrm{U_B}(6)\times\mathrm{U_F}(12)$ model under discussion. Each such band splits into two intrinsic bands, one with $K=K_L+\frac{1}{2}$ and one with $K=K_L-\frac{1}{2}$. The energies of the bands prior to mixing are determined by the term $-2A_LJ_3s_3$ of (7), which splits them an amount $2A_LK_L$ relative to their degenerate intrinsic energies. States with the same J values in the two bands mix via the pseudo-Coriolis interaction, with the mixing matrix elements given by (10). For every such pair of states, an analytic diagonalization of the mixing matrix is possible. Pairs of states are split by an amount $A_L(2J+1)$, and the resulting eigenstates no longer have good K or σ values; they still have good K_L values and furthermore each now has good L.

Exactly the same splitting $A_L(2J+1)$ can be obtained directly from the eigenvalue expression for $H_{\rm rot}$,

$$
E_{\rm rot}=A_LL(L+1)+A_JJ(J+1)\,. \qquad (11)
$$

The purpose of our analysis is to make more evident the connection to the Nilsson model, and thus we have followed an approach that highlights the role of the pseudo-Coriolis interaction.

Now we return to the comment made earlier that in the pseudo-L approach different constants appear in front of the rotational kinetic energy term and the pseudo-Coriolis interaction term. The origin of the different constants is the presence of separate $\boldsymbol{L}\cdot\boldsymbol{L}$ and $\boldsymbol{J}\cdot\boldsymbol{J}$ terms in the rotational hamiltonian. Both contribute to the rotational kinetic energy; only the $\boldsymbol{L}\cdot\boldsymbol{L}$ term contributes to the pseudo-Coriolis interaction.

The rotational hamiltonian (7) can be rewritten in the form

$$
H_{\rm rot}=(A_L+A_J)\boldsymbol{R}\cdot\boldsymbol{R}+\text{extra terms}\,. \qquad (12)
$$

Comparing (12) and (1), we see that the moment of inertia I of the even–even core is given by

$$I = 1/2(A_L + A_J) \,. \tag{13}$$

This is also the moment of inertia of the total Bose–Fermi system, as can be seen directly from (7). We can thus rewrite the coefficient in front of the pseudo-Coriolis interaction as

$$A_L = (1/2I)[A_L/(A_L + A_J)] \,. \tag{14}$$

Note that A_L enters as a multiplicative constant in both the decoupling matrix elements for $K = \frac{1}{2}$ bands (9) and the band mixing matrix elements (10). Depending on the relative signs of A_L and A_J, the quantity $A_L/(A_L + A_J)$ will either be greater than or less than unity. The net result will be an *effective* suppression or enhancement of pseudo-Coriolis matrix elements, relative to the values they would have if the multiplicative constant were simply $1/2I$. It is a well-known feature of the Nilsson model that the Coriolis coupling matrix elements required to phenomenologically describe the mixing of K bands are usually much weaker than those that arise theoretically [9]. The above discussion suggests a possible mechanism *within the pseudo-L scheme but outside the standard adiabatic Nilsson scheme* for suppressing (or enhancing) such coupling matrix elements.

Note that such a modification of coupling matrix elements in the pseudo-L scheme occurs whenever the rotational hamiltonian contains both $\boldsymbol{L}\cdot\boldsymbol{L}$ *and* $\boldsymbol{J}\cdot\boldsymbol{J}$ terms. The expressions for both the decoupling parameter and the mixing matrix elements become identical to those of the Nilsson model whenever the $\boldsymbol{J}\cdot\boldsymbol{J}$ term vanishes. Such a term arises whenever the pseudo-orbital doublets $j = l \pm \frac{1}{2}$ are not exactly degenerate in energy. Further work on this proposed mechanism for modifying coupling matrix elements is clearly called for.

There is one further important distinction between the pseudo-L scheme and the Nilsson scheme. This deals with the structure of the states in the two schemes. The conventional Nilsson treatment couples deformed states of the odd particle to the ground band of the even–even core only. In a more general adiabatic treatment, excited bands of the core also enter; however, there is still no mixing between different intrinsic core configurations. Such a restriction is not mandated by the pseudo-L approach. Here, physical states in general contain admixtures of different core configurations. Moreover, such mixing leads naturally to a fragmentation of single-particle transfer strength and significant fragmentation has indeed been observed in odd-mass deformed nuclei in the Hf–W–Os region [12].

It should be remarked that many of the features of the treatment described above, including the approximate existence of pseudo-L symmetry and associated pseudo-K values, arose in earlier studies based on the pseudo-SU(3) scheme [2] for deformed nuclei. The crucial difference is that in our approach, no attempt is made to describe the core in an explicit fermionic basis. Instead, the core is treated phenomenologically, as in the Nilsson scheme, and it is this feature that permits an explicit comparison of rotation–particle coupling effects in the two approaches to be carried out.

In summary, we have discussed in this letter the pseudo-L scheme as it pertains to strongly deformed odd-mass systems described by an SU(3) Bose–Fermi symmetry, and have considered its relation to the usual Nilsson-model treatment. We have seen that the pseudo-L scheme incorporates two important extensions to the Nilsson framework. First, core states other than those of the ground band can be incorporated without departing from a pseudo-L symmetry. Second, the collective hamiltonian associated with a pseudo-L symmetry includes separate $\boldsymbol{L}\cdot\boldsymbol{L}$ and $\boldsymbol{J}\cdot\boldsymbol{J}$ interactions, which can modify the effective matrix elements that couple intrinsic bands. Of course, the ultimate goal of any theory of odd-mass deformed nuclei, whether purely fermionic or based on a IBFA model, is to describe *real* deformed nuclei. Although SU(3) may provide a meaningful starting point, it is clear that symmetry-breaking effects will have to be taken into account. In such cases, pseudo-L need no longer remain a good quantum number of the system, and the exact decomposition of the hamiltonian given by (4) may no longer apply. An important question to address in the future is whether a better starting point for a description of such nuclear systems is provided by an adiabatic approach or by a pseudo-L approach.

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