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Scattering of neutralinos from niobium

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We calculate cross sections for the scattering of neutralino dark-matter candidates from ^{93}Nb at all potentially relevant values of the momentum transfer q. The results will aid the interpretation of data from niobium dark-matter detectors.

Cold dark matter may exist in the form of heavy supersymmetric fermions called neutralinos [1]. Traveling at $v \approx 10^{-3}c$ relative to the earth and weighing $\approx$ 10–1000 GeV, these particles will transfer only keV's of energy when they hit a nucleus in a terrestrial detector. The development of technology sensitive to such low recoil energies is proceeding rapidly at several locations [2]. Some detectors will be supercooled so that they can measure the small rise in temperature or other macroscopic changes produced by a recoiling nucleus. Silicon and germanium are each the basis of detector programs that are well underway. Another element – niobium – is also being studied [3].

In order to evaluate the results of these future experiments, we need to understand the interaction of hypothesized particles with detector nuclei. Neutralino cross sections depend on the distribution of spin in the detector nucleus, an observable that is difficult to model accurately because to lowest order nucleons move as pairs with no net spin. Silicon is light enough so that shell-model calculations or a straightforward argument from existing data [4] can reliably predict deviations from this simple picture. Germanium, on the other hand, is so complicated in structure that a good calculation appears very difficult. Niobium is an intermediate case: heavy but with few enough valence protons and neutrons to make a relatively simple shell-model treatment sensible. Here we calculate cross sections for neutralinos on ^{93}Nb.

A few basic facts about the elastic scattering of neutralinos are important: The most general neutralino in the minimal supersymmetric extension of the standard model [5] is a linear combination of Bino Wino and two higgsinos, written as

$$\chi = Z_1 \tilde{B} + Z_2 \tilde{W}^3 + Z_3 \tilde{H}_1 + Z_4 \tilde{H}_2 , \quad (1)$$

where the coefficients Z_i depend on a few free parameters, and in turn determine the constants a_p and a_n that specify the strength of the "spin-dependent" neutralino couplings to protons and neutrons. The cross section also contains a "spin-independent" piece that to good approximation does not interfere with the spin-dependent cross section. The relative size of the two pieces is a function of model parameters that are not a priori determined; spin-independent scattering is apparently significant only in heavy nuclei [6]. Finally, as was first pointed out in ref. [7], if the neutralinos are heavier than 30 or 40 GeV, the three-momentum transfer q can be large enough so that both kinds of scattering will be suppressed by form factors that depend on the distribution of nucleons inside the nucleus. Recent data from LEP constrain light neutralinos and make this possibility likely [6,8]. The framework for calculating the form fac-

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tors was presented in ref. [9] and will be applied here to ^{93}Nb.

Spin-independent scattering is a coherent process and therefore easy to model; its form factor, the Fourier transform of the ground state nuclear density (under the assumption of proportional proton and neutron densities), is well approximated [9] by the function

$$F_a(q) = \frac{3j_1(qR_0)}{qR_0} \exp[-\tfrac{1}{2}(qs)^2] , \qquad (2)$$

with $s=1$ fm, and $R_0^2 = R^2 - 5s^2$ ($R = 1.2A^{1/3}$ fm). The square of this form factor for ^{93}Nb is plotted up to $q_{max} = 2M_{Nb}v$ in fig. 1, alongside that of a simpler (but more convenient) gaussian approximation [7], $F_b(q) = \exp[-\frac{1}{10}(qR)^2]$.

Spin-dependent scattering is much subtler because it takes place largely near the nuclear Fermi surface, where it is affected by the details in the behavior of relatively few nucleons. In the simplest picture, the single-particle model, only the last nucleon contributes to the cross section. Ref. [4] points out that this picture is often inadequate and describes an "odd-group" model in which the single-particle assumption is relaxed and the proton and neutron spins, which determine neutralino cross sections at $q=0$, are related to measured magnetic moments. Unfortunately, the technique cannot be simply extended to non-zero q, and a more detailed description of the nuclear wave function must be employed. Ref. [9] uses a BCS-based model of the Fermi surface to compute the spin-dependent cross sections for ^{131}Xe. Even that method cannot be applied in ^{93}Nb because the valence shells contain only a few nucleons and pairing correlations are therefore much weaker. For precisely this reason, however, the dynamics of the valence nucleons can be naturally described by a straightforward shell-model diagonalization [10]. Large single-particle gaps occur at Z, $N=38$ and (especially) 50 and to a reasonable approximation only three valence protons and two valence neutrons in ^{93}Nb need be considered [#1].

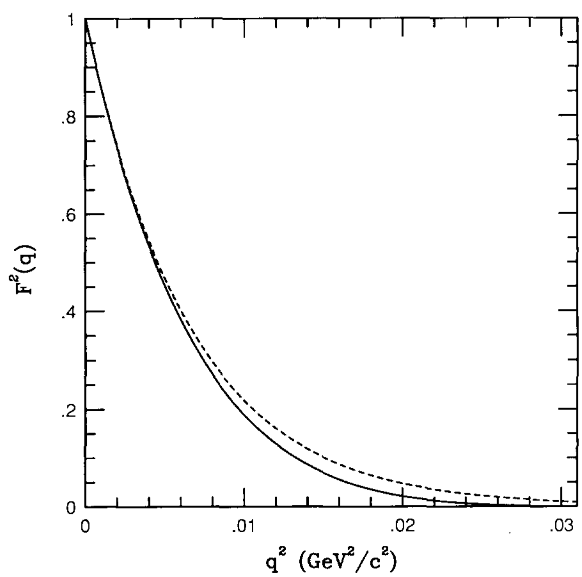

Fig. 1. The squares of the form factors $F_a(q) = [3j_1(qR_0)/qR_0] \times \exp[-\frac{1}{2}(qs)^2]$ (solid line) and $F_b(q) = \exp[-\frac{1}{10}(qR)^2]$ (dashed line) as a function of q^2 for ^{93}Nb, with R, R_0, and s as defined in the text.

With this in mind we construct a basic shell-model space (to be referred to as "the small space" as it contains only twenty $\frac{9}{2}^+$ states) corresponding to three protons in the $1p_{1/2}$ or $0g_{9/2}$ levels and two neutrons in the $1d_{5/2}$ level. We determine the single-particle energies of these levels from nearby nuclei (^{89}Sr, ^{87}Sr, and ^{89}Y) with only one particle or hole outside the doubly magic ^{88}Sr core. We use harmonic-oscillator wave functions for the single-particle orbits, and a residual two-body surface-delta interaction with isoscalar and isovector strengths $A_0 = 0.6$ MeV and $A_1 = 0.35$ MeV, as given in ref. [12]; in the small space this force successfully models many features of the dynamics.

We would like to reproduce at least roughly the measured magnetic moment of the $\frac{9}{2}^+$ ground state. With the ingredients just outlined, free nucleon g-factors, and the aid of the OXBASH shell-model code [13], we obtain a magnetic moment $\mu = 6.36$ Bohr magnetons – to be compared with the single-particle moment 6.79 and the experimental moment 6.17. Additional quenching of our result will come from states outside the small space; it is well known [14] that the excitation of a single particle from the core can significantly affect μ even if barely present in the wave function. We therefore add to our basis all states

[#1] A recent discussion of the effects of nuclear deformation on magnetic moments in this region of the periodic table appears in ref. [11].

in which one proton or neutron is excited from the small space (or from the full neutron $0g_{9/2}$ level) to any level in the sdg shell. The resulting space has about 2700 states, some 1100 of which – those with three protons in $0g_{9/2}$ and one neutron excited to $0g_{7/2}$ – we are forced by complexity to treat in perturbation theory. (For the same reason, we omit about 650 states in which a neutron is promoted from $0g_{9/2}$ to $1d_{3/2}$; the size of the effects from configurations we do include leads us to believe that the resulting error is small.) In this "large space", the magnetic moment is 5.88, a value that is too small but not unrealistic considering that meson-exchange currents can renormalize orbital proton g-factors upward by about 10% [14], affecting the magnetic moment without altering the nuclear spins. Furthermore, though single-particle occupation numbers change somewhat when we omit selected levels (e.g., the $2s_{1/2}$), spin-dependent quantities like the magnetic moment change much less. The large-space wave functions is therefore our best estimate, and we use it in the discussion of neutralino scattering to follow.

As mentioned above, the cross section at $q=0$ for a given type of neutralino depends only on the total proton and neutron spins S_p and S_n. Our full calculation yields $S_p=0.46$, $S_n=0.08$. The corresponding small-space results, 0.48 and 0.04, provide an indication of the amount of nuclear-physics uncertainty in these numbers. Both results are surprisingly close to the naive single-particle predictions $S_p=0.5$, $S_n=0$. The nominally more sophisticated odd-group model, which assumes only that the neutrons carry no angular momentum and uses the experimental magnetic moment as input, yields [4] $S_p=0.36$, $S_n=0$! Something has gone wrong here in the odd-group model, even though the neutron spin S_n is indeed fairly small. The problem is related to the fact that ^{93}Nb is an odd-proton nucleus with a large ground-state angular momentum ($\frac{9}{2}$). Both S_p and the proton orbital angular momentum L_p contribute to the magnetic moment, and in this instance both are quenched. In the small space alone, in fact, one can easily show that they must both be quenched by the same multiplicative factor. Since L_p is so large, a reduction by a few percent from $L_p=4$ is enough to reproduce the experimental magnetic moment, which really is not so different from the single-particle moment. In the odd-group model, however, any quenching of S_p from $\frac{1}{2}$ must be compensated for by an increase in L_p, since J is fixed at $\frac{9}{2}$ and the neutrons are assumed to carry no angular momentum. S_p must therefore be reduced substantially – too much in fact – in order to obtain the experimental magnetic moment. The valence shell structure of ^{93}Nb makes it a sort of "worst case" for the odd-group model. In mirror nuclei with $A \lesssim 50$, where beta-decay data allow an accurate determination of S_p and S_n, the odd-group model is almost universally better than the single-particle model, and its failure in niobium is probably best viewed as an isolated event.

The expressions for the spin-dependent neutralino–nucleus cross sections at general q, to which we now turn, appear in some detail in ref. [9]. The differential cross section can be expressed in the form

$$\frac{d\sigma}{dq^2} = \frac{8G_F^2}{(2J+1)v^2} S(q) , \qquad (3)$$

where $S(q)$ is a structure function defined in ref. [9] that reflects the spatial distribution of spin in the nuclear ground state. Fig. 2 shows $S(q)$ for a higgsino on ^{93}Nb. The different curves represent the single-

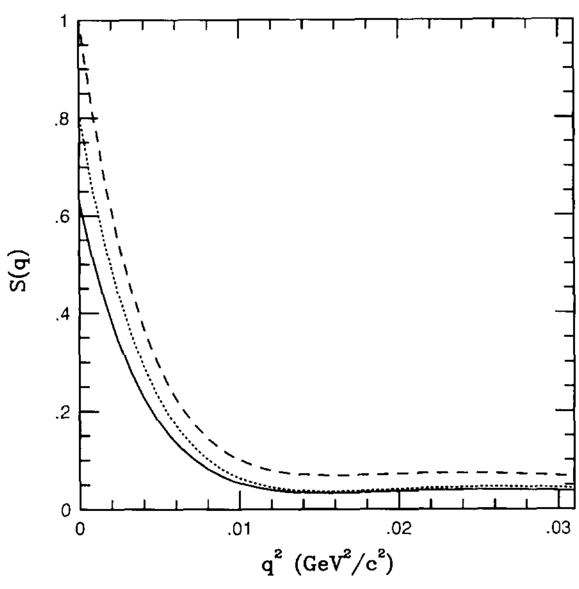

Fig. 2. The quantity $S(q)$ from eq. (3) versus q^2, for a higgsino on ^{93}Nb. The dashed line is the prediction of the single-particle model, the dotted line is the small-space prediction, and the solid line is the full result. The normalization has been adjusted so that the single-particle $S(0)=1$.

particle, small-space, and full results. To facilitate comparison with earlier work, the normalization has been adjusted so that the single-particle structure function is 1 at $q=0$. The small-space curve approaches the full result at large q because, as was the case [9] in ^{131}Xe, the additional components in the wave function are small and contribute significantly only at low q. We should note that, in contrast to the ^{131}Xe case, a simple rescaling of the single-particle result (by an amount that unfortunately is not well estimated by the odd-group model) is a good approximation to the full form factor.

The cross section for the most general neutralino can be constructed from the "partial structure functions" shown in fig. 3. One can always write the full structure function in the form [9]

$$S(q)=a_0^2\,S_{00}(q)+a_1^2\,S_{11}(q)+a_0a_1\,S_{01}(q)\,, \tag{4}$$

where $a_0=a_{\rm p}+a_{\rm n}$ and $a_1=a_{\rm p}-a_{\rm n}$ (we use isospin-scheme coefficients because virtual pions affect the isovector scattering so that S_{11} falls off faster than S_{00}). Interestingly, for an isoscalar neutralino ($a_1=0$) the cross section is actually a few percent larger everywhere than the single-particle prediction, since the magnitude of the neutron spin is slightly greater than the amount by which the proton spin is quenched. An isovector neutralino behaves more or less like the higgsino in fig. 2, which has [#2] $a_0/a_1\approx 0.12$.

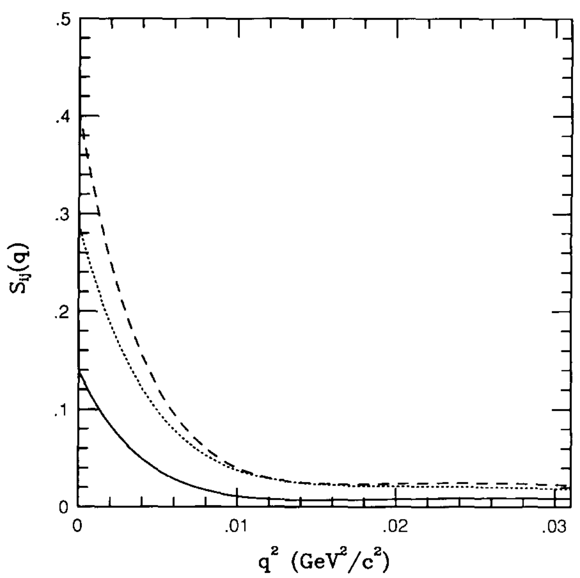

Fig. 3. The partial structure functions $S_{00}(q)$ (dotted line), $S_{11}(q)$ (solid line), and $S_{01}(q)$ (dashed line) versus q^2 in ^{93}Nb. When $a_0=-a_1$ the cross section is small because the neutrons are mostly in angular-momentum zero pairs.

Many of the above conclusions are peculiar to ^{93}Nb; the variety of structural features in the periodic table makes it difficult to generalize about the form factors. The following few remarks, though, should apply essentially everywhere: Only nucleons near the Fermi surface contribute significantly to spin-dependent scattering. Because their orbits extend further out than those in the core (this feature was not captured by the infinite square-well model of ref. [6]), the form factor near $q=0$, which reflects the mean square radius of the contributing nucleons, will always fall off faster for spin-dependent scattering than for spin-independent scattering. By the same token though, the Fermi-surface nucleons have higher momentum on the average, and the spin-dependent form factors will therefore be larger at high q than their spin-independent counterparts.

We can demonstrate these points explicitly in a simple one-dimensional model in which an odd number A of nucleons move independently in a harmonic oscillator potential with length parameter $b\equiv 1/\sqrt{m\omega}$, and couple their spins pairwise to zero. In one dimension the momentum transfer q takes the value $2M_{\rm r}v$ where $M_{\rm r}$ is the neutralino–nucleus reduced mass. Neglecting pionic currents [9], the spin-dependent cross section – normalized for convenience to 1 at $q=0$ – is just the square of the form factor for the last nucleon

$$G_n(q)=\langle n|\exp(\mathrm{i}qx)|n\rangle$$
$$=\exp[-\tfrac{1}{4}(bq)^2]L_n^0(\tfrac{1}{2}(bq)^2)\,, \tag{5}$$

where $n=A-1$ (so that $|n\rangle$ is the Ath oscillator level) and L_k^α is a Laguerre polynomial. The spin-independent form factor is obtained by summing over all occupied orbitals:

[#2] This result assumes EMC values for the spin content of the proton and neglects squark exchange, which is important only if the higgsino is purely symmetric or antisymmetric [15].

$$F_n(q)=\frac{1}{n+1}\sum_{m=0}^{n}G_m(q)$$
$$=\frac{1}{n+1}\exp[-\tfrac{1}{4}(bq)^2]L_n^1(\tfrac{1}{2}(bq)^2)\,. \qquad (6)$$

With these results we have

$$\frac{|G_n(q)|^2}{|F_n(q)|^2}=1-\tfrac{1}{4}(2n+1)(bq)^2+O(q^4) \qquad (7)$$

for small q, and

$$\frac{|G_n(q)|^2}{|F_n(q)|^2}\to(n+1)^2=A^2 \qquad (8)$$

for $q\to\infty$, both as expected. Eq. (8) can be understood as a loss of coherence in the spin-independent form factor. At very high q, F_n (like G_n) depends only on the motion of the last nucleon and, because of the normalization at $q=0$, is down by $1/A$ from G_n.

In three dimensions the two effects illustrated by eq. (7) and eq. (8) compete in determining the total cross section and average recoil; these quantities and more subtle features of the form factors can only be obtained by addressing each nucleus individually. ^{93}Nb is relatively tractable. Many other heavy nuclei are more complicated in structure and still await an adequate treatment.

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