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Uncertainties in nuclear matrix elements for neutrinoless double-beta decay

Jonathan Engel

Department of Physics and Astronomy, CB3255, University of North Carolina, Chapel Hill, NC 27599-3255, USA

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Abstract

I briefly review calculations of the matrix elements governing neutrinoless double-beta decay, focusing on attempts to assign uncertainties. At present, systematic error dominates statistical error and assigning uncertainty is difficult. For some purposes, however, statistical assessment of uncertainty is profitable and, after describing the nuclear models in which matrix elements are commonly calculated, I highlight some statistical uncertainty analysis within the quasiparticle random-phase approximation. I also propose, in broad terms, strategies for reducing both systematic and statistical error.

Keywords: double-beta decay, nuclear structure, error analysis

(Some figures may appear in colour only in the online journal)

1. Introduction

Neutrinoless double-beta ($0\nu\beta\beta$) decay, in which two neutrons change into protons and emit two electrons as the sole decay products, is the only process we know of that might allow us to determine whether neutrinos are Majorana particles, i.e. their own antiparticles [1]. The decay also has the potential to provide us the overall neutrino mass scale, a quantity we still sorely lack, and even to uncover new particles and interactions. But though the mere occurrence of the decay will show that neutrinos are Majorana particles [2], extracting quantitative information about masses and new interactions from an observation or limit requires a knowledge of nuclear matrix elements that cannot be independently measured. The only choice is to calculate the matrix elements [3].

A cottage industry devoted to just that has sprung up in the last few decades. Though the industry practices nuclear-structure theory, it does so in an unusual way. Nuclear theorists are used to comparing the results of their calculations directly with data; that is how they judge the quality of their work. They are not accustomed to calculating something that has never been measured, something about which even the order of magnitude is not obvious. Many strong theorists have shied away from double-beta decay for that reason. Those who have not produce matrix elements that disagree with one another by factors of up to two or three [4], and it is not at all clear that any of the calculations is near the truth. Error analysis in this project is still in its infancy.

Yet the error analysis is extremely important. Current and future experiments require a lot of material and are expensive [1]. A factor of two uncertainty in the nuclear matrix elements corresponds to a factor of four in the amount of isotope needed to reach a certain limit on the neutrino mass. Experimentalists are thus keenly interested in reducing theoretical error as much as possible. The absence of a good framework for analyzing uncertainties makes error reduction difficult, even as models improve.

This paper offers a brief review of the errors associated with the nuclear matrix elements, touching on the resulting uncertainty in the neutrino mass and other parameters in models of fundamental physics. The range of results produced by different nuclear models means that the largest uncertainties are systematic; most and probably all of the models are deficient in the way they handle the important physics. For this reason, work on statistical uncertainty in the matrix elements is sparse and generally more primitive than in other fields. Nonetheless, the literature contains some fairly sophisticated statistical analysis, and it will be discussed briefly here. Because the review is short and the topic broad, however, we do not try to mention all significant work on the subject.

The next section discusses the matrix elements that govern $0\nu\beta\beta$ decay in a few of the most commonly considered models of new physics. Section 3 describes the nuclear models used to calculate the matrix elements. Section 4 discusses systematic error and section 5 presents some examples of statistical error analysis. The last section proposes steps to reduce uncertainty.

2. Nuclear matrix elements and the connection to fundamental physics

There is a version of double-beta decay ($2\nu\beta\beta$ decay, in which the emitted electrons are accompanied by two antineutrinos) that is violates none of the rules of the standard model. $2\nu\beta\beta$ decay has been observed in many of the nuclei in which it is energetically allowed and in which single-beta decay is energetically forbidden or highly retarded, i.e. in the candidate isotopes for $0\nu\beta\beta$ experiments. Double-beta decay without neutrinos, on the other hand, violates lepton number, and requires neutrinos to be Majorana particles. Here, briefly, are some of the mechanisms through which $0\nu\beta\beta$ decay may occur:

2.1. Light-neutrino exchange

We don't know of any new particles or interactions beyond the standard model, but we do know that there are three light but not massless neutrinos. For that reason, most work in the field has focused on the mediation of $0\nu\beta\beta$ through the exchange of these neutrinos, which are assumed to be created and absorbed via the ordinary left-handed weak interaction. The rate has the form [5]

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}(Q, Z)\left|M_{0\nu}\right|^2 \left\langle m_\nu\right\rangle^2, \qquad (1)$$

where

$$\langle m_\nu \rangle = \sum_i U_{ei}^2 m_i, \tag{2}$$

is an average of the three neutrino masses, weighted through their mixing with the electron neutrino, $G_{0\nu}(Q, Z)$ is a phase-space factor (values of which can be found, e.g., in [6]) that depends on the number of protons Z in the final nucleus and on and the energy release Q, and in the 'closure approximation', accurate to about 10% [7], $M_{0\nu}$ is given by

$$\begin{aligned} M_{0\nu} = \langle f| \, \frac{2R}{\pi g_A^2} \sum_{a,b} \tau_a^+ \tau_b^+ \int_0^\infty \mathrm{d}q \, \frac{q}{q + \overline{E} - \left(E_i + E_f\right)/2} \\ \times \Big(j_0(qr_{ab}) \big[h_F(q) + h_{GT}(q) \vec{\sigma}_a \cdot \vec{\sigma}_b \big] \\ - j_2(qr_{ab}) h_T(q) \big[3\vec{\sigma}_a \cdot \hat{r}_{ab} \vec{\sigma}_b \cdot \hat{r}_{ab} - \vec{\sigma}_a \cdot \vec{\sigma}_b \big] \Big) |i\rangle. \end{aligned} \tag{3}$$

Here r_{ab} is the distance between nucleons a and b, E_i and E_f are the energies of the initial and final states $|i\rangle$ and $|f\rangle$, $\overline{E}$ is an average intermediate-nucleus excitation energy to which the matrix element is not sensitive, $g_A \approx 1.27$ is the weak axial-vector coupling constant (inserted in the denominator by convention), and $R \equiv 1.2A^{1/3}$ is the nuclear radius. The functions h contain g_A and the weak vector coupling g_V, and reflect the presence of forbidden one-body currents and nucleon form factors. Definitions can be found, e.g., in [5]. The form above neglects many-body currents [8], which will be discussed later.

Unless it is explicitly stated otherwise, terms such as '$0\nu\beta\beta$ matrix elements' in subsequent sections of this article will refer to the light-neutrino-exchange matrix elements.

2.2. Heavy-particle exchange

Despite the absence of evidence thus far from LHC, many particle physicists believe that there are new particles at the TeV scale, the role of which is to keep the Higgs boson from becoming extremely heavy. In addition, new heavy neutrinos can explain the familiar neutrinos' very low mass via the 'seesaw mechanism' [9–11] or other schemes. Such ideas have given rise to a host of models for physics beyond the standard model. In such models, heavy-particle exchange can generate double-beta decay, competing in rate with light-neutrino exchange for natural values of heavy-particle masses and couplings [12, 13].

We will not list the many nuclear decay operators that arise in extra-standard models (see, e.g., [14] for such a list). Most of them involve the exchange only of heavy particles and therefore have essentially zero range. In such cases, diagrams involving one and (especially) two pions, resulting in an effective longer-range operator, apparently are the most important [15].

3. Nuclear models for double-beta decay

The aforementioned cottage industry works with many nuclear models for double-beta decay, but most calculations have been carried out in one of the following: (a) the quasiparticle random phase approximation (QRPA) or its renormalized extension (RQRPA), (b) the nuclear shell model (NSM), (c) the neutron–proton interacting boson model (IBM-2), (d) the projected Hatree–Fock Bogoliubov approximation (PHFB), or (e) the generator coordinate method in conjunction with the Gogny energy-density functional (EDF). Here is a brief description of each, emphasizing the physics the method focuses on:

(a) *QRPA/RQRPA*: The method [16, 17] constructs ground state correlations by iterating two-quasiparticle excitations on top of a BCS or HFB vacuum, then imposes, to a greater or lesser (RQRPA) degree, a quasiboson approximation (equivalent to a harmonic or small-amplitude assumption) on the excitations. The simple form for the ground state allows an essentially complete single-particle Hilbert space, i.e. an arbitrary number of shells. The Hamiltonian is typically based on a realistic G matrix, but modified in the like-particle pairing and $J^\pi = 1^+$ particle-hole channels to reproduce experimental pairing gaps and Gamow–Teller resonance energies. The strength of isoscalar pairing is considered a free parameter, and usually is fit to reproduce the $2\nu\beta\beta$ decay rate of the nucleus in question. The $0\nu\beta\beta$ result is sensitive to isoscalar pairing but may not reflect its effects accurately. The reason is that the interaction strength is generally fixed fairly close to a value that produces a phase transition to an isoscalar pair condensate, a state vastly different from the like-particle BCS vacuum that is the starting point for the quasiparticles excitations.

(b) *NSM:* The physics treated by the shell model [18] is in some ways complementary to that treated by the QRPA. The single-particle Hilbert space is small, typically a few valence orbits, but through direct diagonalization of a Hamiltonian the shell model includes all possible correlations within that space. The valence-shell interaction usually comes from G-matrix perturbation theory or a renormalization-group treatment, but must be tweaked to reproduce spectra. The method should generate an effective decay operator to account for missing single-particle levels, in the same way it tries to generate an effective interaction, but with a few exceptions [19] practitioners have stuck with the bare operator in equation (3). Only two research groups [20, 21] carry out calculations with truly large Hamiltonian matrices.

(c) *IBM-2*: The idea that inspires this approach [22] is a truncation of the very large shell-model space to states built from pairs of nucleons with $J = 0$ and 2, followed by a replacement of those pairs—assumed to be very collective—with bosons. The boson Hamiltonian and electromagnetic transition operators are then fit to low-lying collective states in nuclei near the one of interest. The double-beta transition operators are mapped from shell-model calculations that insert the bare operator between two- and four-valence-nucleon states, which are generated by a schematic interaction. The method is thus based on an approximate mapping of a collective subspace in the shell model, with the Hamiltonian constructed phenomenologically; it does an excellent job in reproducing trends for spectra and E2 transitions involving collective states across isotope and isotone chains.

(d) *PHFB:* In this approach [23] one finds the quasiparticle vacuum that minimizes a schematic Hamiltonian, separable in the pairing, quadrupole, and hexadecupole channels. One then projects that vacuum onto states with well-defined particle number and angular momentum. A large single-particle space can be used but no correlations beyond static deformation and pairing are included.

(e) *EDF:* Like the QRPA, this approach [24] effectively allows all possible single-particle states. It begins with the density-dependent Gogny D1S interaction [25] and a set of HFB calculations, each constrained to yield a different value for the quadrupole moment and pairing gap. Each of the resulting quasiparticle vacua is projected as in the PHFB approach, and the Gogny interaction diagonalized, via a solution to the 'Hill-Wheeler equation,' [26] in the space spanned by the projected (and constrained) vacua. The method thus goes beyond PHFB, including dynamical correlations by mixing many mean fields. It neglects presumably important isoscalar pairing correlations, however (though some exploratory work includes them [27]). Non-collective correlations have also yet to be included.

4. Sources of systematic errors

The distinction between systematic and statistical error can be fuzzy in the context of nuclear models [28], all of which are deficient in some ways. If the source of the deficiency is clear, the resulting errors will usually be called systematic. As we shall see, however, a systematic error such as the quenching of spin can sometimes be profitably treated as statistical. Here, in any event, are the two largest sources of systematic error.

4.1. Omitted structure physics

Figure 1, taken from [4], compares the predictions (as of 2012) of the various methods just described for the light-neutrino-exchange $0\nu\beta\beta$ matrix elements. Two things are apparent right away: the figure contains no error bars and there is a lot of scatter. The lack of error bars reflects systematic uncertainties that cannot be reliably estimated. The scatter is dismaying to experimentalists, but not random; the plot has patterns. The shell model results, where they exist, are always smaller than than the others. The EDF results, especially in nuclei that are close to spherical, are larger than the others. The results of the IBM-2 and the QRPA are typically somewhere between, and relatively close to one another. In which methods are the systematic errors largest?

One cannot really answer that question, but some people are beginning to try. The authors of [32] compare the EDF approach to the shell model in isotopes near the beginning of the fp shell, where shell model calculations include an entire harmonic oscillator shell. They show that the two methods give similar large results when the shell model restricts its basis to seniority-zero states and the EDF method restricts itself to a single mean field. When each method includes (its own kind of) correlations, however, the shell model result becomes significantly smaller. One might conclude that either isoscalar pairing or non-collective correlations (or both) shrink the matrix element.

The RQRPA, which includes the former but not the latter, is difficult to carry out in a single shell, so it is hard to tell how much of the effect is due to isoscalar pairing and how much to non-collective correlations. But unpublished work in the same model space suggests that the isoscalar pairing is the dominant effect [33]. It also appear that increasing the size of single-particle space in the shell model usually makes the matrix element larger. A calculation for the nuclei we really care about in a reasonably large model space—perhaps two major shells—that includes all collective correlations (deformation, which is usually neglected in the QRPA, and all kinds of pairing) might produce something close to the correct result. The considerations above suggest that that result would lie somewhere between the matrix elements of the shell model and the RQRPA for four of the nuclei in the figure. (Three of these, ^{76}Ge, ^{130}Te, and ^{136}Xe, are the active isotopes in the largest-scale experiments.)

Of course, all this is conjecture and, moreover, ignores other sources of systematic error, such as those in the effective interactions and effective decay operator. Perturbation theory suggests that the shell model matrix element should be somewhat larger when the decay operator is corrected for levels outside the model space [19], a conclusion that supports the intuition above, but perturbation theory may not be accurate. Finally, we have yet to mention an even more important systematic effect: the frequent quenching of spin-dependent

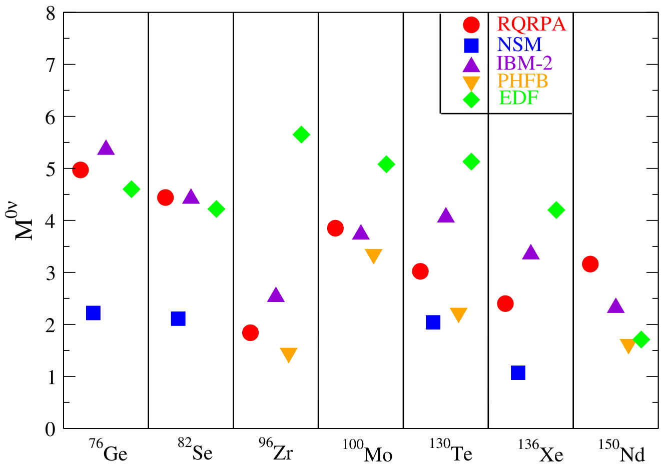

Figure 1. The predictions of five nuclear models for $0\nu\beta\beta$ matrix elements, taken from [4]: renormalized QRPA (red circles) [29, 30], nuclear shell model (blue squares) [31], interacting boson model II (purple triangles) [22], projected Hartree–Fock–Bogoliubov method (orange upside-down triangles) [23], and the generator-coordinate method within nuclear energy-density-functional theory (green diamonds) [24].

operators. Essentially all calculations overestimate single-beta decay and $2\nu\beta\beta$ rates (except for those that fit their parameters to such rates). That issue is the subject of the next subsection.

4.2. Many-body currents and quenching of matrix elements

We have known for a long time that Gamow–Teller strength is almost universally small compared to single-particle values or even the predictions of the shell model [34], and that integrated strength is significantly less than sum rules say it should be. But we've never fully understood why. How much of a role is played by non-nucleonic degrees of freedom, i.e. many-body currents? How much by nuclear structure, which may push strength into a long tail above the giant resonance? The most detailed meson-model calculations indicate that non-nucleonic effects are small [35], but that conclusion has been challenged recently by chiral effective field theory (χEFT) [8].

The important question for double-beta decay: how much are $0\nu\beta\beta$ matrix elements quenched? Are they as far off from our calculations as experimental β and $2\nu\beta\beta$ matrix elements are? Are all the results in figure 1 too large, and if so, by how much? The answers depend on the source of the quenching. The effects of the two-body currents in χEFT decrease as the momentum transfer increases, and so such currents will quench $2\nu\beta\beta$ decay, for which the momentum transfer is essentially zero at each virtual β decay, more than $0\nu\beta\beta$ decay, for which an intermediate neutrino can transfer several hundred MeV c^{-1} of momentum from one decaying nucleon to the other. Thus, if most of the quenching is due to two-body currents, as χEFT suggests it might be, then the $0\nu\beta\beta$ matrix elements will be quenched by a factor on the order of 30% [8, 36]. If it is due to nuclear-structure effects—complicated states that are neglected in our models—than the situation is less certain. The $0\nu\beta\beta$ matrix element has a different form than does the $2\nu\beta\beta$ matrix element, and it may be that whatever structure is quenching $2\nu\beta\beta$ decay has no effect on $0\nu\beta\beta$ decay. On the other hand, it might be that because both processes consist of two virtual β decays, the two matrix elements are quenched by approximately the same amount. That amount is very model dependent. In the QRPA as traditionally practiced, one assumes that all the quenching is captured by the interaction; indeed, the parameters of the Hamiltonian are fit to reproduce $2\nu\beta\beta$ rates. But in the shell model, which fits only to spectra, $2\nu\beta\beta$ matrix elements are too large and must be quenched by hand, typically by 60–70%. In the IBM-2 that number is more like 80 or 85% [37]. A similar quenching of $0\nu\beta\beta$ matrix elements would be disastrous for experiments; the amount of material needed to be sensitive to a given neutrino mass is proportional to the inverse of the square of these numbers! here it is not the uncertainty that would be a killer as much as the actual size of the matrix elements.

Systematic uncertainties, in any event, are quite large, at least 100% given all the sources of uncertainty we have mentioned. The question of what to do about them will be taken up in the conclusion.

5. Statistical uncertainties in the QRPA

5.1. Adjustment of isoscalar pairing and the treatment of g_A as a free parameter

With such large systematic uncertainties in the matrix elements it is difficult to analyze statistical ones. Though there have been attempts to do so in several models (see, e.g., [38] for a discussion of PHFB) the most sophisticated have probably come within the QRPA. The two most important model parameters in that approach are $g_{\rm pp}$, the strength of the interaction in the isoscalar pairing channel, and $g_{\rm A}$, the axial-vector coupling constant of the nucleon (which appears in any approach).

In most QRPA calculations, as mentioned above, $g_{\rm pp}$ is fit to $2\nu\beta\beta$ decay rates. The reason is the work of [29], which showed that the variation in the results of QRPA—due to differences in model space, interaction, degree of QRPA renormalization, etc—nearly disappeared when $g_{\rm pp}$ was adjusted to reproduce the measured $2\nu\beta\beta$ decay rate. The elaboration of the role of $g_{\rm pp}$ in [29] is probably the most influential uncertainty analysis of $0\nu\beta\beta$ matrix elements.

The other important constant $g_{\rm A}$ has a measured value, of course (about 1.27), but practitioners have typically incorporated the quenching discussed above into an 'effective' $g_{\rm A}$ that is smaller, sometimes 1.0. Some recent work, however, has treated g_A as a completely free model parameter alongside $g_{\rm pp}$. Ref. [39] concludes that a least-squares fit of $g_{\rm A}$ and $g_{\rm pp}$, where possible, to the β^- decay rate and β^+/electron-capture rate of the $J^\pi = 1^+$ ground state in the *intermediate* nuclei involved in double-beta decay in addition to the $2\nu\beta\beta$ rates of the initial nuclei, leads to an effective $g_{\rm A}$ of about 0.7 or 0.8. This value, which is comparable to that needed in the shell model to reproduce $2\nu\beta\beta$ rates, is significantly smaller than 1.0–1.25, the range usually used in the QRPA. It turns out, however, that the smaller $g_{\rm A}$ changes the $0\nu\beta\beta$ rate, which nominally is nearly proportional to $g_{\rm A}^2$ (because the Gamow–Teller piece of the matrix element is the largest) by only about 12%. The reason is that a small $g_{\rm A}$ acts to decrease the double-beta matrix elements, while a small $g_{\rm pp}$ increases them. If $g_{\rm A}$ decreases, then $g_{\rm pp}$ must decrease as well to keep the calculated $2\nu\beta\beta$ decay accurate, and the change in the $0\nu\beta\beta$ matrix element is smaller than on might at first expect.

The authors of [39] neglect two-body currents, which may not be negligible and which effectively renormalize $g_{\rm A}$ in a momentum-transfer-dependent way that has a larger effect in $2\nu\beta\beta$ decay than in $0\nu\beta\beta$ decay [8]. As calculations improve and the effects of two-body currents are better quantified, (i.e. as the systematic error in the $0\nu\beta\beta$ matrix element decreases) the statistical treatment of $g_{\rm A}$, which will then encode only the uncertainty due to missing nuclear-structure physics, might become more reliable.

5.2. Effects of multiple measurements on determination of $\langle m_\nu \rangle$

Many people have pointed out that multiple observations of $0\nu\beta\beta$ decay might reduce the uncertainty in the average neutrino mass we are trying to extract. They would, for example, presumably identify the model best able to reproduce all measurements with a single value for the average mass. But even within a single model, multiple measurements might be useful. If the model contains parameters that are fit to data, what better data to use than those we are really interested in? Perhaps two or more measurements would help fix predictions and allow a better extraction of the average mass.

Reference [40], elaborating on ideas presented in [43], argues that this hope is not very well founded, at least at present. The reason is that the uncertainties in matrix elements from different nuclei are highly correlated. The paper carries out many different QRPA-like calculations, varying the method (QRPA and RQRPA), the value of g_A (1.0 and 1.25), the treatment of short range correlations (via the Miller–Spencer Jastrow function [41] and the UCOM method [42, 44], which is now, by the way, known to be better), and the size of the single-particle model space (three choices), leading to 24 distinct results. For each of these the parameter g_{pp} is varied to be consistent with the uncertainty in measured $2\nu\beta\beta$ rates. For each 'variable' i (except for g_{pp}) the standard deviation σ_i is chosen to just contain the maximum and minimum choice. The results, plotted as the log of the nuclear matrix element in one nucleus versus the log of the matrix element in a second, for all possible pairs of nuclei, appear in figure 2. Each of the 24 points in each plot is the result of a calculation, with the error bar on each point representing the uncertainty in g_{pp}. The figure also contains the one-standard-deviation ellipses in the matrix-element error.

The shapes of the ellipses, except in cases including ^{96}Zr (for which the matrix element varies wildly with g_{pp}), mean that the errors are highly correlated. That result in turn means that multiple measurements will at present not significantly reduce the error in the average neutrino mass extracted from experiment; the smallest theoretical uncertainty among the calculated matrix elements essentially determines the uncertainty in $\langle m_\nu \rangle$, no matter how many measurements are made. To see why, it may help to imagine the limit of perfect correlation, in which the ellipses become lines with a 45° slope. In that limit, a lifetime measurement in one nucleus will simply correlate every point on the line segment with a value of $\langle m_\nu \rangle$, yielding a range for the latter. A second measurement will do the same and the correlation will either be consistent with the first, in which case it yields no new information about $\langle m_\nu \rangle$, or inconsistent, in which case unknown systematic errors are at play and, again, we can extract no new information. If systematic errors are under control, the segment with the smallest extent, corresponding to the case in which one of the nuclei has the smallest error in its matrix element, will determine the uncertainty in $\langle m_\nu \rangle$.

Another way of seeing the same fact is to note that the correlation means that all the matrix elements are affected in more or less the same way by variation of the parameters. Several measurements will thus not restrict the values of those parameters much more than a single measurement. Reference [40] notes that this situation need not be permanent. Better and new kinds of data, e.g. single-beta-decay rates and strength distributions, might change the situation. Reproducing these as well as $2\nu\beta\beta$ rates may require values of g_A that depend on the nucleus (see, e.g., [45]). Different values for g_A in the two nuclei in any given panel of figure 2 could move points outside the corresponding ellipse. Or, in other words, the new data may not correlate with the $0\nu\beta\beta$ rate in the same way the $2\nu\beta\beta$ rate does.

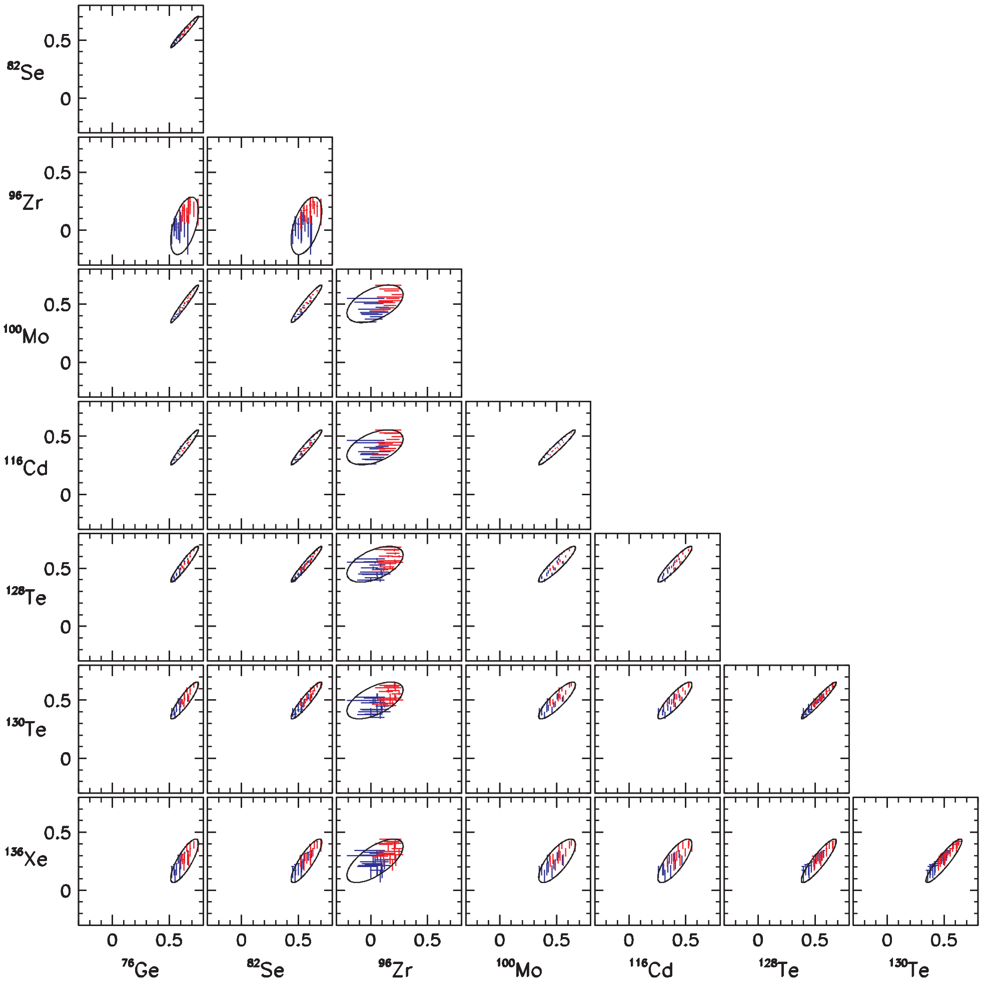

Figure 2. Scatter plot of estimated QRPA values for logarithms of the $0\nu\beta\beta$ matrix element for pairs of decaying nuclei, together with 1σ error ellipses. Reprinted with permission from [40]. Copyright 2009 by the American Physical Society. Error bars on points indicate uncertainty in the QRPA parameter g_{pp}. Blue points are calculated with the Miller–Spencer treatment of short-range correlation functions [41] and red points with the UCOM treatment [42].

5.3. Multiple measurements and exotic new physics

Suppose heavy-particle exchange induces $0\nu\beta\beta$ decay at a rate comparable to that induced by light-neutrino exchange. Can multiple measurements distinguish the two? Reference [46], again following earlier work [43, 47–49], examines the question in three specific models of new physics: heavy neutrino exchange (labeled N in the next figure), R-parity-breaking supersymmetry with heavy gluino exchange (labeled $\not{R}$), and the same supersymmetry with the short-range exchange of a squark (in place of a W) together with the long-range exchange of light neutrinos (labeled $\tilde{q}$).

Figure 3 summarizes the results of the statistical analysis. The work again makes use of several QRPA variants, with slight differences from those in the previous subsection. This

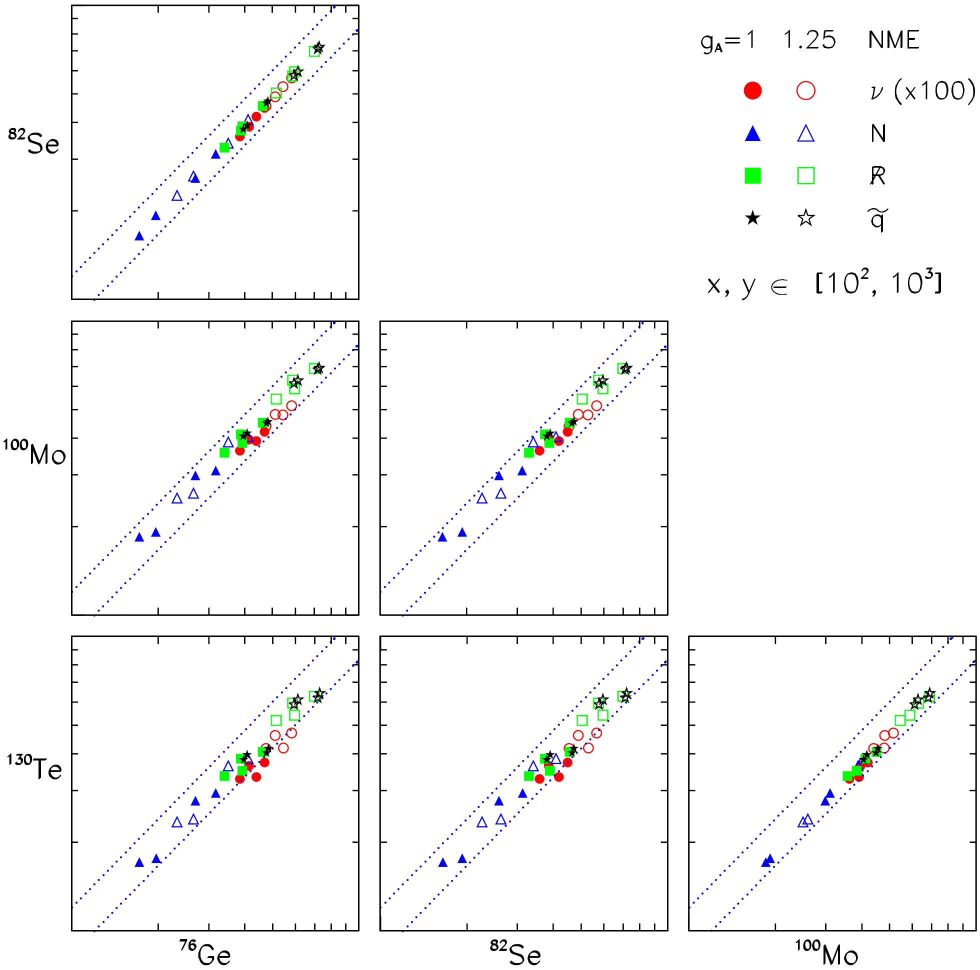

Figure 3. Scatter plot of QRPA predictions for the logarithms of $0\nu\beta\beta$ matrix elements associated with several different kinds of fundamental physics, for pairs of decaying nuclei. Reprinted with permission from [46]. Copyright 2011 by the American Physical Society. Red circles (scaled by 100) refer to the usual light-neutrino exchange, blue triangles to heavy-neutrino exchange, green squares to gluino exchange in *R*-parity-breaking supersymmetry, and black stars to light-neutrino exchange in combination with the exchange of a squark (a supersymmetric particle). Solid symbols are calculated with $g_A = 1$ and open symbols with $g_A = 1.25$. Dotted lines mark the places at which the x and y coordinates differ by 20%.

time there are eight separate calculations for each mechanism, utilizing two distinct single-particle model spaces, two values of g_A (1.0 and 1.25), and two kinds of short-range correlations (but now much closer to one another than before; the phenomenological Miller–Spence Jastrow function is not used). The parameter g_{pp} has this time been fixed at a single number for each calculation by the central value of the measured $2\nu\beta\beta$ rate.

The figure shows the logarithms of the matrix elements (with the light-neutrino exchange scaled by 100) in pairs of nuclei. The points cluster along the diagonal and the variation from one mechanism to the next is similar in magnitude to the variation among QRPA variants for a fixed mechanism. For the same reason as in the previous section, the clustering along a line makes it difficult to determine the mechanism from more than one measurement. The matrix elements for two different mechanisms seem to have nearly the same ratio in all nuclei, despite the difference in operators that produce them.

The situation is not irreparable here, either. Once again, other nuclear models have been ignored (though similar, if less statistically analyzed, conclusions apply when they are considered). More importantly, other extra-standard models, e.g. left–right symmetric models [46], appear (for reasons that are obscure) to be easier to distinguish from ordinary light-neutrino exchange; the matrix elements would fall a bit outside the bands in the figure 3. And other data from $0\nu\beta\beta$ experiments—angular and energy distributions for the electrons—could better distinguish all the models discussed here [50].

6. Discussion and conclusion: reducing uncertainties in the future

Despite the useful statistical studies just described, large systematic errors inhibit, for the most part, the kind of careful error analysis advocated in [28]. What can be done to reduce systematic errors? First, we must work hard so that each method includes all the physics known to be important, physics emphasized by one or the other of the existing approaches. Thus, shell-model practitioners should do everything possible to expand the size of their single-particle spaces and to incorporate correlations from outside those spaces (as was attempted in [19]). The EDF approach should be generalized to include isoscalar pairing [27], spin–isospin correlations, and at least some of the non-collective correlations included in the shell model, and we must work to produce more accurate EDFs. The IBM-2 should include isoscalar pair correlations as well, and its practitioners should work towards a better mapping from fermions to bosons. The QRPA needs to be extended in a way that allows more complex correlations. Two-body currents need to be carefully analyzed and incorporated into all models if they turn out to have significant effects. These are minimal requirements. *Ab initio* methods (described briefly, e.g., in [51]) and increasing computing power should allow good progress toward satisfying most of them in the next few years. Preliminary steps with *ab initio* techniques and large-scale computers to address issues in double-beta decay could in fact form the subject for an entire article.

At the same time as methods are developed and improved, it is possible, as we have seen in a few examples from the QRPA, to use statistical uncertainty within existing models to make important statements about what can be learned from double-beta decay. The statistical job ahead of us, however, is to focus on nuclear-structure uncertainties: to fit parameters to other observables and analyze the correlations between those observables and the $0\nu\beta\beta$ matrix elements. The QRPA already incorporates crude fits; the strength of like-particle pairing, the spin–isospin interaction, and isoscalar pairing are adjusted to reproduce experimental pairing gaps, Gamow–Teller resonance energies, and $2\nu\beta\beta$ decay rates. But fits should include both more observables—ordinary beta decay, excited-state spectra (at least in the intermediate nucleus, where the QRPA can make predictions), occupation numbers and other observables from transfer reactions [52] etc—and more parameters, e.g., perhaps g_A once the effects of two-body currents are reliably quantified. The interactions of the shell model and IBM-2 are fit to many observables, but a correlation analysis, particularly one that includes double-beta matrix elements, is still lacking. Such analysis does not require new many-body technology and could begin immediately.

In the end, though, systematic errors must be significantly reduced. Only then can we really be confident that we have accurate matrix elements. At present, one can really only say that matrix elements are probably correct to within factors of two or three.

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References

[1] Avignone F T III, Elliott S R and Engel J 2008 Double beta decay, majorana neutrinos, and neutrino mass *Rev. Mod. Phys.* **80** 481
[2] Schechter J and Valle J W F 1982 Neutrinoless double beta decay in SU(2) x U(1) theories *Phys. Rev.* D **25** 2591
[3] Vergados J D, Ejiri H and Šimkovic F 2012 Theory of neutrinoless double-beta decay *Rep. Prog. Phys.* **75** 106301
[4] Vogel P 2012 Nuclear structure and double beta decay *J. Phys. G: Nucl. Part. Phys.* **39** 124002
[5] Šimkovic F, Faessler A, Rodin V, Vogel P and Engel J 2008 Anatomy of the $0\nu\beta\beta$ nuclear matrix elements *Phys. Rev.* C **77** 045503
[6] Kotila J and Iachello F 2012 Phase space factors for double-β decay *Phys. Rev.* C **85** 034316
[7] Pantis G and Vergados J D 1990 Double beta decay without invoking closure *Phys. Lett.* B **242** 1
[8] Menéndez J, Gazit D and Schwenk A 2011 Chiral two-body currents in nuclei: Gamow–Teller transitions and neutrinoless double-beta decay *Phys. Rev. Lett.* **107** 062501
[9] Mohapatra R and Senjanovic G 1980 Neutrino mass and spontaneous parity violation *Phys. Rev. Lett.* **44** 912
[10] Gell-Mann M *et al* 1979 *Supergravity* (North-Holland: Amsterdam) p 315
[11] Yanagida T 1979 Horizontal symmetry and masses of neutrinos *Conf. Proc.* **C7902131** pp 95–99
[12] Mohapatra R N 1999 Particle physics implications of neutrinoless double beta decay *Nucl. Phys. Proc. Suppl.* **77** 376–85
[13] Cirigliano V, Kurylov A, Ramsey-Musolf M J and Vogel P 2004 Neutrinoless double beta decay and lepton flavor violation *Phys. Rev. Lett.* **93** 231802
[14] Bonnet F, Hirsch M, Ota T and Winter W 2013 Systematic decomposition of the neutrinoless double beta decay operator *J. High Energy Phys.* JHEP03(2013)055
[15] Faessler A, Kovalenko S, Šimkovic F and Schweiger J 1997 Dominance of pion-exchange in R-parity violating supersymmetry contributions to neutrinoless double beta decay *Phys. Rev. Lett.* **78** 183
[16] Engel J, Vogel P and Zirnbauer M R 1988 Nuclear structure effects in double-beta decay *Phys. Rev.* C **37** 731
[17] Toivanen J and Suhonen J 1995 Renormalized proton–neutron quasiparticle random-phase approximation and its application to double beta decay *Phys. Rev. Lett.* **75** 410
[18] Caurier E, Martínez-Pinedo G, Nowacki F, Poves A and Zuker A P 2005 The shell model as unified view of nuclear structure *Rev. Mod. Phys.* **77** 427–88
[19] Holt J D and Engel J 2013 Effective double-β-decay operator for ^{76}Ge and ^{82}Se *Phys. Rev.* C **87** 064315
[20] Caurier E, Menéndez J, Nowacki F and Poves A 2008 The influence of pairing on the nuclear matrix elements of the neutrinoless double beta decays *Phys. Rev. Lett.* **100** 052503
[21] Horoi M and Stoica S 2010 Shell model analysis of the neutrinoless double-β decay of ^{48}Ca *Phys. Rev.* C **81** 024321
[22] Barea J and Iachello F 2009 Neutrinoless double-β decay in the microscopic interacting boson model *Phys. Rev.* C **79** 044301
[23] Chaturvedi K, Chandra R, Rath P K, Raina P K and Hirsch J G 2008 Nuclear deformation and neutrinoless double-β decay of ^{94,96}Zr, ^{98,100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te, and ^{150}Nd nuclei within a mechanism involving neutrino mass *Phys. Rev.* C **78** 054302
[24] Rodríguez T R and Martínez-Pinedo G 2010 Energy density functional study of nuclear matrix elements for neutrinoless $\beta\beta$ decay *Phys. Rev. Lett.* **105** 252503

[25] Berger J F, Girod M and Gogny D 1984 Microscopic analysis of collective dynamics in low energy fission *Nucl. Phys.* A **428** 23
[26] Ring P and Schuck P 2004 *Texts and Monographs in Physics* (*The Nuclear Many-Body Problem*) (Berlin: Springer)
[27] Hinohara N and Engel J 2014 Proton–neutron pairing amplitude as a generator coordinate for double-β decay *Phys. Rev.* C **90** 031301
[28] Dobaczewski J, Nazarewicz W and Reinhard P-G 2014 Error estimates of theoretical models: a guide *J. Phys. G: Nucl. Part. Phys.* **41** 074001
[29] Rodin V A, Faessler A, Šimkovic F and Vogel P 2003 On the uncertainty in the $0\nu\beta\beta$ decay nuclear matrix elements *Phys. Rev.* C **68** 044302
[30] Fang D L, Faessler A, Rodin V and Simkovic F 2010 Neutrinoless double-beta decay of ^{150}Nd accounting for deformation *Phys. Rev.* C **82** 051301
[31] Menéndez J, Poves A, Caurier E and Nowacki F 2009 Disassembling the nuclear matrix elements of the neutrinoless double beta decay *Nucl. Phys.* A **818** 139–51
[32] Menéndez J, Rodríguez T R, Martínez-Pinedo G and Poves A 2014 Correlations and neutrinoless $\beta\beta$ decay nuclear matrix elements of *pf*-shell nuclei *Phys. Rev.* C **90** 024311
[33] Menéndez J, Hinohara N, Engel J and Martínez-Pinedo G 2014 in preparation
[34] Brown B A and Wildenthal B H 1988 Status of the nuclear shell model *Annu. Rev. Nucl. Part. Sci.* **38** 29–66
[35] Towner I S 1987 Quenching of spin matrix elements in nuclei *Phys. Rep.* **155** 263
[36] Engel J, Simkovic F and Vogel P 2014 Chiral two-body currents and neutrinoless double-beta decay in the QRPA *Phys. Rev.* C **89** 064308
[37] Barea J, Kotila J and Iachello F 2013 Nuclear matrix elements for double-β decay *Phys. Rev.* C **87** 014315
[38] Rath P K, Chandra R, Chaturvedi K, Raina P K and Hirsch J G 2010 Uncertainties in nuclear transition matrix elements for neutrinoless $\beta\beta$ decay within the projected-Hartree–Fock–Bogoliubov model *Phys. Rev.* C **82** 064310
[39] Faessler A, Fogli G L, Lisi E, Rodin V, Rotunno A M and Šimkovic F 2008 Overconstrained estimates of neutrinoless double beta decay within the QRPA *J. Phys. G: Nucl. Part. Phys.* **35** 075104
[40] Faessler A, Fogli G L, Lisi E, Rodin V, Rotunno A M and Šimkovic F 2009 QRPA uncertainties and their correlations in the analysis of $0\nu\beta\beta$ decay *Phys. Rev.* D **79** 053001
[41] Miller G A and Spencer J E 1976 Pion exchange reactions with nuclei *Ann. Phys., NY* **100** 562
[42] Kortelainen M and Suhonen J 2007 Improved short-range correlations and $0\nu\beta\beta$ nuclear matrix elements of ^{76}Ge and ^{82}Se *Phys. Rev.* C **75** 051303(R)
[43] Deppisch F and Päs H 2007 Pinning down the mechanism of neutrinoless double beta decay with measurements in different nuclei *Phys. Rev. Lett.* **98** 232501
[44] Kortelainen M, Civitarese O, Suhonen J and Toivanen J 2007 Short-range correlations and neutrinoless double beta decay *Phys. Lett.* B **647** 128
[45] Delion D S and Suhonen J 2014 Effective axial-vector strength and β-decay systematics *Europhys. Lett.* **107** 52001
[46] Faessler A, Fogli G L, Lisi E, Rotunno A M and Šimkovic F 2011 Multi-isotope degeneracy of neutrinoless double beta decay mechanisms in the quasi-particle random phase approximation *Phys. Rev.* D **83** 113015
[47] Simkovic F, Vergados J and Faessler A 2010 Few active mechanisms of the neutrinoless double beta-decay and effective mass of majorana neutrinos *Phys. Rev.* D **82** 113015
[48] Gehman V M and Elliott S R 2007 Multiple-isotope comparison for determining $0\nu\beta\beta$ mechanisms *J. Phys. G: Nucl. Part. Phys.* **34** 667
[49] Fogli G L, Lisi E and Rotunno A M 2009 Probing particle and nuclear physics models of neutrinoless double beta decay with different nuclei *Phys. Rev.* D **80** 015024
[50] Ali A, Borisov A V and Zhuridov D V 2007 Probing new physics in the neutrinoless double beta decay using electron angular correlation *Phys. Rev.* D **76** 093009
[51] Bogner S *et al* 2013 Computational nuclear quantum many-body problem: the UNEDF project *Comput. Phys. Commun.* **184** 2235–50
[52] Freeman S J and Schiffer J P 2012 Constraining the $0\nu2\beta$ matrix elements by nuclear structure observables *J. Phys. G: Nucl. Part. Phys.* **39** 124004