Neutralino inelastic scattering with subsequent detection of nuclear γ rays

J. Engel
Department of Physics and Astronomy, CB3255, University of North Carolina, Chapel Hill, North Carolina 27599

P. Vogel
Physics Department 161-33, Caltech, Pasadena, California 91125
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We consider the potential benefits of searching for supersymmetric dark matter through its inelastic excitation, via the ''scalar current,'' of low-lying collective nuclear states in a detector. If such states live long enough so that the γ radiation from their decay can be separated from the signal due to nuclear recoil, then background can be dramatically reduced. We show how the kinematics of neutralino-nucleus scattering is modified when the nucleus is excited and derive expressions for the form factors associated with exciting collective states. We apply these results to two specific cases: (1) the $I^{\pi}=5/2^{+}$ state at 13 keV in ^{73}Ge, and (2) the rotational and hence very collective state $I^{\pi}=3/2^{+}$ at 8 keV in ^{169}Tm (even though observing the transition down from that state will be difficult). In both cases we compare the form factors for inelastic scattering with those for elastic scattering. The inelastic cross section is considerably smaller than its elastic counterpart, though perhaps not always prohibitively so.

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A number of groups are trying to detect weakly interacting dark matter, one of the most promising candidates for what is the supersymmetric ''lightest neutralino.'' A popular approach is to try to observe the scattering of these particles on nuclear targets in low-background laboratory experiments. The signature of neutralino-nucleus scattering is the low-energy recoil of the nucleus in a detector. Since the scattering rate is expected to be tiny, the background is the main factor limiting sensitivity, even when low itself.

Supersymmetric dark matter is reviewed in Ref. [1]. Here we are interested only in the nuclear physics aspects of this problem, and in particular in the possibility of detecting inelastic scattering, thereby dramatically reducing the background. (The nuclear physics of dark matter detection is reviewed in Ref. [2].) The work was inspired by questions from researchers in the field [3,4].

Though inelastic scattering of neutralinos has been considered before, notably in Ref. [5], the focus was on spin-dependent scattering. The authors discussed low-lying excited states in stable nuclei with large measured $M1$ matrix elements; later, Ref. [6] reported an upper limit of 9.8×10^{-2} counts/kg/day (at 90% C.L.) for the inelastic excitation of the $7/2^{+}$ state at 57.6 keV in ^{127}I. It has since become clear, however [7], that spin-independent scattering will almost always occur with greater probability than its spin-dependent counterpart. We therefore focus here on the possibility of excitation by the scalar current, where the relevant multipole is $E2$ instead of $M1$. Collective $E2$ transitions, of which there are many, may allow the scalar current to be even more effective.

Of course there is a price to pay for the extra γ ray in the signal from inelastic scattering: the cross section is noticeably smaller than the elastic one. As we explain below, this is caused here not so much by the kinematics discussed in Ref. [5] — $E2$ excitations can often be found lower in the spectrum than $M1$ excitations — or by the factor qR that enters higher multipoles, but rather by a considerable reduction in ''coherence'' from elastic scattering, even when collective nuclear states are excited. Collective excitations of the nucleus generally involve valence nucleons, of which there are more than the (effectively) one that participates in spin-dependent scattering, but still far fewer than the A that are involved in elastic scattering. Thus, though we gain in some ways by considering the scalar current, we will still not obtain cross sections that approach those from elastic scattering. We quantify this remark below.

Let us consider kinematics first. A particle of mass M_X moves with velocity v and scatters on a stationary target of mass M_A. After the scattering the target has $E_{\rm exc}$ of excitation energy, i.e., its mass is $M_f=M_A+E_{\rm exc}$. The momentum transfer is

$$\vec{q}^{\,2}=M_X^2|\vec{v}-\vec{v}\,'|^2=M_X^2[v^2+v'^2-2vv'\cos(\theta)], \quad (1)$$

where θ is the scattering angle and v' is the final velocity of the scattered particle. The energy transfer is

$$\omega=M_X(v^2-v'^2)/2=E_{\rm recoil}+E_{\rm exc}=\frac{\vec{q}^{\,2}}{2M_f}+E_{\rm exc}. \quad (2)$$

The minimum and maximum momentum transfer, and thus also the minimum and maximum recoil energy $E_{\rm recoil}=q^2/2M_f$, correspond to $\cos(\theta)=\pm1$. Eliminating v' we obtain a quadratic equation for q^2 which gives

$$q_{\rm min}^{\rm max}=\mu v\left(1\pm\sqrt{1-\frac{2E_{\rm exc}}{\mu v^2}}\right), \quad (3)$$

where $\mu=M_XM_f/(M_X+M_f)$ (we can neglect the small difference between M_A and M_f here) is the reduced mass. Thus, for the inelastic process to occur at all, we must have $E_{\rm exc}<\mu v^2/2$. (Note that $\mu v^2/2$ is less than the neutralino kinetic energy, since $\mu<M_X$.) To obtain the scattering rate of neutralinos with some velocity distribution at a fixed momentum

transfer q (or recoil energy E_{recoil}), we have to integrate over the velocity distribution from minimum velocity

$$v_{\min}=\frac{q}{2\mu}+\frac{E_{\mathrm{exc}}}{q}. \tag{4}$$

At the same time, for inelastic scattering there is an absolute minimum of momentum transfer, $q=\sqrt{2\mu E_{\mathrm{exc}}}$.

Turning to the nuclear matrix elements that govern the cross section, we have, from Eqs. (4.24) and (4.25) of Ref. [2] (generalized to transitions from $J\to J'\neq J$),

$$\frac{d\sigma}{dq^2}=\frac{8G_F^2}{(2J+1)v^2}S_S(q), \tag{5}$$

where the form factor for initial and final states of the same parity[1] is

$$S_S(q)=\sum_{L\ \mathrm{even}}|\langle J'||\mathcal{C}_L(q)||J\rangle|^2, \tag{6}$$

and

$$\mathcal{C}_{LM}(q)=\sum_i c_0 j_L(qr_i)Y_{L,M}(\hat{r}_i). \tag{7}$$

The summation over L is restricted by $|J-J'|\leqslant L\leqslant J+J'$ and the lowest allowed L generally contributes most. For appropriate values of J and J' this value will correspond to the $L=2$ quadrupole mode, which also has the advantage of producing collective excitations of the nuclear surface; we denote the associated form factor by $S_2(q)$. We have lumped into the constant c_0 all the particle physics aspects of the problem except the overall scaling G_F^2. In the ratio of inelastic to elastic form factors the constant c_0 drops out.

To calculate the matrix elements in Eq. (7) we have to know something about the structure of the initial and final states. The $q\to 0$ limit of the matrix element in Eq. (7) for $L=2$ can be measured in the Coulomb excitation or electromagnetic decay of the excited state. The rates of these processes are usually expressed in terms of the quantity

$$B(E2,J\to J')=|\langle J'||er^2Y_2||J\rangle|^2/(2J+1). \tag{8}$$

Let us first consider the attractive $9/2^+\to 5/2^+$ excitation in ^{73}Ge. That isomeric excited state at 13 keV has a long half-life (2.95 μs) and a rather large $B(E2)$ (23 Weisskopf units for the γ-decay transition $5/2^+\to 9/2^+$). We make one crude but reasonable assumption here: that the transition density for the excitation is concentrated at the nuclear surface, as if the excited state were a vibration. Then we have

$$B(E2,J\to J')\simeq e^2\rho_0^2R^4\langle A_{\mathrm{ang}}\rangle^2/(2J+1), \tag{9}$$

[1]One might imagine $E1$-like transitions between low-lying states of opposite parity, but for nuclear-structure reasons their strengths are notoriously small.

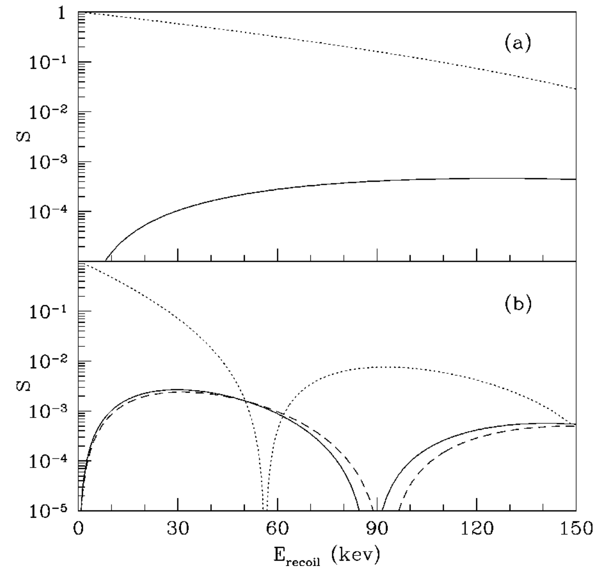

FIG. 1. The quantities $S=\sigma(q)/\sigma_{\mathrm{elastic}}(q=0)$ for elastic (dotted lines) and inelastic (full lines) neutralino scattering. The upper panel (a) is for ^{73}Ge and the lower panel (b) for ^{169}Tm. The dashed line in (b) is the inelastic S evaluated with Eq. (11), which is less accurate than Eq. (19).

where R is the nuclear radius, ρ_0 is the proton density, and A_{ang} is the matrix element of the angular factors.

With the same assumptions we can write the form factor $S_2(q)$ for the inelastic neutralino $J\to J'$ scattering as

$$S_2(q)\approx c_0^2|\langle J'||j_2(qr)Y_2||J\rangle|^2=c_0^2\left(\frac{A}{Z}\rho_0\right)^2 j_2(qR)^2\langle A_{\mathrm{ang}}\rangle^2, \tag{10}$$

where the factor A/Z comes from the additional assumption that the neutron and proton densities are proportional. Using the known $B(E2)$ we can rewrite the above as

$$S_2(q)=c_0^2\frac{A^2}{Z^2}(2J+1)j_2(qR)^2\frac{B(E2)}{e^2R^4}. \tag{11}$$

The S_2 form factor can then be compared to the form factor for elastic scattering, which is governed by the operator $\mathcal{C}_{00}\equiv c_0\Sigma_i j_0(qr_i)Y_{00}(\hat{r}_i)$. A constant density inside the nuclear radius and the relation

$$\int_0^R j_0(qr)r^2dr=\frac{R^2}{q}j_1(qR) \tag{12}$$

give

$$S_{el}(q)=c_0^2(2J+1)^2A^2\frac{9j_1(qR)^2}{4\pi(qR)^2}. \tag{13}$$

The ratio of inelastic to elastic cross sections, $S_2(q)/S_{el}(q)$, from Eqs. (11) and (13), is independent of the constant c_0.

Figure 1 (the upper panel) shows the elastic and inelastic form factors as a function of the recoil energy E_{recoil}, normalized to the elastic form factor at $q=0$ (i.e., $E_{\mathrm{recoil}}=0$).

The inelastic form factor in fact begins at a finite $E_{\rm recoil}$ related to the minimum momentum transfer in Eq. (3). The largest $E_{\rm recoil}$ we consider, 140 keV, corresponds to neutralinos of mass ≈ 60 GeV, (the mass indicated by a recent experiment [8]) moving with the galactic escape velocity, 650 km/s. For inelastic scattering, Eq. (3) restricts the $E_{\rm recoil}$ to less than about 127 keV. At low recoil energies, $E_{\rm recoil} \leqslant 30$ keV, the inelastic form factor is small because the spherical Bessel function $j_2(x)$ is proportional to $x^2/15$ for small x. Even at larger recoil energies, however, the inelastic form factor is down from the elastic one by a factor of 100–1000. Only near the zero of the function $j_1(x)$, which corresponds to $E_{\rm recoil} \sim 220$ keV in Ge, is the inelastic cross section larger than the elastic one. The small inelastic cross section is caused by the absence of the coherence factor A^2 [which appears divided by Z in Eq. (11) only to renormalize the density]. The collectivity of the $E2$ transition, which as noted above is restricted to the nuclear surface, cannot fully compensate this loss. Thus, while the sharp γ ray in the signal is undeniably beneficial, the expected count rate is substantially smaller than in elastic scattering. To further quantify this statement we evaluate the total elastic and inelastic cross sections for neutralinos with $M_X = 60$ GeV and a Maxwellian velocity distribution ($\overline{v} = 220$ km/s) terminated at the galactic escape velocity (650 km/s). The result for an ideal detector is

$$\frac{\langle \sigma^{\rm inelastic} \rangle}{\langle \sigma^{\rm elastic} \rangle} = 2.8 \times 10^{-5}. \tag{14}$$

A real detector will have some threshold in recoil energy below which it is not sensitive. The elastic form factor is largest at low recoil while the inelastic form factor is completely negligible there; excluding events with energies below the lower limit will therefore increase the ratio above. In a detector with a 10 keV threshold, the ratio is

$$\frac{\langle \sigma^{\rm inelastic} \rangle}{\langle \sigma^{\rm elastic} \rangle}_{\text{from 10 keV}} = 5.7 \times 10^{-5}, \tag{15}$$

still a rather small number.

Are there circumstances in which the reduction is not so dramatic and an experiment more desirable? For this to be the case, there must exist a low-lying (not much more than 20 keV) excited state with a very collective $E2$ transition. This state must live sufficiently long so that its deexcitation can be separated in time from the signal caused by the recoil kinetic energy. Finally, to eliminate the need for isotope enrichment, the target nucleus should be the only stable isotope of the element it represents.

A quick search of the Table of Isotopes [9] reveal that these conditions are not so easy to fulfill. In fact, we found only one nucleus, ^{169}Tm, that comes close. Its rotational $3/2^+$ state at 8.4 keV has a half-life of 4.1 ns and a very collective $B(E2; 3/2^+ \rightarrow 1/2^+_{g.s.})$ of 226 Weisskopf units. Detecting inelastic scattering to this state will be difficult; its excitation energy is too low and its half-life too short. Nevertheless, we evaluated the corresponding form factor to see what kind of count rates we could expect.

In nuclei with permanent deformation the $B(E2)$ values are related to the expectation value of $r^2 Y_{20}$ in the intrinsic frame of the nucleus, which in turn follows from the deformation parameter β:

$$\langle r^2 Y_{20}^{\rm intr} \rangle = \frac{3ZeR_0^2}{4\pi} \beta \left(1 + \frac{2}{7} \sqrt{\frac{5}{\pi}} \beta + \cdots \right). \tag{16}$$

We can also write the intrinsic-frame expectation value of the operator $\mathcal{C}_{20}$ in Eq. (7) in terms of β:

$$\langle \mathcal{C}_{20}^{\rm intr}(q) \rangle = \frac{3Ac_0}{4\pi} \beta \left(j_2(qR_0) + \frac{1}{14} \sqrt{\frac{5}{\pi}} \times [qR_0 j_1(qR_0) - j_2(qR_0)] \beta + \cdots \right). \tag{17}$$

To relate the inelastic form factor to the $B(E2)$ value, we use the expressions for rotational states:

$$B(E2; J,K \rightarrow J',K) = \langle r^2 Y_{20}^{\rm intr} \rangle^2 \langle JK20 | J'K \rangle^2, \tag{18}$$

and

$$S_2(q; J,K \rightarrow J',K) = \langle C_{20}^{\rm intr}(q) \rangle^2 (2J+1) \langle JK20 | J'K \rangle^2. \tag{19}$$

(The quantum number K, the angular-momentum projection on the nuclear symmetry axis, is 1/2 for ^{169}Tm.) To leading order in β, these relations give the same result as Eq. (11). The terms of order β^2 supply about a 10% correction in ^{169}Tm, which has $\beta \approx 0.3$.

The form factors for ^{169}Tm appear in the lower panel of Fig. 1. The maximum recoil energy for a 60 GeV neutralino with the galactic-escape velocity is now 112 keV. The inelastic form factor, as expected, is not as suppressed compared to its elastic counterpart as in ^{73}Ge; the factor is less than 100 in the broad maximum of the inelastic form factor at ≈ 30 keV recoil energy. The ratio of the total cross sections integrated from the lowest possible momentum transfer is now

$$\frac{\langle \sigma^{\rm inelastic} \rangle}{\langle \sigma^{\rm elastic} \rangle} = 1.5 \times 10^{-3}, \tag{20}$$

and increases to

$$\frac{\langle \sigma^{\rm inelastic} \rangle}{\langle \sigma^{\rm elastic} \rangle}_{\text{from 10 keV}} = 5.9 \times 10^{-3} \tag{21}$$

when integrated from a 10-keV threshold. To relate these results to those in Ge, one must recall that the normalizing factor, $S_{\rm el}(q=0)$, scales like A^2, i.e., it is larger for ^{169}Tm than for ^{73}Ge by $(169/73)^2$. With a 10 keV threshold, the integrated inelastic cross section per kg of material in ^{169}Tm is therefore suppressed with respect to the elastic cross section in ^{73}Ge by less than 100, a number that may not be so intimidating.

In conclusion, we have examined the neutralino inelastic scattering to collective states with large $B(E2)$ values. We have shown how to evaluate the form factors and presented examples. While the search for inelastic neutralino scattering offers an opportunity to suppress most background, it also leads to a considerable reduction of the expected signal.

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[1] G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rep. **267**, 195 (1996); J. R. Primack, D. Seckel, and B. Sadoulet, Annu. Rev. Nucl. Part. Sci. **38**, 751 (1988); P. F. Smith and J. D. Lewin, Phys. Rep. **187**, 203 (1990).

[2] J. Engel, S. Pittel, and P. Vogel, Int. J. Mod. Phys. E **1**, 1 (1992).

[3] R. Gaitskell (private communication).

[4] F. Avignone (private communication).

[5] J. Ellis, R. A. Flores, and J. D. Lewin, Phys. Lett. B **212**, 375 (1988).

[6] H. Ejiri, K. Fushimi, and H. Ohsumi, Phys. Lett. B **317**, 14 (1993).

[7] M. Drees and M. M. Nojiri, Phys. Rev. D **47**, 4226 (1993).

[8] R. Bernabei *et al.*, Phys. Lett. B **424**, 195 (1998).

[9] *Table of Isotopes*, 7th. ed., edited by C. M. Lederer and V. S. Shirley (Wiley, New York, 1978).