Effective Lagrangians and parity-conserving time-reversal violation at low energies

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Using effective Lagrangians, we argue that any time-reversal-violating but parity-conserving effects are too small to be observed in flavor-conserving nuclear processes without dramatic improvement in experimental accuracy. In the process we discuss other arguments that have appeared in the literature. [S0556-2821(96)06509-7]

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The discrete spacetime symmetries of parity (P) and time reversal (T) have played a crucial role in our understanding of fundamental interactions. Parity violation is a general feature of weak interactions, observed in a wide range of phenomena. By contrast, time-reversal violation has been seen only in the neutral kaon system. Yet measurements in the kaon system alone are insufficient to determine whether the Kobayashi-Maskawa (KM) [1] mechanism of explicit T violation is operating or whether extra-standard-model physics is at play. Even the B-meson factories under construction may not be able to tell us if the source of T violation is really the KM mechanism.

The possibility that T violation might arise outside the standard model has motivated a number of recent low-energy (MeV range or less) experiments. These measurements, which do not test KM-based T violation but may be sensitive to other sources, are classified according to whether or not the measured observables violate P as well as T [2]. Electric-dipole moments, both of elementary particles and atoms, are T-violating and P-violating (TVPV) observables. The quantities we focus on here are T violating but P conserving (TVPC) and flavor-conserving. They include correlations both in γ decay [3] and neutron scattering [4] as well as quantities extracted from nuclear tests of detailed balance [5]. Some observables in β decay [6] are TVPC but are flavor changing, and will not be considered here.

The reason the TVPC experiments [2] are interesting is that limits on the quantities they measure are still quite weak (much weaker than the limits on similar TVPV quantities), raising the possibility that TVPC effects could be relatively large. The experiments are not very sensitive in part because of the inability of a single pion, which is largely responsible for the strong force between nucleons in a nucleus, to transmit a TVPC interaction [7]. Though the experiments are improving, the best published limit on the effective TVPC coupling of the nucleon to the ρ, the lightest relevant meson, is still only about 10^{-2} times the normal strong ρNN coupling [8,9].

Is it possible that large T violation from outside the standard model is lurking just below current limits, that a TVPC effect could appear at 10^{-3} or 10^{-4} times the strong coupling g? Prior work has addressed this and related issues. Herczeg *et al.* [10] have shown that in any renormalizable gauge theory, with the θ_{QCD} term neglected, Feynman graphs representing a TVPC flavor-conserving quark-quark interaction must contain more than two non-QCD, non-QED vertices. This theorem suggests that low-energy TVPC effects will be strongly suppressed in theories that generate T violation through the weak coupling of heavy bosons to quarks. In other kinds of models, however, the theorem leaves the issue open. If, for example, the bosons that break T are strongly coupled and confined, the TVPC interaction might not be perturbative, and an examination of its graphical structure might not provide constraints. Conti and Khriplovich [11] have approached the issue in a different way, arguing that measured limits on electric dipole moments, which are TVPV, constrain TVPC vertices through graphs that also contain parity-changing Z bosons. They obtain a limit of 10^{-10} for the ratio of TVPC to strong interactions. Here we argue that TVPC effects are small but not necessarily that small. More specifically, under very conservative assumptions, regardless of how time-reversal invariance is broken, effective TVPC couplings lie at or below 10^{-8} times the strong coupling g.

To address these matters in a systematic fashion, we draw on effective field theory, which makes possible very general conclusions about low-energy phenomena despite our ignorance of physics at high energies [12]. Explicit dependence on any physics at scales much larger than a typical momentum transfer in a low-energy experiment can be removed from the full theory. This leaves an effective Lagrangian consisting of a sum of nonrenormalizable (dimension greater than four) operators involving standard-model fields. The effects of the high-energy physics appear in factors multiplying each of the operators in the effective Lagrangian. If the unknown physics is associated with some high-energy scale Λ, the effective Lagrangian that represents its low-energy limit can be written as

$$\mathscr{L}_{\mathrm{eff}}=\mathscr{L}_0+\frac{1}{\Lambda}\mathscr{L}_1+\frac{1}{\Lambda^2}\mathscr{L}_2+\cdots, \tag{1}$$

where each $\mathscr{L}_i$ contains a series of operators of dimension $i+4$, each multiplied by a dimensionless coefficient expected to be of order one. (Enhancements of coefficients beyond the expectations of ''naive dimensional analysis'' [13] are possible, however; an example is the $\Delta I = 1/2$ rule, where the coefficient responsible is about 20). The results of any calculation using this Lagrangian will be a power series in (p/Λ), where p is a typical momentum transfer in the process under consideration. If $p \ll \Lambda$, the series will in general converge, and truncating it at any order in $1/\Lambda$ provides a theory with a finite number of terms, predictive to that order in $1/\Lambda$.

To use this formalism to bound low-energy TVPC effects without knowing anything about physics at higher scales, we must first identify a maximum quark-gluon momentum p relevant to the experiments we consider and a minimum scale Λ with which T-violating physics could be associated. To be conservative we choose the proton mass, $m_p \approx 1$ GeV to be the momentum scale p. The appropriate value for Λ is less obvious, but it should be at least the mass of the Z^0. Undiscovered gauge particles could conceivably be lighter, but precision tests of the standard model impose strong constraints on the couplings and/or masses such particles can have. The contribution of any gauge boson of mass m coupled to up and down quarks (q) with strength f will contain a factor of f^2/m^2, which must be smaller than the analogous factor for Z^0 exchange to avoid conflict with the measurement of the partial width for $Z^0 \rightarrow \overline{q q}$ [14]. It therefore seems unlikely that particles can be lighter than about 100 GeV and still yield effects that are not extremely suppressed [15]. One can argue that Λ should be much larger, but the conservative estimate we use is $\Lambda \sim 100$ GeV.

To estimate the size of TVPC physical effects we must identify the lowest-dimensional TVPC operators in Eq. (1). There are no gauge-invariant TVPC flavor-diagonal operators of dimension six or less in standard-model fields [16]. Dimension-six $SU(2)_L$-noninvariant operators can in principle contribute to physical effects [17], but these operators all contain weak gauge bosons that suppress their contributions to low-energy processes beyond those of higher-dimensional operators. Therefore, it would appear that the largest local TVPC flavor-diagonal $SU(3)_C \times U(1)_Q$-invariant operators in the standard model have dimension seven. All such four-quark operators can be written in the form [18]

$$C_7 \left(\frac{1}{\Lambda}\right)^3 \overline{q_1} \gamma_5 D^\mu q_2 \overline{q_3} \gamma_5 \gamma_\mu q_4 + \text{H.c.}, \qquad (2)$$

where C_7 is a dimensionless constant expected to be of order one, and $q_1 = q_2$, $q_3 = q_4 \neq q_1$ or $q_1 = q_4$, $q_2 = q_3 \neq q_1$. (The Gordon decomposition can be used to replace D^μ by the expression $\sigma^{\mu\nu} q_\nu$.) There is also a quark-gluon-photon operator of the form

$$C_7' \left(\frac{1}{\Lambda}\right)^3 \overline{q} \sigma_{\mu\nu} \lambda_a q G_a^{\mu\rho} F_\rho^\nu, \qquad (3)$$

where $G_a^{\mu\rho}$ is the gluon field strength tensor, the λ_a's are color SU(3) matrices, and F_ρ^ν is the electromagnetic field strength tensor [19]. The existence of these operators suggests that the largest TVPC flavor-conserving term in the expansion of $\mathscr{L}_{\text{eff}}$ is in $\mathscr{L}_3$, and that experimental effects should occur at a scale of order $(p/\Lambda)^3$. With conservative estimates for p and Λ, this naive result implies that the effects of any flavor-conserving TVPC operator in low-energy experiments must be suppressed by at least 10^{-6} relative to strong interactions. For our order of magnitude estimates, we take the hadron-level interactions to have roughly the same strength as the corresponding quark-level interactions. This means, for instance, that $\overline{g_\rho}$, the ratio of the TVPC ρNN coupling to the strong ρNN coupling, is at most about 10^{-6}.

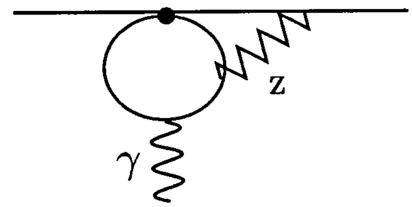

FIG. 1. One diagram that could give rise to a neutron EDM at matching. Solid lines are quarks, the jagged line is a Z boson, and the wavy line is a photon. The solid dot indicates the insertion of TVPC physics.

Reference [11] shows, however, that the stringent experimental limits on the neutron electric dipole moment (EDM) imply that the effects of the dimension-seven operators are even smaller. The diagram in Fig. 1 shows a potential contribution of TVPC physics to the low-energy dimension-five quark-EDM operator, where the Z-boson exchange makes the diagram P violating. Matching this diagram to the effective theory valid at EDM scales results in an estimate for the coefficient C_5 in the dimension-five EDM operator

$$\frac{C_5}{\Lambda} \overline{q} \sigma_{\mu\nu} \gamma_5 q F^{\mu\nu} \qquad (4)$$

on the order of

$$C_5 \sim \frac{4\pi\alpha}{(16\pi^2)^2} C_7 \sim 4 \times 10^{-6} C_7. \qquad (5)$$

The measured limit [20,21] on the neutron EDM, $d_n/e \lesssim 10^{-25}$ cm, gives $C_5 \lesssim 5 \times 10^{-10}$ and therefore implies that $C_7 \lesssim 10^{-4}$. This indirect bound on the magnitude of C_7 reduces the expected effects of the dimension-seven TVPC four-quark operators in flavor diagonal nuclear experiments to at most 10^{-10} the size of strong effects, well beyond the reach of current or anticipated experiments. The coefficient C_7' in Eq. (3) can be bounded at a similar level because the associated operator contributes to the neutron EDM via another two-loop diagram containing a Z boson.

The suppression of the coefficients in the dimension-seven operators, however, does not translate into an equivalent suppression of TVPC effects; larger ones can come from operators of dimension *eight*, provided they do not contribute significantly to the neutron EDM. An example of such an operator is

$$\frac{C_8}{\Lambda^4} \overline{q} \gamma_\mu \gamma_5 q \overline{q} \gamma_\nu \gamma_5 \lambda_a q G_a^{\mu\nu}, \qquad (6)$$

which represents the TVPC interaction of four quarks with a gluon. The interactions giving rise to this operator could contribute to the neutron EDM, but at a level much lower than in the case of the dimension-seven operators. The reason is that the dimension-eight operator does not itself flip chirality, and so when inserted into a diagram like that of Fig. 1 (with the end of the gluon line attached to a quark line), it must be accompanied by a quark mass on an external quark line to contribute to the chirality-changing dimension-five operator in Eq. (4). The coefficient C_8 can therefore assume its ''natural'' value of order 1 without generating a dipole moment larger than the measured limit (it cannot be significantly enhanced, however, without doing so). The suppression of operators that change chirality relative to those that do not is plausible; it may be that all chirality-changing operators at low energies originate from fermion-mass insertions in the full theory. Such insertions would add factors such as m_q/Λ (about 10^{-4} here) to the dimension-seven chirality-changing operators without affecting the dimension-eight operator in Eq. (6). The reasonableness of this scenario, and in particular the lack of an experimental constraint on C_8, means that the most natural bound on TVPC effects in low-energy flavor-conserving experiments compared to strong effects is not 10^{-6} or 10^{-10}, but $(m_p/\Lambda)^4 = 10^{-8}$.

Without knowing the source of physics beyond the standard model that may induce TVPC couplings, we may use an effective Lagrangian valid at low energies and estimate the size of its largest terms through dimensional analysis. Without any experimental input, we can conclude that since $\overline{g_\rho}$ is generated only by operators of at least dimension seven, it is very likely less than 10^{-6}. Existing limits on the neutron EDM appear to constrain the effects of these operators, however, so that the largest effects consistent with experiment arise from operators of dimension eight. This results in a natural upper bound on $\overline{g_\rho}$ of about 10^{-8}. (If a measurement of a TVPC effect were obtained between 10^{-6} and 10^{-8}, this would suggest that either the high-energy theory possesses some unknown symmetry, or that accidental cancellations prevent the TVPV physics from contributing to the neutron EDM.) Our estimates are conservative, and we conclude that a dramatic improvement in sensitivity is required for low-energy experiments to have a good chance of seeing TVPC effects.

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