Quantization of Asymmetric Shapes in Nuclei

J. Engel and F. Iachello
A. W. Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut 06511
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We propose a description of asymmetric shapes in nuclei in terms of a bosonic U(16) group, and discuss some properties of rotation-vibration spectra in the SU(3) limit of the model.

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Even-even nuclei in the mass region $A \approx 226$ display low-lying collective states of negative parity. Several explanations in terms of reflection asymmetry in the ground state have been put forth to account for the properties of these nuclei, including one involving the separation of nuclei into two unequal clusters.[1] Recently, Leander *et al.*[2] have concluded from a study of odd-even nuclei in the same mass region that the reflection asymmetry is generated by an octupole-shaped nuclear mass distribution. The purpose of this note is to present a possible scheme for the *quantitative* description of even-even nuclei with octupole shapes. By comparing the predictions inherent in this scheme with those of the cluster model,[1] one may be able to determine which interpretation best describes the experimental data.

Octupole shapes ($\lambda^P = 3^-$) may be described classically by parametrization of the nuclear radius in the form[3]

$$R = R_0[1 + \textstyle\sum_\mu \alpha_{3\mu} Y_{3\mu}(\theta,\phi)]. \quad (1)$$

In practice these shapes are always superimposed on the usual symmetric deformation of multipolarity $\lambda^P = 2^+$ (quadrupole). The presence of octupole deformation causes a shift in the nuclear center of mass that must be balanced by addition of a dipole deformation $\lambda^P = 1^-$, so that in realistic situations

$$R = R_0[1 + \textstyle\sum_\mu \alpha_{1\mu} Y_{1\mu}(\theta,\phi) + \sum_\mu \alpha_{2\mu} Y_{2\mu}(\theta,\phi) + \sum_\mu \alpha_{3\mu} Y_{3\mu}(\theta,\phi)], \quad (2)$$

where the α_1's are determined by the α_2's and α_3's and are, in lowest order, proportional to $\alpha_2\alpha_3$. It appears from experiment that the deformations are not rigid, but rather are soft against vibrations. Preliminary discussions of such behavior have been given by Rohozinski[4] in terms of Bohr-type Hamiltonians involving the α variables of Eq. (2). Here we present an alternative scheme, based on the introduction of bosonic quanta.

Quadrupole collective states have been described by consideration of a set of N interacting bosons having angular momentum and parity $J^P = 0^+$ and $J^P = 2^+$, called s and d, respectively.[5] The numbers of s and d bosons may vary so long as the total number N is conserved. We suggest that quadrupole-octupole collective spectra may be similarly described via a set of N bosons with angular momentum and parity $J^P = 0^+$, 2^+, 1^-, 3^-, which we denote by s, d, p, and f. In what follows, we discuss a particular limit of this model that may be appropriate for the description of strong axially symmetric deformation.

We begin by neglecting the quadrupole degree of freedom and considering a set of N s, p, and f bosons. It is noteworthy that, contrary to our initial expectations, we were unable to produce rotational bands with energies increasing with angular momentum as $L(L+1)$ using s and f bosons alone. [The s,f model, which naturally leads to bands with energies increasing as $L(L+15)$, may be of interest in molecular physics.] However, incorporating a p boson makes the production of rotational bands with $L(L+1)$ behavior straightforward. To show why this is so, we note that this model has a dynamical U(11) group, 11 being the dimension of the space spanned by the s, p, and f boson operators $(1+3+7)$. States are therefore characterized by the totally symmetric representations $[N]$ of U(11). A study of that group reveals the existence of, among others, a dynamical symmetry corresponding to the group chain

$$U(11) \supset U(10) \supset SU(3) \supset O(3) \supset O(2). \quad (3)$$

Standard techniques allow one to construct Hamiltonians with eigenvalues

$$E(N,N_b,\omega_b,\lambda_b,\mu_b,K_b,L,M) = \alpha + \beta N_b + \gamma N_b^2 + \kappa_b C(\lambda_b,\mu_b) + \kappa' L(L+1), \quad (4)$$

with

$$C(\lambda,\mu) = \lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu, \quad (5)$$

where α, β, γ, κ_b, and κ' are arbitrary constants and N, N_b, ω_b, λ_b, μ_b, K_b, L, and M label the states in chain (3). N is the total number of bosons, N_b is the number of negative-parity $(p+f)$ bosons, (λ_b,μ_b) label the SU(3)

representations, L and M are the angular momentum and its z component, respectively, and ω_b and K_b are additional quantum numbers needed to completely specify the states. In determining the allowed values of the quantum numbers (λ_b,μ_b) one needs the branching rules for $U(10)\supset SU(3)$; these have been tabulated in part by Elliott[6] and will not be given explicitly here. The spectrum resulting from Eq. (4) resembles that associated with a strongly deformed, axially symmetric, reflection-asymmetric shape, except for the fact that the parity doubling expected from simple geometric arguments[2] does not occur. Since positive-parity states arise from the manifold with N_b even, and negative-parity states from that with N_b odd, bands of opposite parity are displaced with respect to one another. Though exact parity doubling does not occur in real even-even nuclei, we may obtain it here, if we wish, by introducing signature-dependent interactions, which act differently on states with N_b even and N_b odd. Allowing these, we may modify Eq. (4) to

$$E(N,N_b,\omega_b,\lambda_b,\mu_b,K_b,L,M) = \alpha+\beta N_b+\gamma[1-(-)^{N+N_b}/N_b]N_b^2+\kappa_b[1-(-)^{N+N_b}/N_b]C(\lambda_b,\mu_b)+\kappa' L(L+1). \quad (6)$$

The low-lying part of the spectrum of Eq. (6) is shown in Fig. 1. Particularly interesting is the structure of the lowest (parity doubled) vibrations; they have quantum numbers $K^P=0^\pm,2^\pm$ and $K^P=1^\mp,3^\mp$, and we have called them σ, δ, and π,ϕ, respectively. In a reflection-symmetric body, represented by s and d bosons, the bands with $K^P=1^+$ and 3^+ as well as all those with negative parity are missing. Of the bands above, only the $K^P=0^+,2^+$ are present.

We now proceed to a discussion of a more realistic case, including d bosons along with the s, p, and f bosons. The dynamical group is now U(16); states are characterized by the totally symmetric representations $[N]$ of that group and can be written as linear combinations of configurations of the type $(s,d)^{N_a}(p,f)^{N_b}$, where $N_a+N_b=N$. The model has the interesting feature that both the s,d and p,f subgroups U(6) and U(10) contain the same subgroup chains U(5), SU(3), and O(6).[7] Since we are interested here in describing axially symmetric rotations, we discuss only one of these chains. Denoting by $SU(3)_a$ and $SU(3)_b$ the groups built from (s,d) and (p,f) boson operators, we consider the chain

$$U(16)\supset U(6)\otimes U(10)\supset SU(3)_a\otimes SU(3)_b\supset SU(3)\supset O(3)\supset O(2), \quad (7)$$

where SU(3) denotes the group formed by addition of the generators of $SU(3)_a$ and $SU(3)_b$. As before, standard techniques allow one to construct Hamiltonians with eigenvalues

$$E(N,N_a,N_b,\omega_b,\lambda_a,\mu_a,\lambda_b,\mu_b,\lambda,\mu,K,L,M) = \alpha+\beta N_b+\gamma N_b^2+\kappa_a C(\lambda_a,\mu_a)+\kappa_b C(\lambda_b,\mu_b)+\kappa C(\lambda,\mu)+\kappa' L(L+1), \quad (8)$$

where the quantum numbers $N,N_a,N_b,\ldots$ label the states in the group chain (7). In addition to the branching rules for $U(6)\supset SU(3)_a$ and $U(10)\supset SU(3)_b$, we also need here rules for decomposing the tensor product $(\lambda_a,\mu_a)\otimes(\lambda_b,\mu_b)$. These can be obtained by consideration of the Young tableaux.[6]

As in the case discussed above, parity doubling does not occur naturally. If we wish its presence, we may again introduce signature-dependent interactions to yield

$$E(N,N_a,N_b,\omega_b,\lambda_a,\mu_a,\lambda_b,\mu_b,\lambda,\mu,K,L,M) = \alpha+\beta N_b+\gamma[1-(-)^{N+N_b}/N_b]N_b^2+\kappa_a[1+(-)^{N+N_b}/N_a]C(\lambda_a,\mu_a)+\kappa_b[1-(-)^{N+N_b}/N_b]C(\lambda_b,\mu_b) +\kappa[1-(-)^{N+N_b}/(3N_b+2N_a)]C(\lambda,\mu)+\kappa' L(L+1). \quad (9)$$

The spectrum corresponding to Eq. (9) is rather complex, and we show in Fig. 2 only a small portion of the vibrational bandheads. Note the presence of several distinct kinds of bands, corresponding to quadrupole vibrations, octupole vibrations, and relative oscillations of the quadrupole and octupole deformations. These are the bands that should appear experimentally if the nucleus has permanent octupole and quadrupole deformation.

We have shown how the U(16) model can generate decoupled rotation-vibration spectra, and analyzed the structure of its vibrational modes in this limit. The results shown in Figs. 1 and 2 represent idealized situations, and the obtaining of them has necessitated the use of somewhat cumbersome signature-dependent Hamiltonians. The main advantage of our model, however, is not so much that it can reproduce these idealized spectra, as that it may be used to treat quantitatively properties of real nuclei. Since these do not show exact parity doubling, the introduction of signature-dependent terms may not be necessary. In actual nuclei, the octupole deformations seem to be very soft. The potential,

$$V(\epsilon_3)=\tfrac{1}{2}C\epsilon_3^2+D[\exp-\epsilon_3^2/a^2)-1], \quad \epsilon_3=\alpha_{30}, \quad (10)$$

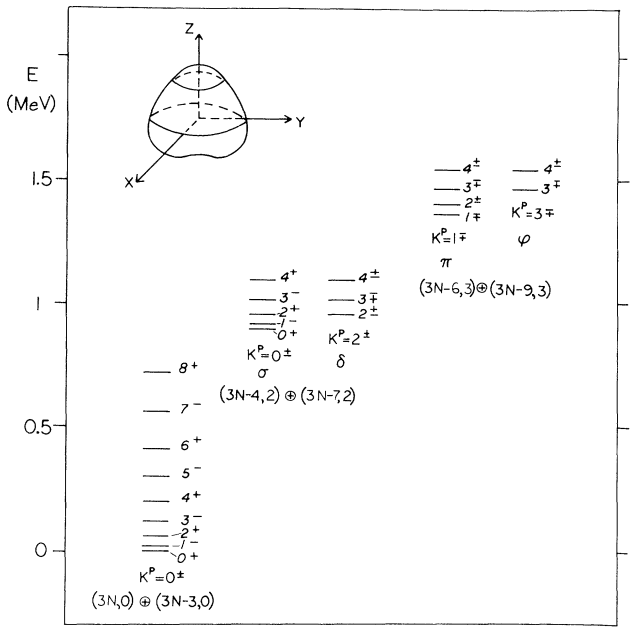

FIG. 1. Schematic representation of the spectrum of a nucleus with octupole deformation in the algebraic approach. The spectrum is calculated from Eq. (5) with $\alpha=\beta=\gamma=0$ in the large-N limit. The rotational constants are chosen to be $\kappa_b=(-50/N)$ keV and $\kappa'=10$ keV.

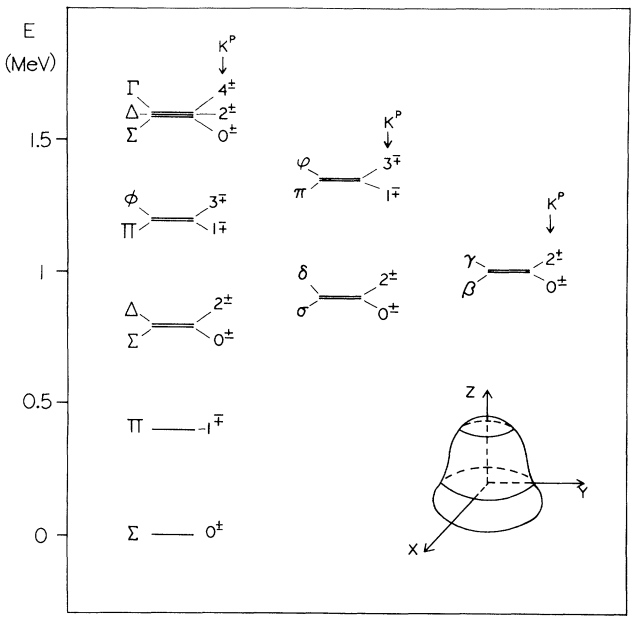

FIG. 2. Schematic representation of the vibrational spectrum of a nucleus with both quadrupole and octupole deformation in the algebraic framework. The spectrum is calculated from Eq. (9) in the large-N limit with appropriately selected parameters. There are three types of vibrations, with scales set by κ, κ_b, and κ_a. The first of these is labeled with molecular spectroscopic symbols ($\Sigma, \Pi, \Delta, \ldots$) in addition to the parity and value of K. The second type is labeled as in Fig. 1 and the third type according to the usual nuclear quadrupole notation (β and γ). Other sets of vibrational bands, corresponding to different values for N_a and N_b, have been pushed higher up in the spectrum and are not shown in the figure. On top of each vibrational state shown here, there is built a rotational band as in Fig. 1.

used in Ref. 2 corresponds in the algebraic framework to a transition from the rotational chain discussed above [U(10)⊃SU(3)] to the vibrational chain U(10)⊃U(3) ⊗ U(7). One of us (J.E.) has written a computer code to perform the calculations for these transitional Hamiltonians.

One question raised by our work is to what degree the model may be justified microscopically. In particular, the dipole collectivity implied by the p boson may be difficult to obtain once spurious states corresponding to center-of-mass motion are removed. Another issue is the extent to which our model can be related to the Bohr-type model of Rohozinski.[4] We are at present investigating the connection in detail. Finally, we hope that the availability of a technique to treat quantitatively both quadrupole and octupole degrees of freedom in even-even nuclei will stimulate further experimental and theoretical efforts to understand the precise nature of the low-lying negative-parity states in nuclei. At present, experimental information does not allow us to distinguish conclusively between the clustering[1] and octupole[2] interpretations; we feel that experimental work on the structure of the excited vibrational bands can help resolve the question.

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