DOUBLE BETA DECAY IN THE GENERALIZED-SENIORITY SCHEME

J. ENGEL, P. VOGEL
Norman Bridge Laboratory of Physics 161-33, California Institute of Technology, Pasadena, CA 91125, USA

Xiangdong JI
W.K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, CA 91125, USA

and

S. PITTEL
Bartol Research Institute, University of Delaware, Newark, DE 19716, USA

Received 1 March 1989; revised manuscript received 5 June 1989

We employ a generalized-seniority-based truncation scheme to calculate double beta decay rates in ^{76}Ge, ^{82}Se, ^{128}Te, and ^{130}Te, avoiding problems associated with other shell-model calculations and the quasiparticle random phase approximation (QRPA). Our two-neutrino matrix elements are small, reflecting a suppression mechanism first observed in the QRPA. The new results should be most accurate for neutrinoless decay because the closure approximation is reliable and the matrix elements less suppressed. We set limits on the Majorana mass of the neutrino that differ only by factors of two or three from those of previous methods, suggesting that major difficulties in predicting double beta decay are restricted to the two-neutrino process.

The rate of neutrinoless (0ν) nuclear double beta (ββ) decay is related to the (Majorana) mass of the neutrino by matrix elements that depend delicately on the complicated structure of the nuclei involved in the decay. A variety of techniques have been used in attempts to calculate these matrix elements and their two-neutrino (2ν) counterparts [1–5], with results of uncertain accuracy. While the work of refs. [3–5] is suggestive, there is still no quantitative understanding of the seemingly very suppressed 2ν decay of rock-bound ^{130}Te [6].

One many-body method, the quasiparticle random phase approximation (QRPA), points [3–5] to a suppression mechanism – the particle–particle (pp) component of the $T=0$ nuclear force. But despite this success, the QRPA has some shortcomings. Its equations violate particle number conservation, require a choice between the initial and final nuclei as the "ground state" on which excited states in the intermediate odd–odd nucleus are based, and most importantly, become unstable for nuclear forces not much stronger than those actually assumed. This latter fact is reflected in the rapid variation of calculated matrix elements with the strength of the pp force. Other methods avoid these problems, but are deficient in their own ways. Shell model calculations, for example, generally have resulted in larger matrix elements than the QRPA produces, despite the presumably correct inclusion of pp forces. Assuming that the observed 2ν suppression in ^{130}Te is a real effect, we may surmise that the stringent truncations imposed on shell model spaces exclude configurations responsible for the reduction. In heavy nuclei some sort of drastic truncation is unavoidable, but many traditional shell-model approximations, including the omission of spin–orbit partners of certain single-particle orbitals and the use of a weak coupling scheme that selects states from separate consideration of low-lying proton and neutron spectra, seem unsuited for ββ transitions. Here, using insight from QRPA work, we diagonalize in a shell-model space that should be more appropriate for the operators governing ββ decay.

0370-2693/89/$ 03.50 © Elsevier Science Publishers B.V.
(North-Holland Physics Publishing Division)

In the closure approximation (about which we will say more shortly), the ββ nuclear matrix elements have the form

$$M^{2\nu}_{\rm GT}({\rm cl}) = \langle 0^+_{\rm f} \mid \sum_{k,l} \boldsymbol{\sigma}(k)\cdot\boldsymbol{\sigma}(l)\, t^+(k) t^+(l) \mid 0^+_{\rm i} \rangle \,, \qquad (1)$$

$$M^{0\nu}_{\rm GT} = R\langle 0^+_{\rm f} \mid \sum_{k,l} H(\bar{E}, r_{k,l}) \boldsymbol{\sigma}(k)\cdot\boldsymbol{\sigma}(l)\, t^+(k) t^+(l) \mid 0^+_{\rm i} \rangle \qquad (2)$$

and

$$M^{0\nu}_{\rm F} = R\langle 0^+_{\rm f} \mid \sum_{k,l} H(\bar{E}, r_{k,l}) t^+(k) t^+(l) \mid 0^+_{\rm i} \rangle \,, \qquad (3)$$

where $|0^+_{\rm i}\rangle$ ($|0^+_{\rm f}\rangle$) is the ground state of the initial (final) nucleus, R is the nuclear radius, and $H(\bar{E}, r_{k,l})$, defined in ref. [3], is an approximately $1/r$ "neutrino potential" that includes the effects of short-range correlations in the wave functions. The physics behind the QRPA-induced suppression of these quantities is the following: First, the BCS procedure smears the nuclear Fermi surface over a relatively large number of orbitals, representing the states in eqs. (1)–(3) as condensates of S-pairs ($J=0$, $T=1$). Without any further modification, this pairing serves to open up the 2ν decay (the effect in 0ν processes is similar, though less pronounced) that in zeroth order is usually completely Pauli-blocked. The decay amplitudes are then reduced by RPA proton–neutron correlations, which result in the admixing of 4, 8, etc. quasiparticles into the ground states. If the correlations are *too* strong the solutions can become unstable, indicating that the BCS state is not a good lowest order approximation. Unfortunately, the RPA is known to frequently overestimate correlations, leading to instabilities when no phase transitions are in fact nearby. Here we wish to incorporate the physics of the RPA discussed above while avoiding the difficulties associated with number non-conservation and the possibly false instabilities.

All this can be accomplished within the framework of the shell model if spin–orbit partners are consistently included in the single-particle basis and an appropriate truncation scheme is employed. The concept of generalized seniority [7] guides us here in restricting the huge number of configurations allowed in large shell-model spaces. We start by replacing the BCS vacuum by a number-conserving "generalized seniority$=0$" state involving a condensate of $J=0$ S-pairs for protons and a corresponding condensate for neutrons, viz:

$$|w_\pi = w_\nu = 0\rangle = (S^\dagger_\pi)^{N_\pi} (S^\dagger_\nu)^{N_\nu} |0\rangle \,, \qquad (4)$$

where ($\rho = \pi, \nu$)

$$S^\dagger_\rho = \sum_j \tfrac{1}{2}(2j+1)^{1/2} \alpha^\rho_j [\rho^\dagger_j \rho^\dagger_j]^{(0)} \qquad (5)$$

creates a coherent $J=0$ pair, and the α^ρ_j are quantities to which we turn later. The quasiparticle correlations generated by the QRPA correspond to *proton–neutron* pairs; while the RPA equations are non-perturbative, in lowest order they mix two of these pairs into the BCS ground state. This is the level at which we work here, in a number-conserving scheme. By recoupling the particles, we can represent the most general $J=0^+$ state with two proton–neutron pairs in the form

$$|w_\pi = w_\nu = 2\rangle = \sum_{klmn,L} C^L_{klmn} \{ [\pi^\dagger_k \pi^\dagger_l]^{(L)} [\nu^\dagger_m \nu^\dagger_n]^{(L)} \}^{(0)} \times (S^\dagger_\pi)^{N_\pi - 1} (S^\dagger_\nu)^{N_\nu - 1} |0\rangle \,. \qquad (6)$$

The state (6) (after projecting out $w=0$ pieces) has generalized seniority $w=2$ in both neutrons and protons. The basis for our shell-model calculations in the initial and final nuclei consists of all such vectors, together with the fully paired state (4).

Aside from our desire to mimic the QRPA, there is further rationale for choosing such a set. In lowest order, the ground state of an undeformed heavy even–even nucleus can be represented by a suitably chosen $w_\pi = w_\nu = 0$ state of the form (4). The double beta decay operator changes two neutrons into two protons, resulting (in the same approximation) in a final-nucleus $w_\pi = w_\nu = 2$ state of the form (6). By including these in the basis, we are taking a step towards closing our space under the action of ββ operators. Higher seniority states may of course be generated from those with $w_\pi = w_\nu = 2$, but will be mixed into the nuclear ground states with increasingly small amplitudes in nuclei (such as those considered here) that are not strongly deformed. Our belief is that the dominant effects are produced by states with generalized seniority zero and two, and that because some of these lie high in the like-particle spectrum, a weak-coupling truncation can miss important contributions.

To fully specify our generalized seniority basis we still need to determine the appropriate values of α_j^π and α_j^ν that enter into the dominant S-pairs. The procedure we use is similar to one outlined in ref. [8], where an iteration scheme was employed to minimize the $w=0$ state energy-expectation value for one kind of particle, while the other was treated in a boson approximation. Here, we adjust the α's for protons and neutrons simultaneously so as to find the minimum energy state of the form (4). The optimal pairs determined in this way vary significantly from nucleus to nucleus, and are noticeably different in the initial and final nuclei in any given $\beta\beta$ decay.

Diagonalizing in the low-seniority basis and then evaluating $\beta\beta$ decay amplitudes is time-consuming but straightforward. We are able to choose reasonably large single-particle spaces without neglecting spin–orbit partners, and to calculate matrix elements of hamiltonians and transition operators by adapting formulae developed earlier [9] in connection with the interacting boson model. We have applied the method to the $\beta\beta$ decay of ^{76}Ge, ^{82}Se, ^{128}Te, and ^{130}Te. In the two lighter nuclei our single-particle space comprises the full 1f–2p shell, plus the $1g_{9/2}$ and $1g_{7/2}$ orbits for both protons and neutrons; the resulting basis contains 561 states. In Te we include the 3s–2d–1g shell and the $1h_{11/2}$ and $1h_{9/2}$ orbits, resulting in a low-seniority basis with 1147 states. The chief uncertainty in our calculations is the effective nucleon–nucleon force. Ideally, one would choose the effective interaction along with the single particle energies so as to optimally reproduce observables in the nuclei of interest. The large number of orbitals we include makes this impractical. We have instead obtained our single particle levels from a Woods–Saxon well [10], and used as a two-body interaction a parametrized fit to a Reid-soft-core [11] (or Paris [12]) G-matrix. Because we are dealing with open-shell nuclei and have obtained the single particle energies and two-body interaction from the different sources, there is a danger of double-counting single particle effects by including the proton–neutron monopole–monopole part of the interaction in the calculation. This latter piece is difficult to determine reliably, and so we have performed each calculation twice, once with the monopole–monopole force and once without it; the true result probably falls somewhere between. By restricting the states in the intermediate odd–odd nuclei to have generalized seniority $w_\pi=1$, $w_\nu=1$, we can verify that with these forces pairing gaps and giant Gamow–Teller resonances are reproduced fairly well.

The $w=1$ truncation in the odd–odd nuclei ignores much of the effect of the $w=2$ components in the even–even nuclei. That restriction is too severe to allow reliable evaluation of suppressed quantities like β^+ strength distributions and the full 2ν amplitudes that involve intermediate-nucleus wave functions. We have therefore resorted to the closure approximation [1], both in 0ν decay, where it is reliable, and in 2ν decay, where it probably is not. Nevertheless, we present our 2ν values (obtained with the force of ref. [11] with and without the proton–neutron monopole–monopole (component) in table 1, together with the shell model numbers of ref. [1]. The current matrix elements are smaller, and the extra suppression we see in Te is particularly encouraging.

As in the QRPA, we find here that the smallness of $M_{\mathrm{GM}}^{2\nu}$ (cl) stems from the cancellation of a relatively large "pairing" matrix element by contributions from correlations in the wave function. For example, in ^{128}Te and ^{130}Te (with the monopole–monopole piece) approximately 80% of the wave functions have $w=0$. If we eliminate the other components, we find in ^{128}Te (^{130}Te) a matrix element of -1.76 (-1.54). The states with seniority $w=2$ by themselves contrib-

Table 1
Calculated matrix elements $|M_{\mathrm{GT}}^{2\nu}(\mathrm{cl})|$ as given in eq. (1).

	^{76}Ge	^{82}Se	^{128}Te	^{130}Te
present work (with monopole proton–neutron interaction)	0.46	0.43	0.26	0.18
present work (without monopole proton–neutron interaction)	1.14	0.90	0.87	0.68
ref. [1]	2.6	1.9	2.9	3.0

ute an additional −0.44 (−0.32). Interference terms connecting $w=0$ and $w=2$ states, however, contribute +1.94 (+1.68), values that are only 15% apart, but the effect of which is disproportionately large in the final suppressed result.

To establish a connection between the 2ν closure matrix elements and decay lifetimes we must somehow determine an "average energy denominator" [1] $\bar{\Delta}_E$. Using our small matrix elements we obtain the known experimental lifetime in ^{82}Se [13], 1.1×10^{20}yr,with $\bar{\Delta}_E$ between 3 and 6 MeV. In ref. [1], by contrast, the denominator was taken to be $1.12A^{1/2} = 10$ MeV, roughly the energy of the giant Gamow–Teller β^- resonance. But this latter prescription is not likely to be correct; there is no reason to assume that the β^+ strength distribution from the final nucleus, which is just as relevant to ββ decay, is peaked at anywhere near the energy of the giant resonance, and in somewhat lighter nuclei (^{48}Ti and ^{54}Fe, for example [14,15]) it is clearly concentrated at much lower energies. The QRPA correctly reproduces this effect and in ββ candidates predicts energy denominators considerably smaller than those obtained from the giant Gamow–Teller peak. While cancellations in the matrix elements make it impossible to nail down $\bar{\Delta}_E$ without actually calculating the full non-closure amplitude, the low energy denominators extracted here from the data and our small matrix elements seem entirely reasonable.

Amplitudes as close to zero as ours are also obviously a little uncertain. States with $w=4$, while barely present in the wave functions, could alter them substantially. Additional uncertainties arise from first-forbidden decay, discussed in ref. [16]. Here we have found as well that liberal variations in the single-particle energies and effective interaction can cause matrix elements to change somewhat (though using the force of ref. [12] makes almost no difference), and that in particular, our insufficient knowledge of the proton–neutron monopole–monopole force introduces an uncertainty. What always survives to some degrees, however, is the cancellation and consequent suppression, and we accordingly take the smallness of our closure matrix elements as an indication that we are on the right track. In 0ν decay, to which we now turn, closure is appropriate and suppression less marked, and our methods should therefore be more reliable.

Table 2 shows values for the 0ν quantity $M_{GT}^{0\nu} - M_F^{0\nu}$, which under the assumption $g_A^{eff}=1$, enters quadratically into the neutrinoless decay lifetime. The same cancellation mechanism as described above operates here, cancellation mechanism as described above operates here, but is less effective. In ^{76}Ge, for

Table 2
Calculated matrix elements $|M_{GT}^{0\nu} - M_F^{0\nu}|$ as given in eqs. (2) and (3).

	^{76}Ge	^{82}Se	^{128}Te	^{130}Te
present work (with monopole proton–neutron interaction)	3.3	1.8	4.5	3.7
present work (without monopole proton–neutron interaction)	5.0	3.7	6.3	5.7
ref. [1]	5.5	4.4	6.7	6.8
ref. [3] $\alpha'_1 = -405$ (MeV fm^3)	1.6	1.2	3.5	3.1
ref. [4]	4.8	4.4	4.4	3.8
$T_{1/2}\times m_\nu^2$ (yr eV^2) present work (with monopole proton–neutron interaction)	9.2×10^{24}	7.3×10^{24}	1.8×10^{25}	1.1×10^{24}

instance, the contributions with the proton–neutron monopole term of the $w=0$, pure $w\neq 0$, and interference terms are -5.0, -1.3, and $+3.1$, respectively. Because the resulting amplitudes are larger, we expect our numbers to be less sensitive to changes in the calculation; the reduced effect of the monopole–monopole force confirms this suspicion. Our present values lie between those of ref. [3] and ref. [4], themselves in reasonably good agreement with each other. We conclude from the variety of calculations producing similar results that these neutrinoless amplitudes are well determined to within a factor 2 or 3; a neutrino-mass limit, therefore, can be extracted to within the same factor.

The approximations presented above appear to be reasonable. We have incorporated in a parameter-free shell-model diagonalization the physics observed in QRPA work while avoiding errors introduced by the extreme collectivity and number non-conservation required in that method. The troublesome order-of-magnitude uncertainties in ββ decay, now clearly a reflection of cancellation, seem restricted to the 2ν process. Neutrinoless double beta decay is the most sensitive available test for Majorana neutrino mass, and the work presented here suggests that the relevant nuclear matrix elements can be accurately determined.

This work was supported by the US Department of Energy under Contract No. DE-FG03-88ER40397 and by the National Science Foundation under Grants Nos. PHY85-05682, PHY86-04197, and PHY86-17198.

References

[1] W.C. Haxton and G.J. Stephenson Jr., Prog. Part. Nucl. Phys. 12 (1984) 409.
[2] H.V. Klapdor and K. Grotz, Phys. Lett. B 142 (1984) 323; K. Grotz and H.V. Klapdor, Nucl. Phys. A 460 (1986) 395.
[3] P. Vogel and M.R. Zirnbauer, Phys. Rev. Lett. 57 (1986) 3148;
J. Engel, P. Vogel and M.R. Zirnbauer, Phys. Rev. C 37 (1988) 731.
[4] O. Civitarese, A. Faessler and T. Tomoda, Phys. Lett. B 194 (1987) 11;
T. Tomoda and A. Faessler, Phys. Lett. B 199 (1987) 475.
[5] J. Engel, P. Vogel, O. Civitarese and M.R. Zirnbauer, Phys. Lett. B 208 (1988) 187.
[6] T. Kirsten et al., in: Nuclear beta decays and neutrino, eds. T. Kotani, H. Ejiri and E. Takasugi (World Scientific, Singapore, 1986) p. 81.
[7] I. Talmi, Nucl. Phys. A 172 (1971) 1.
[8] O. Scholten, Phys. Rev. C 28 (1983) 1783.
[9] O. Scholten and S. Pittel, Phys. Lett. B 120 (1983) 9.
[10] G. Bertsch, The practitioner's shell model (American Elsevier, New York, 1972).
[11] G. Bertsch, J. Borysowicz, H. McManus and W.G. Love, Nucl. Phys. A 284 (1977) 389.
[12] A. Hosaka, K.I. Kubo and H. Toki, Nucl. Phys. A 444 (1985) 76.
[13] S.R. Elliott, A.A. Hahn and M.K. Moe, Phys. Rev. Lett. 59 (1987) 2020.
[14] T. Tsuboi, K. Muto and H. Horie, Phys. Lett. B 143 (1984) 293.
[15] O. Hausser and M.C. Vetterli, Nucl. Phys. A 478 (1988) 559c.
[16] A.G. Williams and W.C. Haxton, University of Washington preprint (1988).