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Boson fermion dynamical SU(3) symmetry for asymmetric deformation in odd mass nuclei

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We show that the SU(3) limit of the U(16) model used to describe asymmetric shape deformation in even-even nuclei may be simply extended to odd-mass nuclei. The extension results in decoupling parameters with opposite signs in opposite-parity bands. We discuss the model in connection with the spectrum of ^{255}Ra.

The nuclei in the Ra–Th region of the periodic table have become the object of increasingly intense experimental and theoretical investigation as the belief that they exhibit stable octupole deformed shapes has gained currency. Most of the theoretical studies thus far have considered the effect of octupole-shaped potentials in mean-field (e.g., Strutinsky or Nilsson) calculations.[1] Attempts are now underway[2,3] to incorporate static reflection-asymmetric deformation into the interacting boson approximation (IBA) model[4] as well, though thus far they have been confined to even-even nuclei. In this note, we discuss their extension to odd-mass nuclei in the framework of the interacting boson fermion approximation (IBFA) model.[5] We show that such an IBFA-type model naturally incorporates the well-known effect in which the decoupling parameters for positive and negative parity bands have opposite signs. We then discuss the possible relevance of such a model to the odd-mass ^{225}Ra nucleus.

The most successful boson description to date of asymmetric deformation in even-even nuclei[2,3] employs the dynamical group U(16), generated by one-body operators for bosons of type $s(L^\pi=0^+)$, $d(L^\pi=2^+)$, $p(L^\pi=1^-)$, and $f(L^\pi=3^-)$. The first two are the familiar ingredients of the usual IBA-1 model.[4] The f bosons are natural elements in any description of octupole shapes and have been used previously to model octupole *vibrations* in rare-earth nuclei.[6] The p bosons, while not *a priori* necessary, have been found to improve the ability of the model to describe, for example, the cascading $E1$ transitions and rotational splittings that characterize octupole-*deformed* nuclei. Furthermore, they allow the construction of analytically solvable Hamiltonians belonging to two distinct group chains—SU(3) and O(4)—that produce spectra with many of the qualitative features expected in octupole-deformed systems. The SU(3) limit was discussed in detail in Ref. 2.

In several recent works, the microscopic foundation of the U(16) model was investigated. Otsuka[7] carried out Nilsson+BCS calculations for particles moving in a field with both quadrupole and octupole deformations. The resulting intrinsic wave functions contained both $L=1$ pairs and $L=3$ pairs, in roughly equal amounts. Catara *et al.*[8] performed schematic shell model calculations in a model space involving $1f_{7/2}$ and $1d_{5/2}$ orbits, but restricted to the dominant $L=0^+$, 2^+, 1^-, and 3^- pairs. Their results showed that the $L=1^-$ pair plays an important role in producing negative parity bands with a rotational structure. Although questions remain as to whether the $L=1$ pairs that enter in these treatments are partially spurious, the results nevertheless suggest the possible relevance of a model involving both p and f bosons.

One advantage of the SU(3) that arises in this new context is its similarity to the SU(3) limit of the ordinary s-d IBA-1 model. The latter has been thoroughly explored in even-even nuclei, and in the past few years has been extended, via the IBFA model, to deformed odd-mass nuclei traditionally described by the Nilsson model.[9] The IBFA picture involves the coupling of a single fermion described by the group $U_f(m_F)$ to a boson *core*. If the fermion occupies certain orbits (e.g., $j=\frac{1}{2},\frac{3}{2},\frac{5}{2}$), then the fermion angular momentum can be decomposed[10,11] into *pseudo-orbital* and *pseudo-spin* $\frac{1}{2}$ pieces to yield

$$U_F(m_F)\supset U_F(\tfrac{1}{2}m_F)\times SU_F(2)\supset SU_F(3)\times SU_F(2)\ . \quad (1)$$

The $SU_F(3)$ group in (1) may then be coupled to the SU(3) subgroup of the boson U(6) core. As discussed in Ref. 11, the resulting boson-fermion dynamical SU(3) symmetry produces spectra similar to those associated with the particle-rotor model.

Exactly the same procedure may be used when the boson group is U(16) instead of U(6). The relevant group

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chain becomes

$$\begin{aligned} U_B(16)\times U_F(m_F) \supset U_B(16)\times U_F(\tfrac{1}{2}m_F)\times SU_F(2)\ , \\ \supset \cdots\ , \\ \supset SU_{B+F}(3)\times SU_F(2)\ , \\ \supset SO_{B+F}(3)\times SU_F(2)\ , \\ \supset Spin(3)\ . \end{aligned} \tag{2}$$

The resulting states again organize themselves into rotational bands, but now there are states with positive and negative parities; in fact, for each K^+ band there is an associated K^- band, just as in the even-even case. The Hamiltonian that governs rotational motion and thus the splittings within rotational bands is, as in Ref. 11, given by

$$H_{\rm ROT} = A_L \mathbf{L}\cdot\mathbf{L} + A_J \mathbf{J}\cdot\mathbf{J}\ , \tag{3}$$

where $\mathbf{L}$ is the total pseudo-orbital angular momentum operator and $\mathbf{J}$ is the total angular momentum operator. Each state in a given rotational band has simultaneously good L and J quantum numbers and thus has rotational energy

$$E_{\rm ROT} = A_L L(L+1) + A_J J(J+1)\ . \tag{4}$$

We now focus on the rotational energy in $K=\frac{1}{2}^+$ and $K=\frac{1}{2}^-$ bands, for which the decoupling parameters in the geometric Nilsson picture have opposite signs. To carry out the analysis, we will need to know the pseudo-orbital angular momentum content of these bands. Note that there are two classes of pseudo-orbital, pseudospin decompositions. For groups in which the pseudo-orbital angular momenta are even, e.g., $U_F(12)$, the leading SU(3) representation for one fermion is of the form $(\lambda_F,\mu_F)=(2n,0)$, where n is a positive integer. On the other hand, for groups in which the pseudo-orbital angular momenta are odd, e.g., $U_F(20)$, the leading SU(3) representation for one fermion is of the form $(2n-1,0)$. The pseudo-orbital angular momentum content of the $K=\frac{1}{2}^+$ and $K=\frac{1}{2}^-$ bands depends on which of the two classes of orbits describes the odd fermion.

In particular, for $K^\pi=\frac{1}{2}^+$ bands, L takes the values $0,2,4,\ldots$ if the fermion orbits involve even pseudo-orbital angular momenta and the values $1,3,5,\ldots$ if they involve odd angular momenta. If we then couple the pseudospin $\frac{1}{2}$ to these L values, we find the following relation between the total angular momentum of a given state and its pseudo-orbital angular momentum:

$$L = J + (-1)^{\lambda_F}\frac{(-1)^{J+1/2}}{2}\ , \tag{5}$$

where λ_F refers to the leading SU(3) representation, as described above. Inserting this into (4) yields the following expression for the rotational energy associated with $K=\frac{1}{2}^+$ bands:

$$\begin{aligned} E^J_{\rm ROT}(K=\tfrac{1}{2}^+) &= (A_L+A_J)J(J+1) \\ &\quad + (-1)^{\lambda_F} A_L (-1)^{J+1/2}(J+\tfrac{1}{2}) + \frac{A_L}{4}\ . \end{aligned} \tag{6}$$

The rotational energy includes a rotational kinetic energy term with moment of inertia

$$I = \frac{1}{2(A_L+A_J)}\ . \tag{7}$$

It also includes a term that is very similar in structure to the Coriolis decoupling term of the particle-rotor model. However, as discussed in Ref. 11, the operator whose matrix elements in the intrinsic basic produce this contribution to the rotational energy is the pseudo-Coriolis interaction and not the usual Coriolis interaction.

Next we consider $K^\pi=\frac{1}{2}^-$ bands. Here, the relation between L and J is slightly different. In these negative-parity bands, L takes the odd-integer values $1,3,5,\ldots$ if the fermion pseudo-orbital angular momenta are even and $0,2,4,\ldots$ if they are odd. Thus, instead of (5), we now have

$$L = J - (-1)^{\lambda_F}\frac{(-1)^{J+1/2}}{2} \tag{8}$$

and the rotational energy has the form

$$\begin{aligned} E^J_{\rm ROT}(K=\tfrac{1}{2}^-) &= (A_L+A_J)J(J+1) \\ &\quad - (-1)^{\lambda_F} A_L (-1)^{J+1/2}(J+\tfrac{1}{2}) + \frac{A_L}{4}\ . \end{aligned} \tag{9}$$

Note that the only difference between (6) and (9) is in the *sign* of the decoupling contribution to the rotational energy. As noted earlier, this is a well-known[9] feature of the Nilsson model; we see that it also emerges naturally in our SU(3) boson-fermion symmetry.

As discussed in detail in Ref. 11, decoupling is restricted to $K=\frac{1}{2}$ bands. Intrinsic bands with higher values of K undergo mixing due to the pseudo-Coriolis interaction but no decoupling.

We have searched for possible manifestations of this SU(3) boson-fermion dynamical symmetry in real odd-mass nuclear systems. The possible relevance of the U(16) model to even-even nuclei in the actinide region was discussed in Refs. 2 and 3. We consider the coupling of this space to a single neutron occupying the shell-model orbitals $4s_{1/2}$, $3d_{3/2}$, $3d_{5/2}$, $2g_{7/2}$, $2g_{9/2}$, and $1i_{11/2}$. We expect this model to reflect some of the features of odd-neutron strongly deformed actinide nuclei, for which octupolelike properties seem to be present at low energies.[12] The intruder $1j_{15/2}$ single-neutron orbital is also relevant in this region; its important contribution to the structure of the negative parity p and f bosons[13] is included in the description of the core. The resulting IBFA model contains an SU(3) boson-fermion symmetry limit of the type we have described. In Fig. 1, we show the spectrum of states in the lowest two bands of ^{225}Ra together with the results of a fit based on this symmetry. We focus on the rotational structure of the bands, which is governed by the rotational Hamiltonian (3), and thus we fit only the energy spacings within the bands. The band heads are determined by the intrinsic Hamiltonian and thus by Casimir operators from subgroups that occur prior to $SO_{B+F}(3)$ and Spin(3) in the group chain (2). There are five SU(3) Casimir operators that can enter and thus plenty of free-

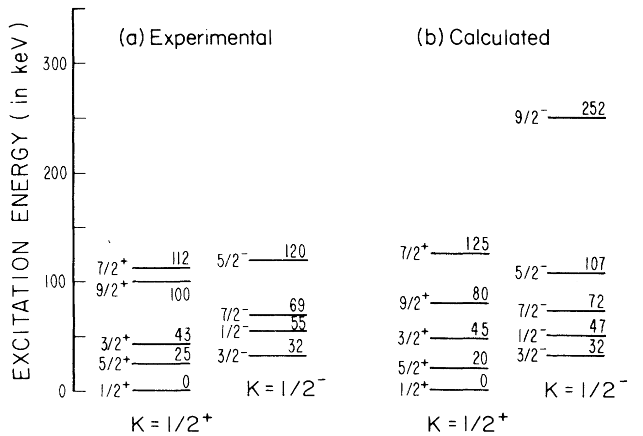

FIG. 1. Comparison of (a) experimental and (b) calculated spectra of low-lying bands in ^{225}Ra. The experimental spectrum is taken from Ref. 12. The calculated spectrum is based on the SU(3) limit of an IBFA model involving a U(16) boson core and the $4s_{1/2}$, $3d_{3/2}$, $3d_{5/2}$, $2g_{7/2}$, $2g_{9/2}$, and $1i_{11/2}$ single-neutron orbitals. The inertial parameters used to calculate the rotational spacings in the two bands are $A_L = 10$ keV and $A_J = -5$ keV.

dom to produce the observed splitting between the two bands. In contrast, all rotational spacings within the bands are determined by only two parameters, A_L and A_J. The calculated spacings shown in Fig. 1 were obtained with the parameters

$$A_L = 10 \text{ keV}, \quad A_J = -5 \text{ keV} . \tag{10}$$

In general, the spacings are well reproduced by the model, including the decoupling inversion effect. It is possible to define from Eqs. (6) and (9) *effective* pseudo-Coriolis decoupling parameters for the two bands according to

$$a_{\rm pC}^{\rm eff}(K=1/2^{\pi}) = \pi(-1)^{\lambda_F}\frac{A_L}{A_L+A_J} , \tag{11}$$

which yield values of $a_{\rm pC}^{\rm eff} = -2$ for the $K=\frac{1}{2}^+$ band and $a_{\rm pC}^{\rm eff} = +2$ for the $K=\frac{1}{2}^-$ band.

Note that we have limited our fit to the two lowest bands. Several higher bands have been observed experimentally, including one built on a $\frac{3}{2}^+$ state at 150 keV, one built on a $\frac{3}{2}^-$ state at 225 keV, and one built on a $\frac{5}{2}^+$ state at 236 keV. Based on the SU(3) symmetry, we would also predict a second $K=\frac{1}{2}^+$ band and a second $K=\frac{1}{2}^-$ band in the same energy region; neither has been observed. Furthermore, were the symmetry exact, the band built on the $\frac{5}{2}^+$ state would lie below the one built on the $\frac{3}{2}^+$ state, and this is not the case. Thus, although ^{225}Ra exhibits some of the features of the SU(3) dynamical symmetry, the description is at best qualitative, and significant symmetry breaking is needed. It is important to note, however, that the reproduction of the properties of the two lowest bands is more a reflection of pseudo-L symmetry than of SU(3) symmetry. We expect that even in the presence of realistic SU(3) symmetry breaking these qualitative features will persist.

We close our discussion with one final point. We have based our analysis of asymmetric shape deformation in odd-mass nuclei on the SU(3) limit of the U(16) model proposed by Engel and Iachello.[2,3] The specific nature of the SU(3) symmetry dictates the quantum numbers associated with low-lying bands and the corresponding bandhead energies. Our principal focus has been on the decoupling inversion effect associated with $K=\frac{1}{2}$ bands, and this would also follow if the core were described by the SU(3) limit of the U(4) alpha-clustering model.[14] Further work to distinguish which model of the core is better able to describe odd-mass nuclei in the light actinides is called for.

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[1]See, e.g., G. Leander, in *Neils Bohr Centennial Conference 1985—Nuclear Structure,* edited by R. Broglia, G. Hagemann, and B. Herskind (North-Holland, Amsterdam, 1985).

[2]J. Engel and F. Iachello, Phys. Rev. Lett. **54**, 1126 (1985).

[3]J. Engel, Ph.D. thesis, Yale University, 1986; J. Engel and F. Iachello (unpublished).

[4]A. Arima and F. Iachello, Phys. Lett. **57B**, 39 (1975); Ann. Phys. (N.Y.) **99**, 253 (1976); **111**, 201 (1978); **123**, 468 (1979).

[5]F. Iachello and O. Scholten, Phys. Rev. Lett. **43**, 679 (1979).

[6]A. F. Barfield, Ph.D. thesis, University of Arizona, 1986.

[7]T. Otsuka, Phys. Lett. **182B**, 256 (1986).

[8]F. Catara, M. Sambataro, A. Insolia, and A. Vitturi, Phys. Lett. **180B**, 1 (1986).

[9]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. II.

[10]A. B. Balantekin, I. Bars, R. Bijker, and F. Iachello, Phys. Rev. C **27**, 1761 (1983); P. van Isacker, A. Frank, and H. Z. Sun, Ann. Phys. (N.Y.) **157**, 183 (1984); R. Bijker, Ph.D. thesis, University of Groningen, 1984.

[11]A. Frank, S. Pittel, D. D. Warner, and J. Engel, Phys. Lett. **182B**, 233 (1986).

[12]R. K. Sheline, D. Decman, K. Nybo, T. F. Thorsteinsen, G. Lovhoiden, E. R. Flynn, J. A. Cizewski, D. K. Burke, G. Sletten, P. Hill, N. Kaffrell, W. Kurcewicz, G. Nyman, and G. Leander, Phys. Lett. **133B**, 13 (1983).

[13]A. Gyurkovich, A. Sobiczewski, B. Nerlo-Pomorska, and K. Pomorska, Phys. Lett. **105B**, 95 (1981).

[14]See, e.g., J. H. Daley, in *Interacting Boson-Boson and Boson-Fermion Systems,* edited by O. Scholten (World Scientific, Singapore, 1984).