$0\nu\beta\beta$ nuclear matrix elements and neutrino potentials

Petr Vogel, Caltech

TC meeting, UNC, Feb. 2, 201

Prepared in collaboration with Fedor Simkovic, who performed most of the presented numerical evaluations

Quite generally the double beta decay nuclear matrix element consists of three parts:

$$M^{0\nu} = M^{0\nu}_{\rm GT} - \frac{M^{0\nu}_F}{g_A^2} + M^{0\nu}_T \equiv M^{0\nu}_{\rm GT}(1 + \chi_F + \chi_T),$$

The Gamow-Teller part M_{GT} is the dominant one. When treated in the closure approximation it is

$$M^{0\nu}_{\rm GT} = \langle f|\Sigma_{lk}\sigma_l \cdot \sigma_k \tau_l^+ \tau_k^+ H(r_{lk}, \bar{E})|i\rangle,$$

The ``neutrino potential" $H(r_{ij},E)$ originates from the corresponding particle physics responsible for the Lepton Number Violation. A large variety of neutrino potentials have been considered in the literature, hence also a large variey of the corresponding nuclear matrix elements.

For the simplest scenario the ``neutrino potential" originating from the light neutrino propagator is

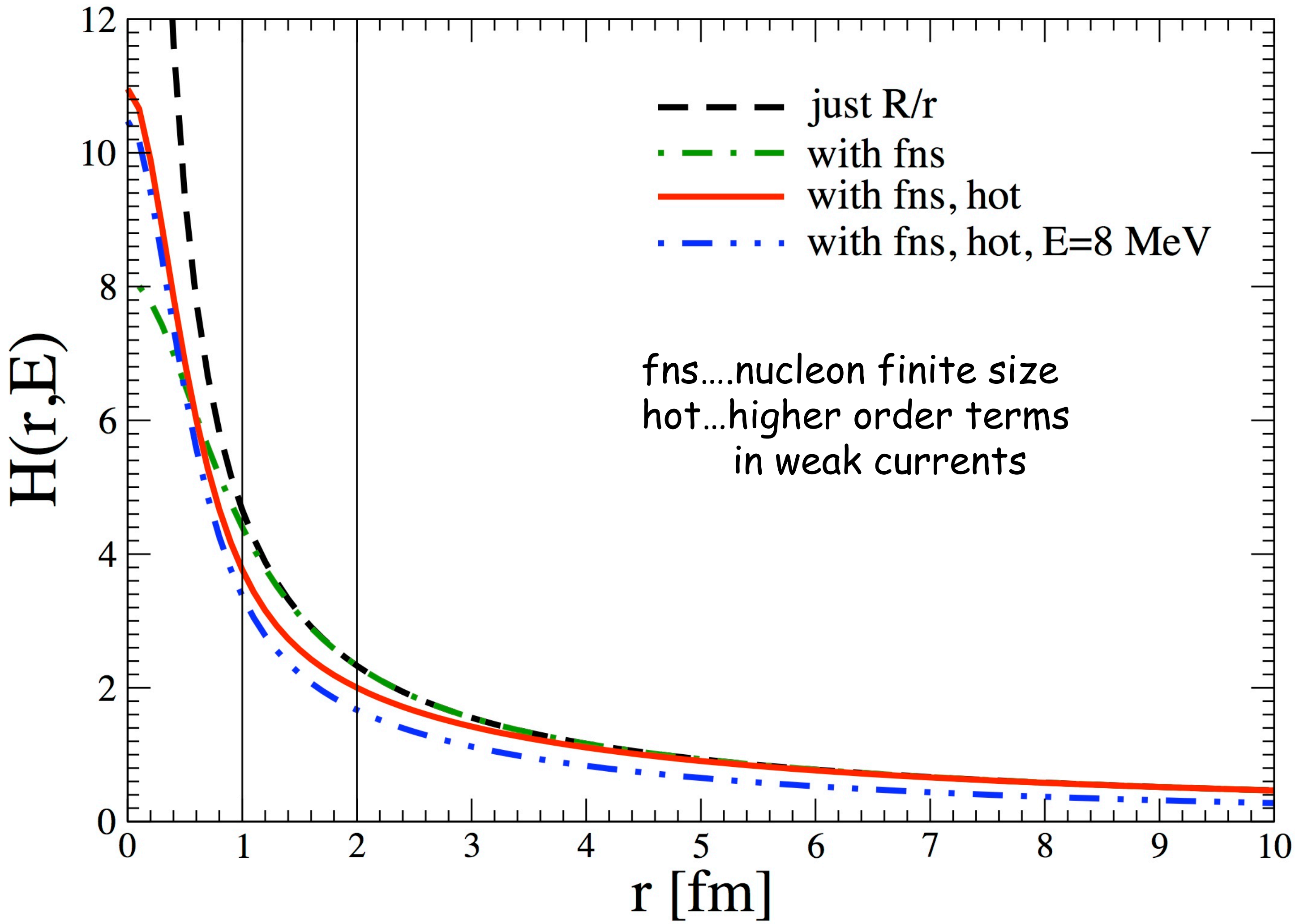
$$H_K\left(r_{12},\, E^k_{J^\pi}\right) = \frac{2}{\pi g_A^2} R \int_0^\infty f_K(q r_{12}) \frac{h_K(q^2) q\, dq}{q + E^k_{J^\pi} - (E_i + E_f)/2}.$$

Where $f_{GT}(qr) = j_0(qr)$ and $h_{GT} = g_A/(1 + q^2/M_A^2)^2$ is the nuclear axial current form factor, $M_A \sim 1$ GeV. The dependence on the excitation energy in the denominator is weak, hence it is replaced by some average constant value E.

We will consider evaluation in the closure approximation. Thus, the matrix elements formally depend only on the initial and final nuclear ground state wave functions.

Tests suggests that the closure approximation is accurate to about 10%.

As we will see, the neutrino momentum is ~200 MeV so the dependence on the nuclear excitation energy is weak. The potential H(r,E) looks like a Coulomb 1/r radial dependence. Finite size and higher order currents remove the singularity at r=0.



How does the matrix element $M^{0\nu}{}_{GT}$ depend on the distance between the two neutrons that are transformed into two protons ? This is determined by the function $C^{0\nu}{}_{GT}(r)$

$$C^{0\nu}_{\rm GT}(r) = \langle f|\Sigma_{lk}\sigma_l \cdot \sigma_k \tau_l^+ \tau_k^+ \delta(r - r_{lk}) H(r_{lk}, \bar{E})|i\rangle,$$

It is normalized by the obvious relation $M^{0\nu}_{\rm GT} = \int_0^\infty C^{0\nu}_{\rm GT}(r)\, dr,$

Thus, if we could somehow determine C(r) we could obtain $M^{0\nu}$.

In order to obtain C(r) consider first the matrix elements of the operator $\sigma_1 \cdot \sigma_2$ between two noninteracting neutrons and two protons coupled to the angular momentum J without the neutrino potential:

$$f^{\mathcal{J}}_{n,n',p,p'}(r) = \langle p(1), p'(2)(r); \mathcal{J} \parallel \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2} \parallel n(1), n'(2)(r); \mathcal{J}\rangle$$

Here are few examples for the $f_{7/2}$ and $f_{5/2}$ orbits. These functions, as expected, typically extend up to the nuclear diameter, peaking near the middle. Some of them, in particular those with J = 0, are asymmetric with larger amplitude at small distances.

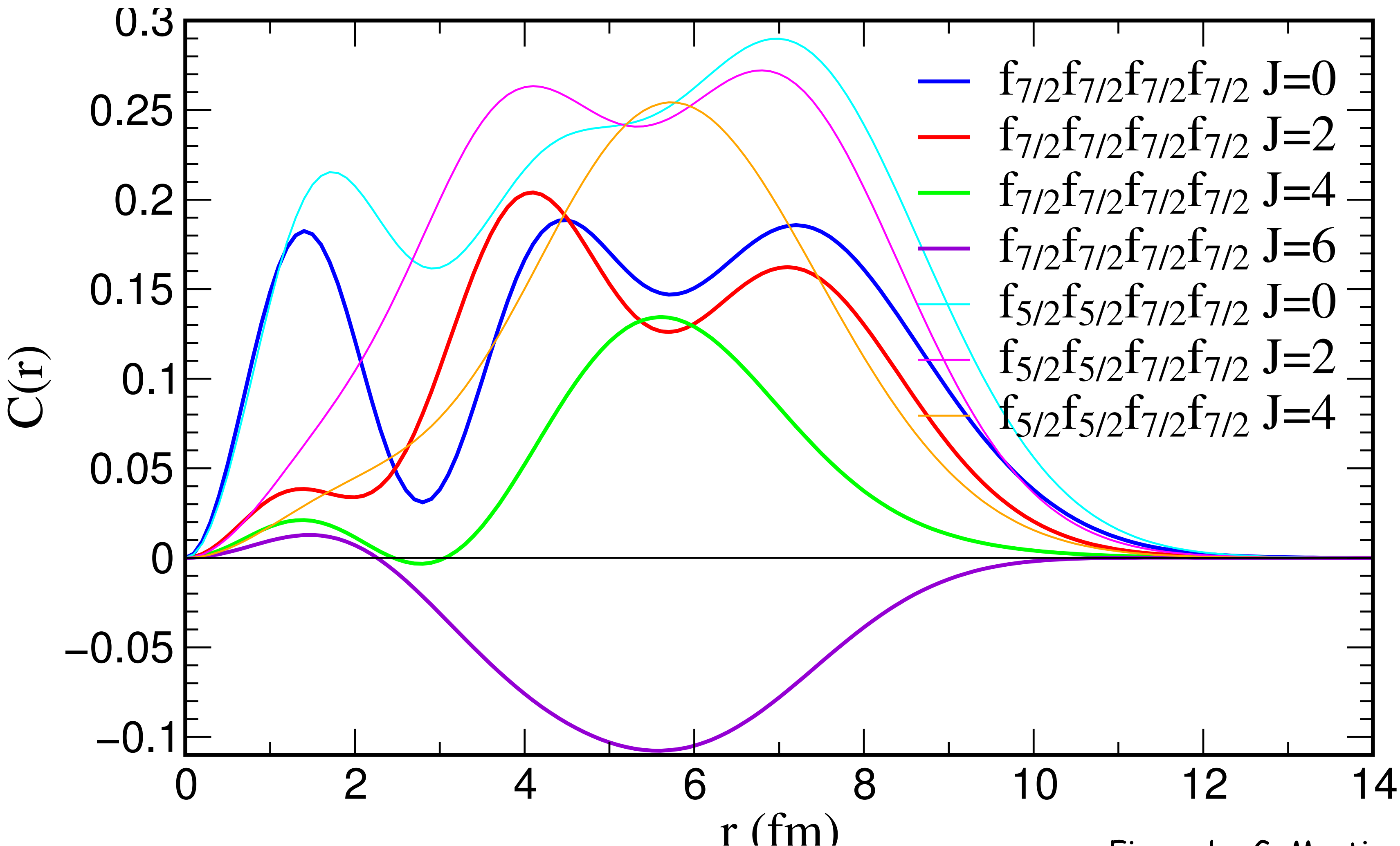
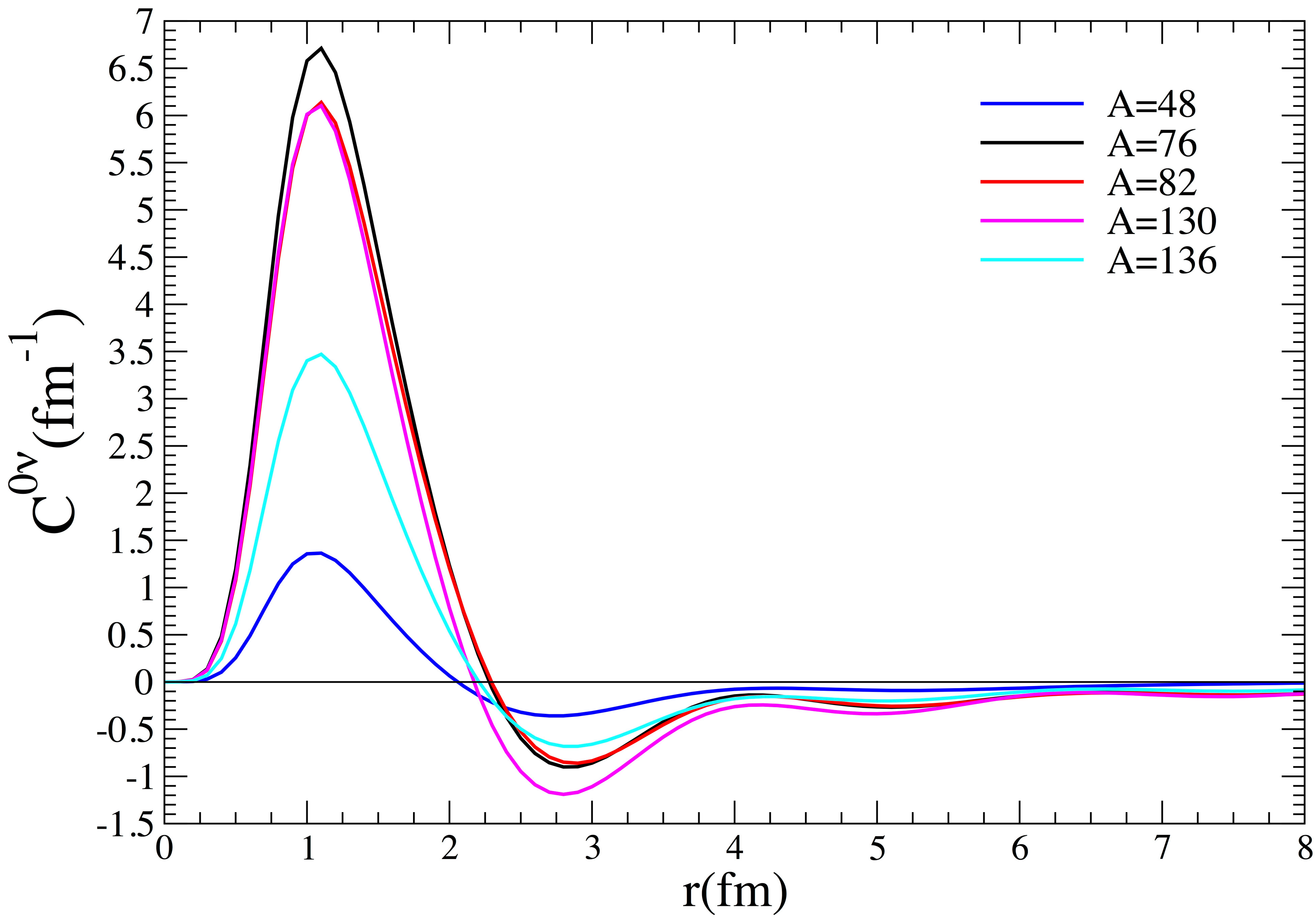
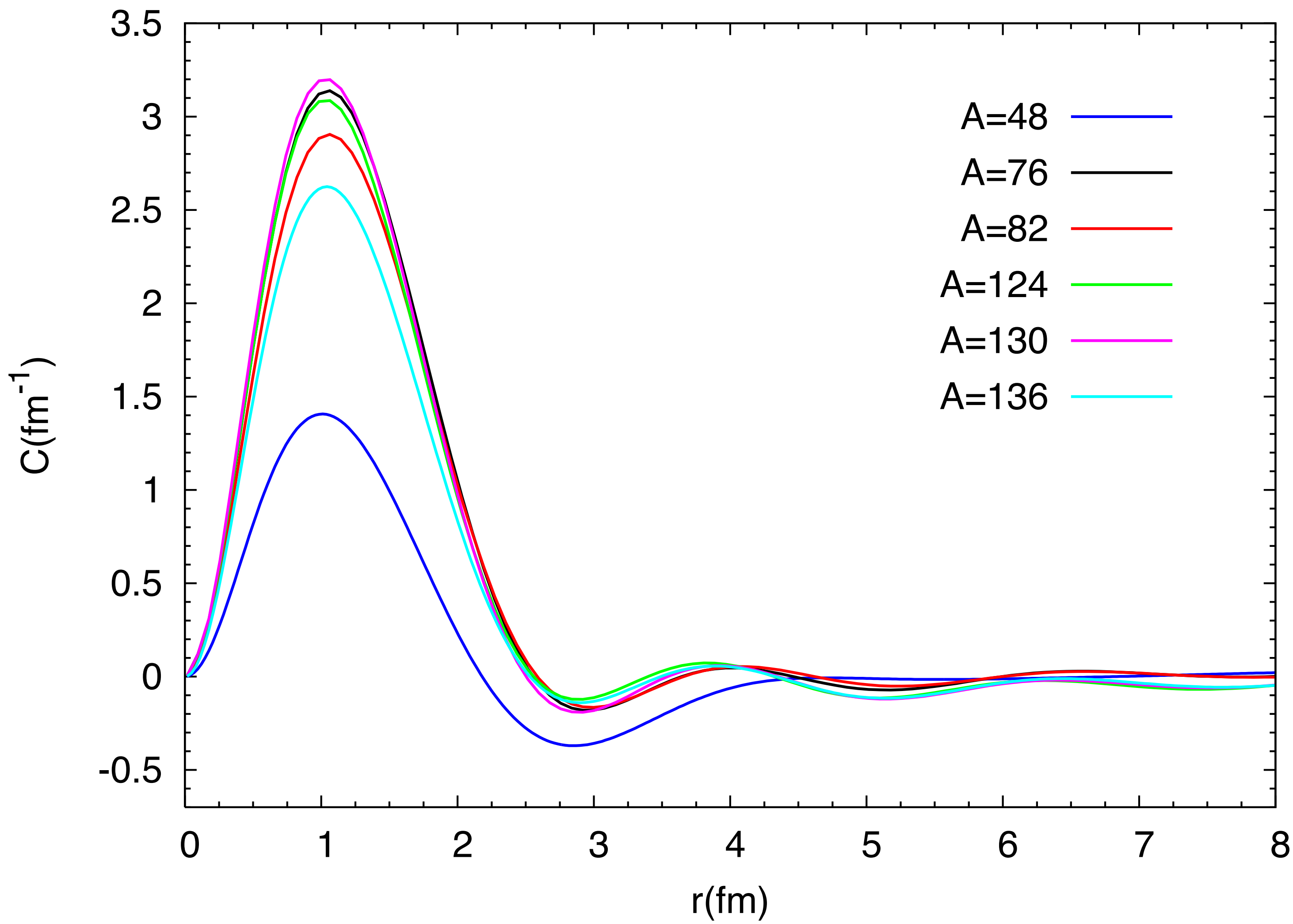


Figure by G. Martinez

Function $C^{0\nu}(r)$ evaluated in QRPA. Note the peak at ~ 1fm. There is little contribution from r > 2-3 fm. And the function for different nuclei look very similar, essentially universal. The magnitude of $M^{0\nu}$ is determined basically by the height of the peak.

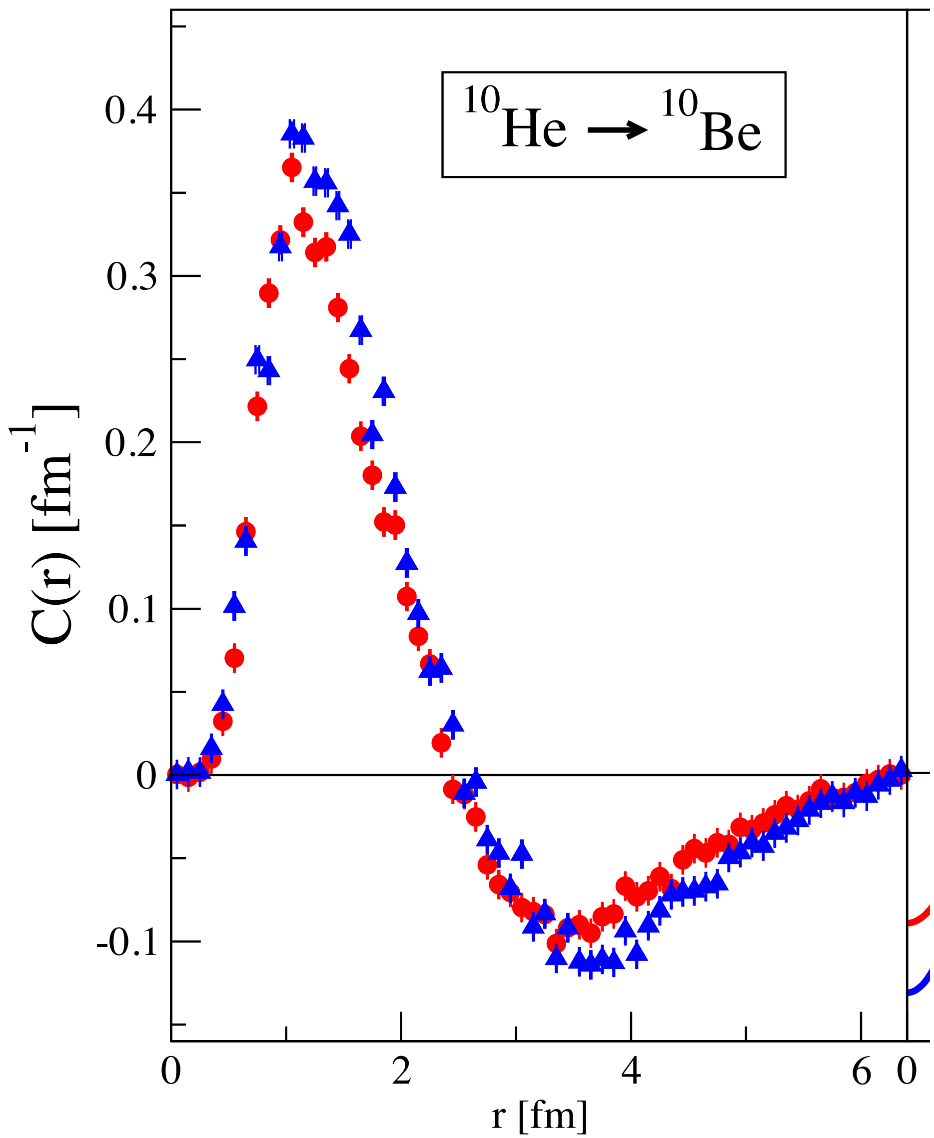


Now C(r) evaluated in the nuclear shell model. All relevant features look the same as in QRPA despite the very different way the equations of motion are formulated and solved.



From Menendez et al, Nucl. Phys. *A* **818**, 130 (2009)

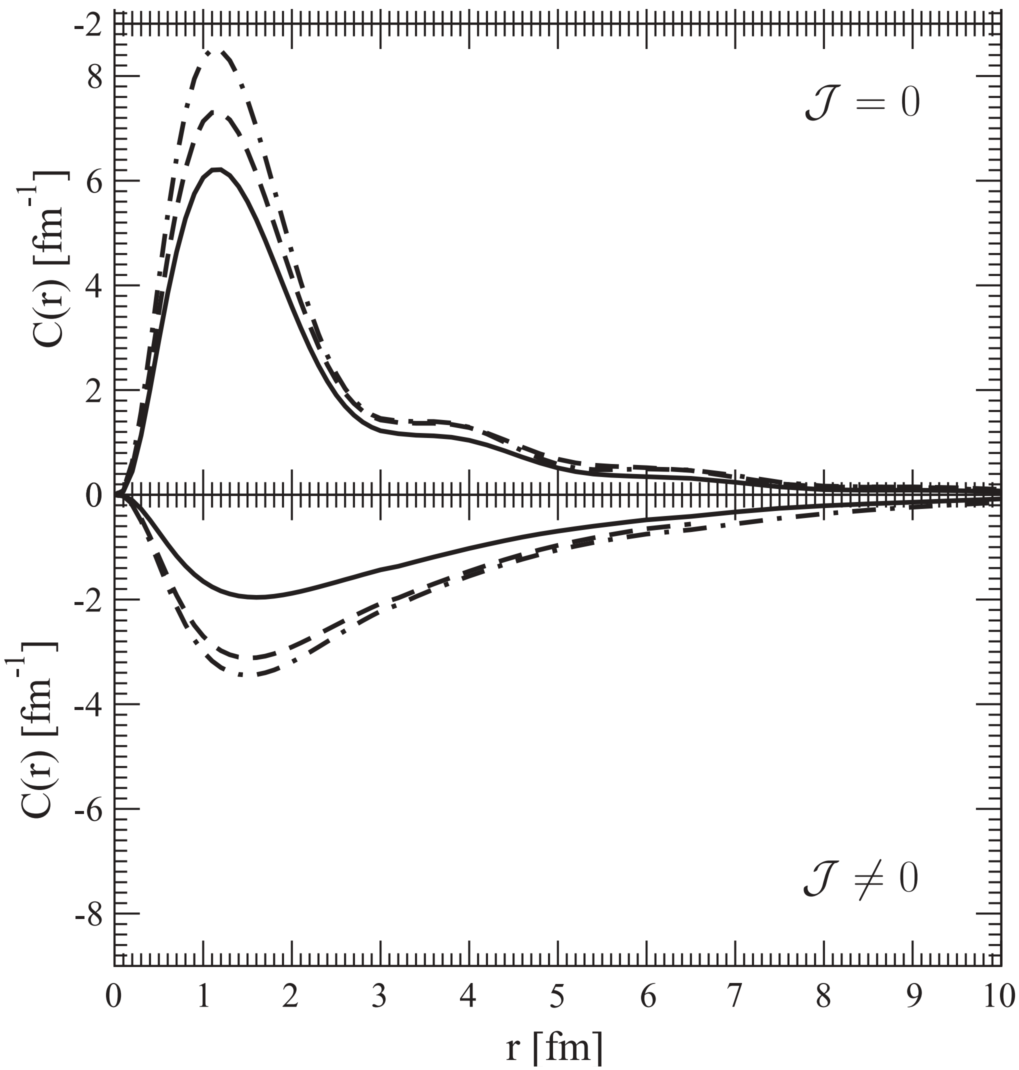
C(r) for the hypothetical $0\nu\beta\beta$ decay of ^{10}He.



The calculation was performed using the *ab initio* variational Monte-Carlo method. So most of the approximations inherent in NSM or QRPA are avoided. Yet the C(r) function looks, at least qualitatively, very similar to the results shown before.

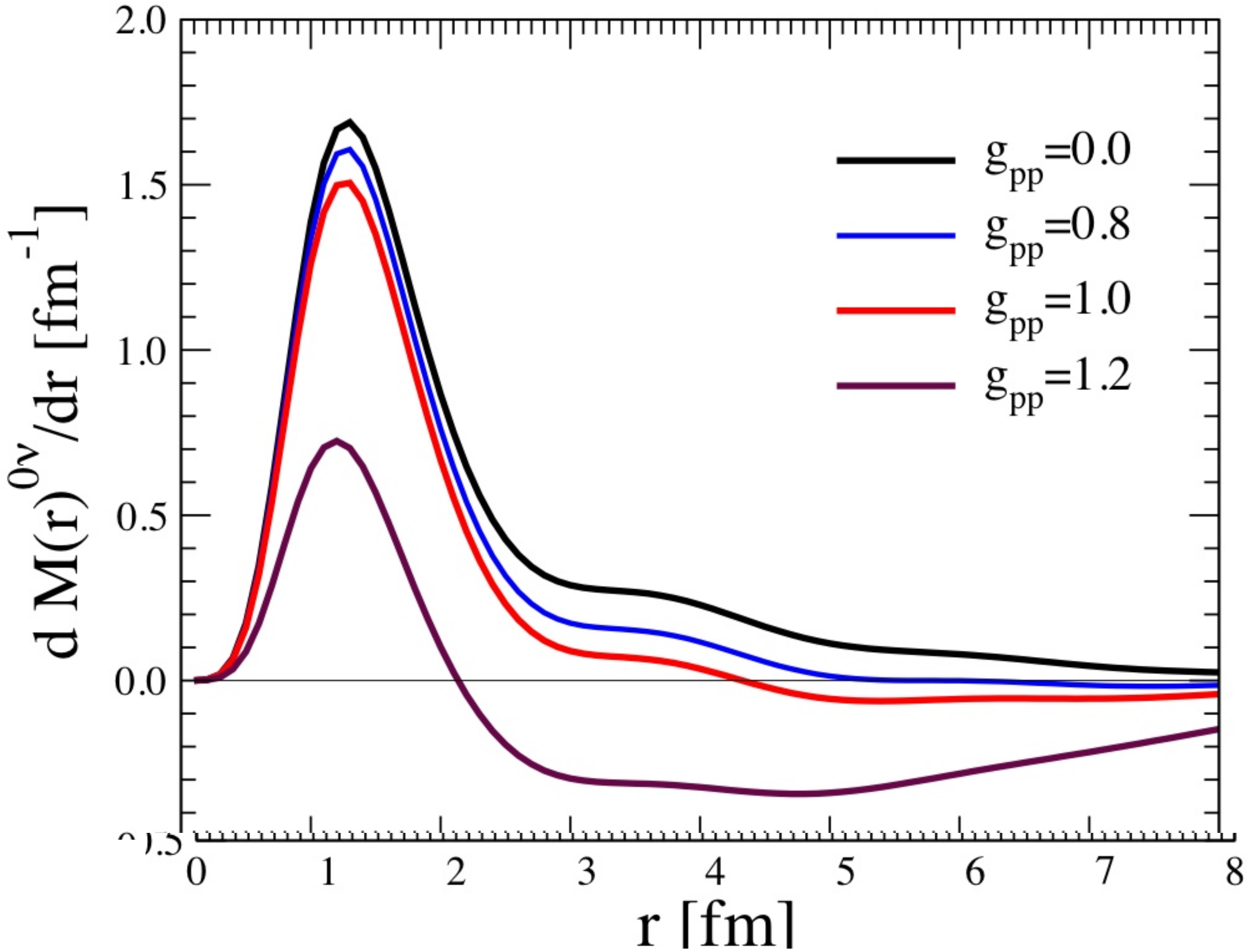
Figure from Pastore et al.,1710.05026

The fact that the resulting C(r) is concentrated at r < ~2fm is the result of cancellation between $\mathcal{J}$ = 0 (pairing) and other values of $\mathcal{J}$ (broken pairs). We have seen the effect of such cancellation before. It is again common to QRPA and NSM.

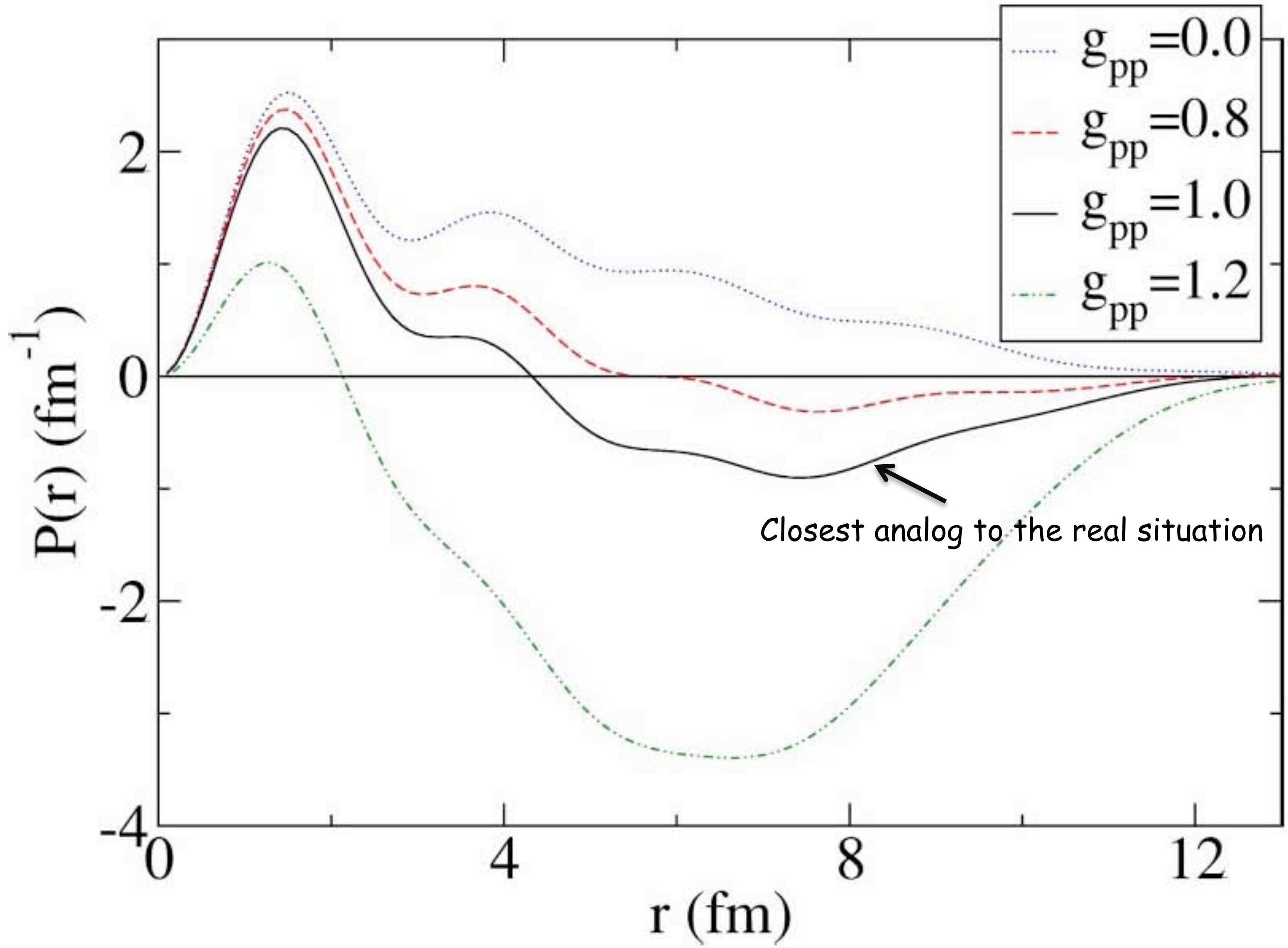


The cancellation of contributions beyond r ~ fm is a consequence of nuclear interaction. It happens only when the strength of the isoscalar pairing has the correct value.

The radial dependence of $M^{0\nu}$ evaluated in the exactly solvable model (Engel & Vogel Phys. Rev. C**69**, 034304 (2004)). Note that the cancellation for r > 2-3 fm appears only near g_{pp} = 1. That happens in real nuclei also.



This is the analog of $C^{2\nu}_{cl}(r)$ evaluated in the exactly solvable model. The curve with g_{pp} = 1.0, which is a closest analog of the realistic situation and it indeed looks quite similar to it.



Lets consider once more the GT m.e. for $0\nu\beta\beta$

$$M^{0\nu}_{\rm GT} = \langle f|\Sigma_{lk}\sigma_l \cdot \sigma_k \tau_l^+ \tau_k^+ H(r_{lk}, \bar{E})|i\rangle,$$

If we remove from the operator the neutrino potential H(r,E) we obtain the matrix element of the double GT operator connecting the ground states of the initial and final nuclei. The same operator would be responsible for the $2\nu\beta\beta$ decay if it would be OK to treat it in the closure approximation. It is also a component of the ``double GT" strength function for the initial nucleus |i>.

$$M^{2\nu}_{\rm cl} \equiv \langle f|\Sigma_{lk}\sigma_l \cdot \sigma_k \tau_l^+ \tau_k^+|i\rangle,$$

In reality, the closure approximation is not good for the $2\nu\beta\beta$ decay, but we can still consider the corresponding value if we somehow can guess the correct average energy denominator.

The correct expression for $M^{2\nu}$ includes energy denominators

$$M^{2\nu} = \Sigma_m \frac{\langle f||\sigma\tau^+||m\rangle\langle m||\sigma\tau^+||i\rangle}{E_m - (M_i + M_f)/2},$$

We can define the radial function $C^{2\nu}{}_{cl}(r)$ the same way as for the genuine $M^{0\nu}$ matrix element, thus

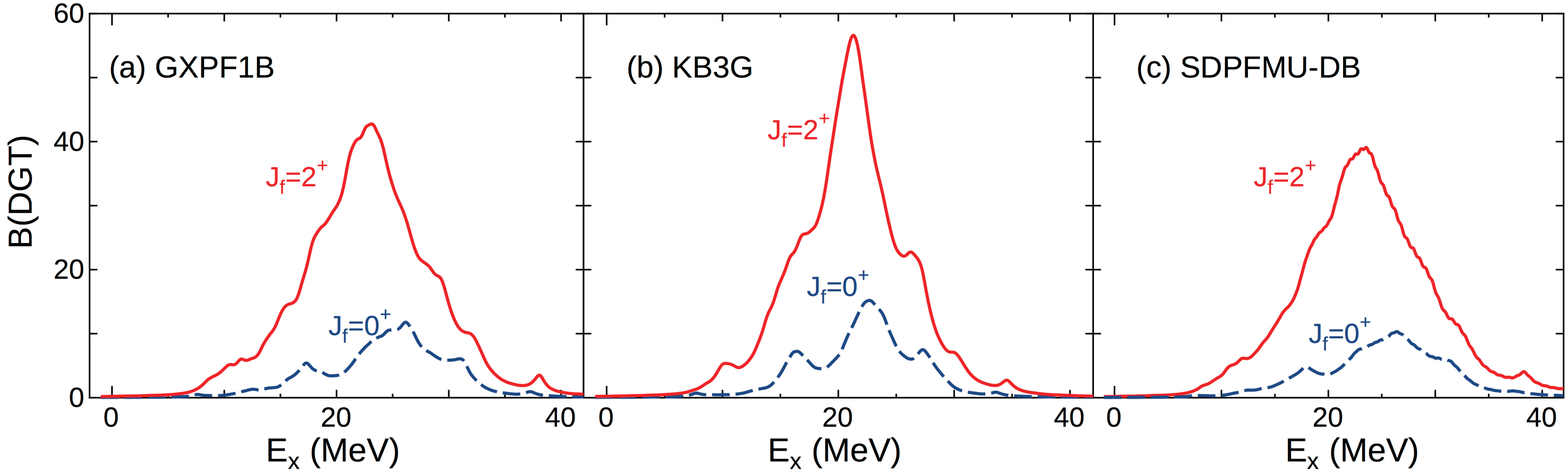
$$C^{2\nu}_{\rm cl}(r) = \langle f | \Sigma_{lk} \sigma_l \cdot \sigma_k \delta(r - r_{lk}) \tau_l^+ \tau_k^+ | i \rangle,$$

$$M^{2\nu}_{\rm cl} = \int_0^\infty C^{2\nu}_{\rm cl}(r)\, dr.$$

It is now clear that, at least formally, the following equality holds: $C^{0\nu}(r) = H(r,E_0)\, C^{2\nu}{}_{cl}(r)$ while

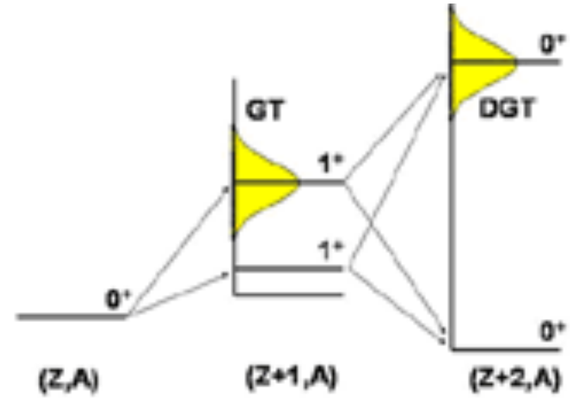
$$M^{0\nu}_{\rm GT} = \int_0^\infty C^{0\nu}_{\rm GT}(r)\, dr,$$

So, if we can somehow determine the function $C^{2\nu}{}_{cl}(r)$ we will be able to determine $C^{0\nu}(r)$ and thus also the ultimate goal, the $M^{0\nu}$. And, moreover, this is so for **any neutrino potential**. Thus, evaluation of $M^{0\nu}$ is reduced to a simple integral, provided any one of the functions $C(r)$ is known. All of such $M^{0\nu}{}_i$ are then consistent and easily evaluated.



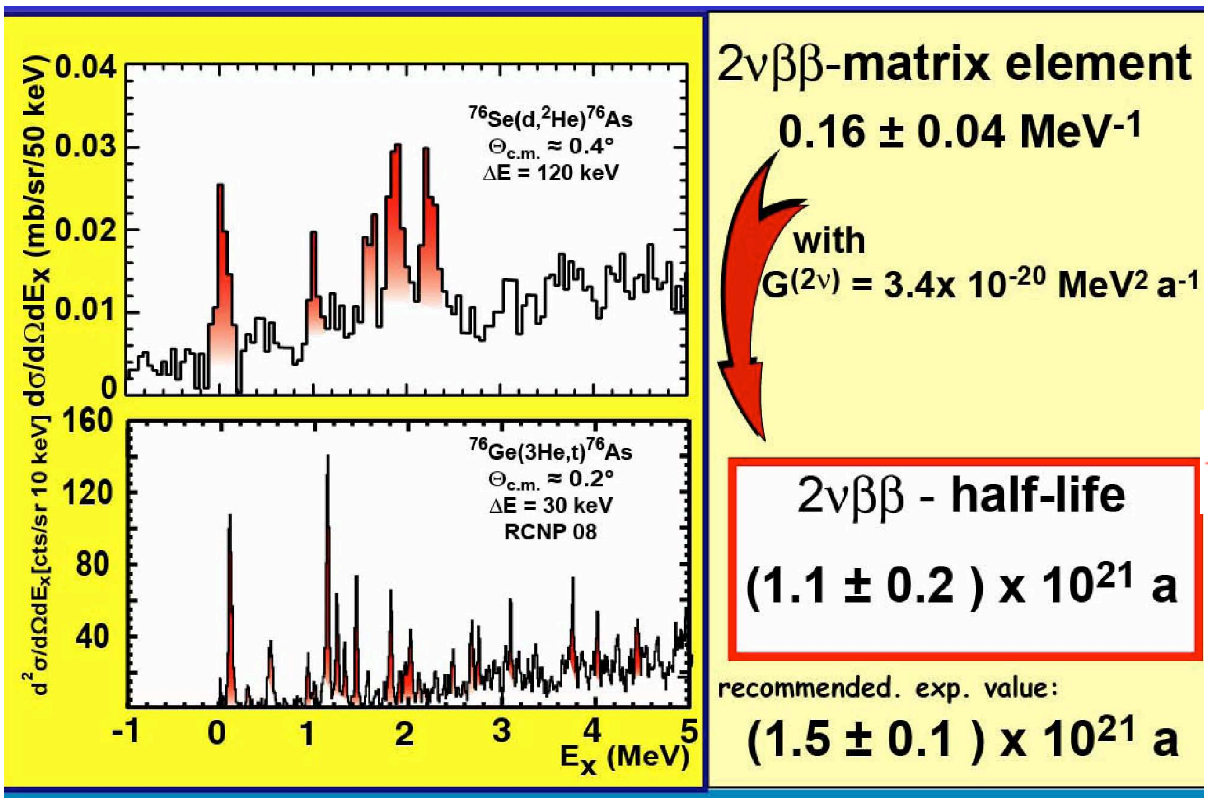
Double GT strength of ^{48}Ca evaluated in the shell model with several model hamiltonians (see Shimizu et al, 1709.01088). The $2\nu\beta\beta$ closure matrix element (i.e. the DGT transition to the ^{48}Ti ground state) that we are interested in represents only about 10^{-4} fraction of the shown total strength.

Cross sections of (t,^{3}He) and (d,^{2}He) reactions give B(GT$^{\pm}$) for β^+ and β^-; products of the amplitudes (B(GT)$^{1/2}$) entering the numerator of $M^{2\nu}_{GT}$



$$M^{2\nu}_{GT} = \sum_m \frac{M^{(+)}_{GT}(m)\ M^{(-)}_{GT}(m)}{Q_{\beta\beta}/2 + m_e + E_x(1^+_m) - E_0}$$

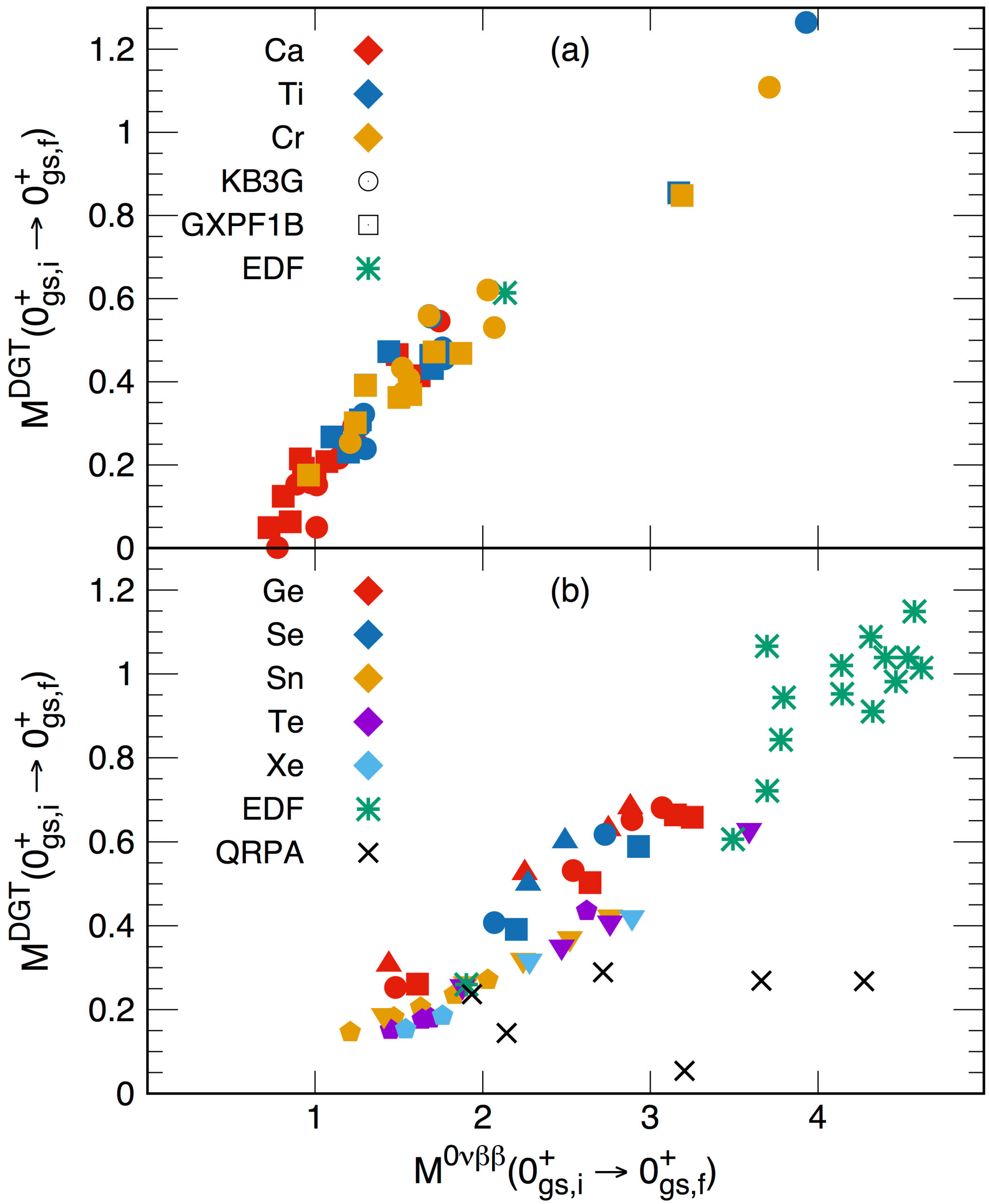
The β^- strength is dominated by the giant GT resonance. However, the β^+ strength is concentrated at low energy, little (but unknown) strength to the giant.



Closure 2νββ-decay NME

$$M^{2\nu}_{GT-cl} = \sum_m M^{(+)}_{GT}(m)\ M^{(-)}_{GT}(m)$$

Grewe, …Frekers at al, PRC 78, 044301 (2008)



Shimizu et al. claim that the $M^{0\nu}$ and $M^{2\nu}{}_{cl}$ matrix elements are proportional to each other.

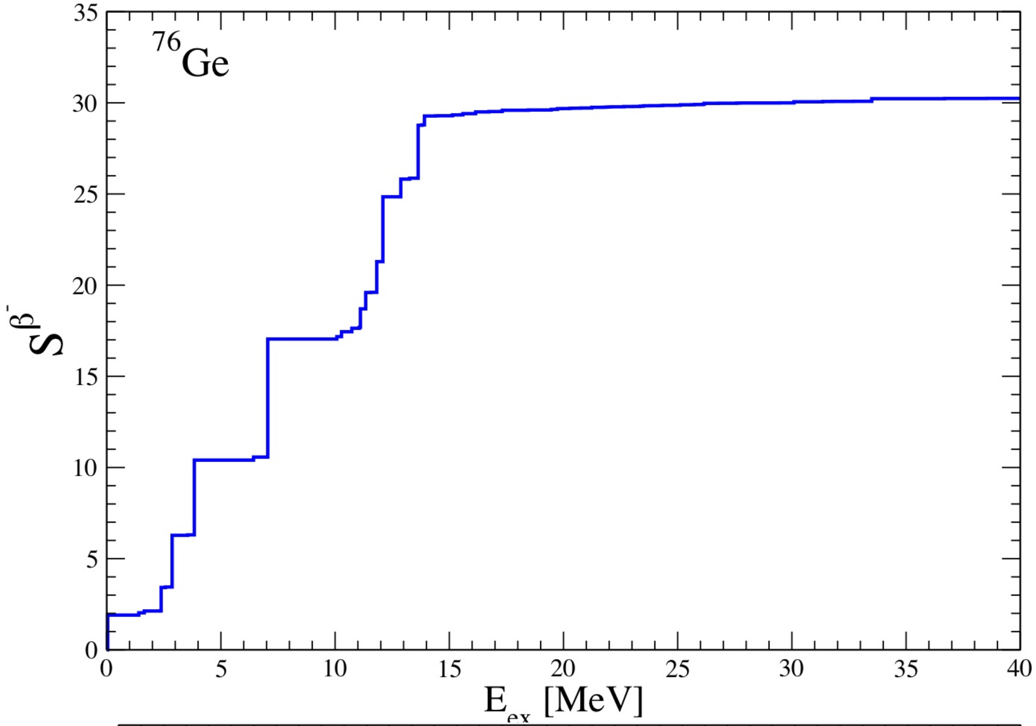
In (a) the calculated m.e. for *pf* shell nuclei are shown.

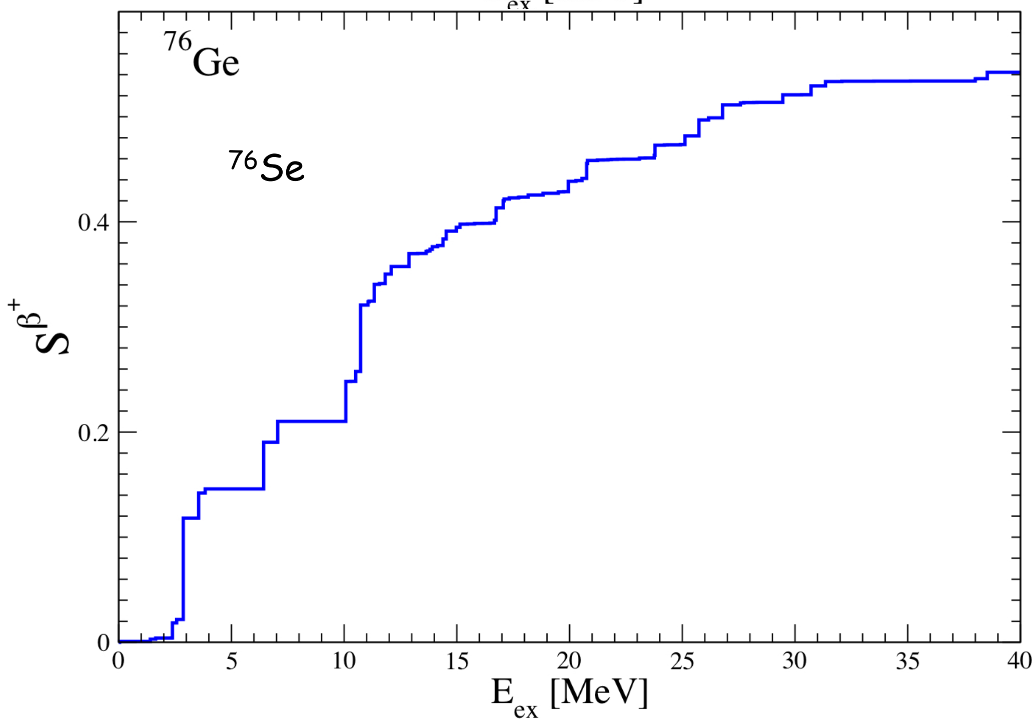
In (b) the heavier nuclei are considered. The proportionality seems to be confirmed, except when QRPA is used.

So, who is right?

Clearly, determination of $M^{2\nu}_{cl}$ is not easy. We do know the value of $M^{2\nu}$, however $M^{2\nu}_{cl}$ cannot be extracted from the known $2\nu\beta\beta$ decay half-life. That's because while both $M^{2\nu}$ and $M^{2\nu}_{cl}$ depend only on the virtual 1^+ states in the intermediate odd-odd nucleus, the weights of individual states are different. Those at higher energies contribute less to $M^{2\nu}$ than to $M^{2\nu}_{cl}$.

This would be OK if the higher energy states have negligibly small either $\langle m| \sigma \tau^+ |i\rangle$ and $\langle m| \sigma \tau^- |f\rangle$. But that is not the case, apparently.





The β^- and β^+ strength function calculated in SRQRPA with 21 s.p. levels. Note the different scales in the two panels.

In the β^- case one can clearly see the giant GT state. Also, the strength saturates at ~15 MeV.

On the other hand, the much smaller β^+ strength, unlike the usual claims, gets substantial contribution from relatively high excitation energies as well.

Whether this high-lying β^+ strength exists or not is the crucial question.

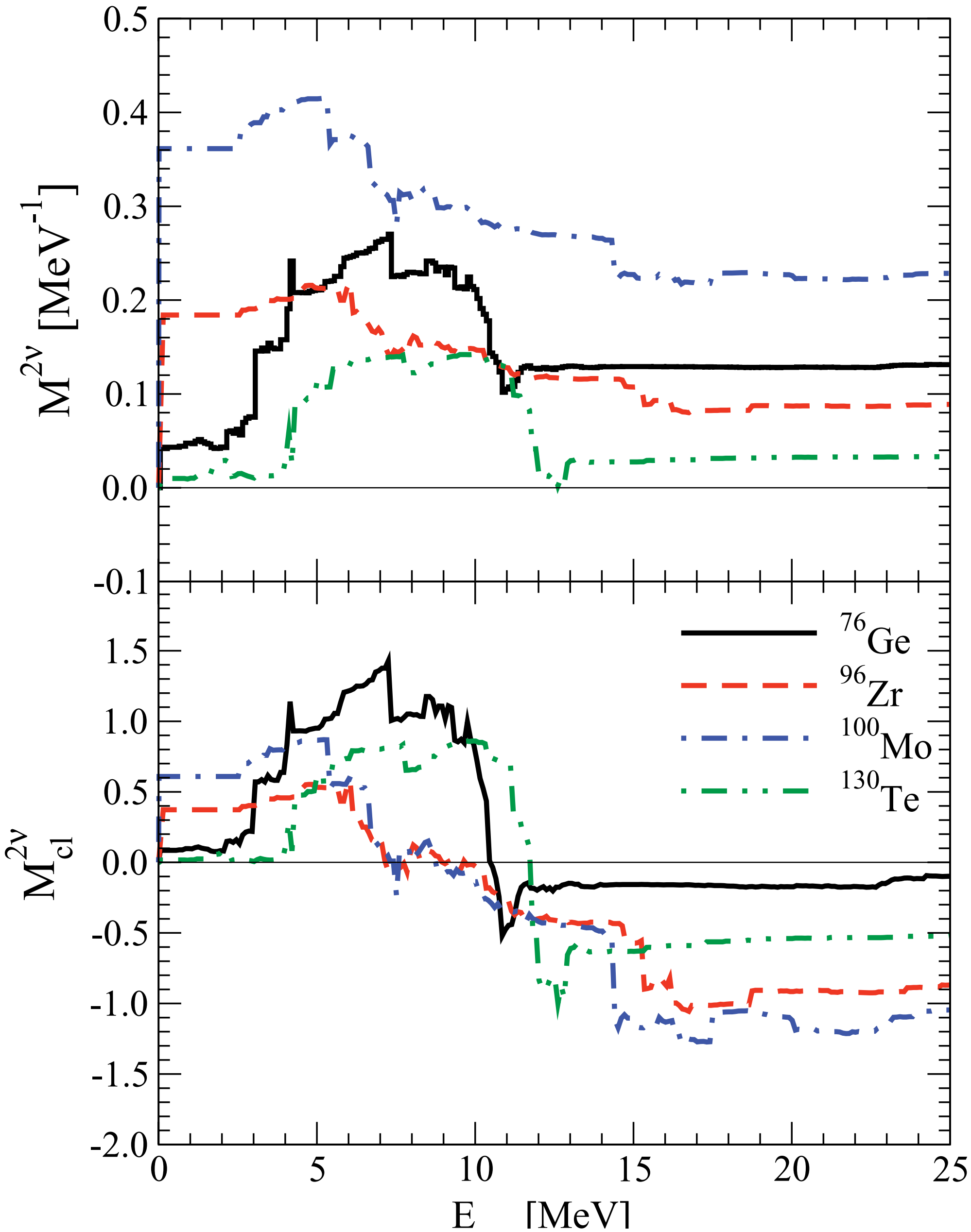
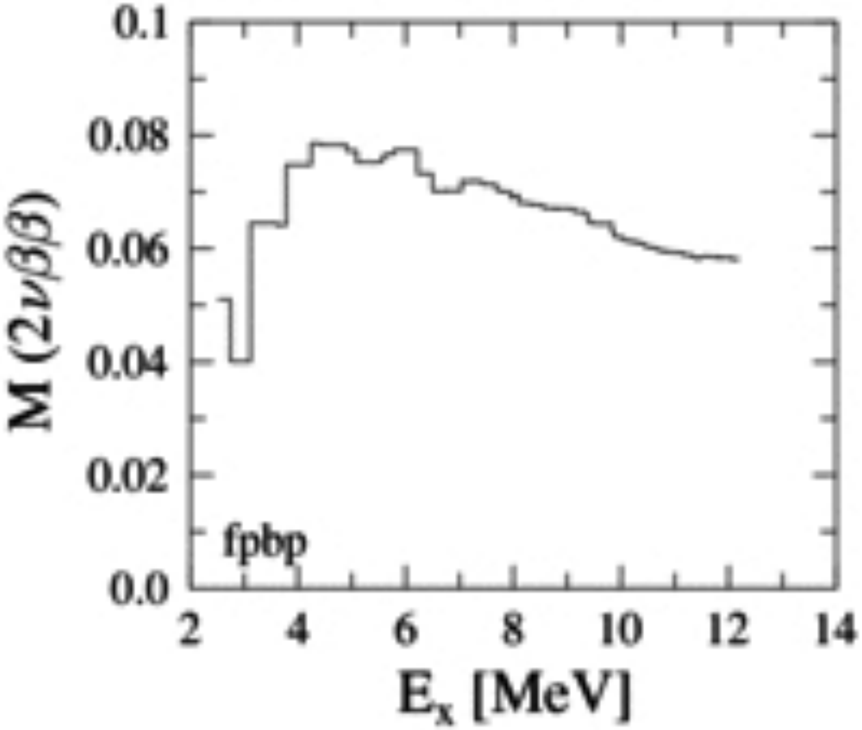
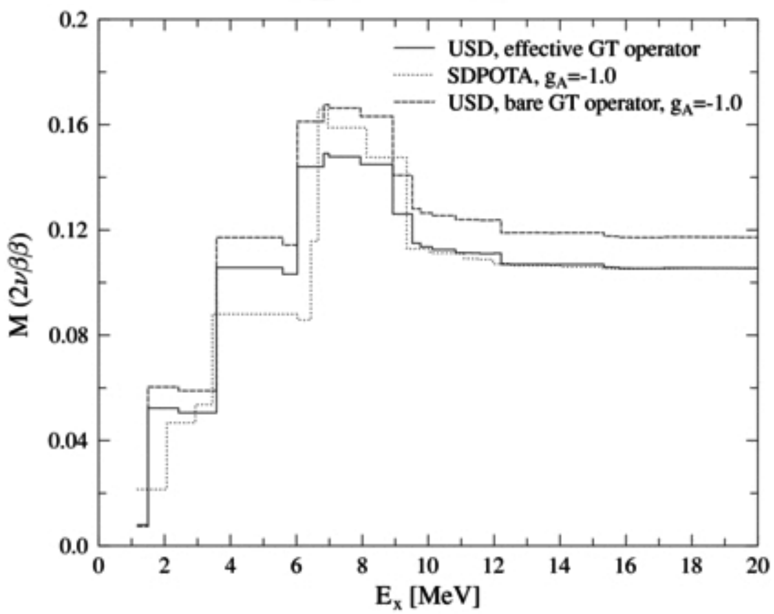


Illustration of the difficulties. In the upper panel are the contributions to the $M^{2\nu}$ from states up to E. Even though the correct value is reached (by design), it is also crossed at lower energies, followed by a drop at ~ 10 MeV.

In the lower panel the same calculation is done for $M^{2\nu}_{cl}$. In this case the high energy drop is much larger because it is not reduced by the energy denominator present in the true $M^{2\nu}$.

While the states up to ~5 MeV can be studied experimentally, the ~ 10 MeV can not. It is not clear whether they exist or not.

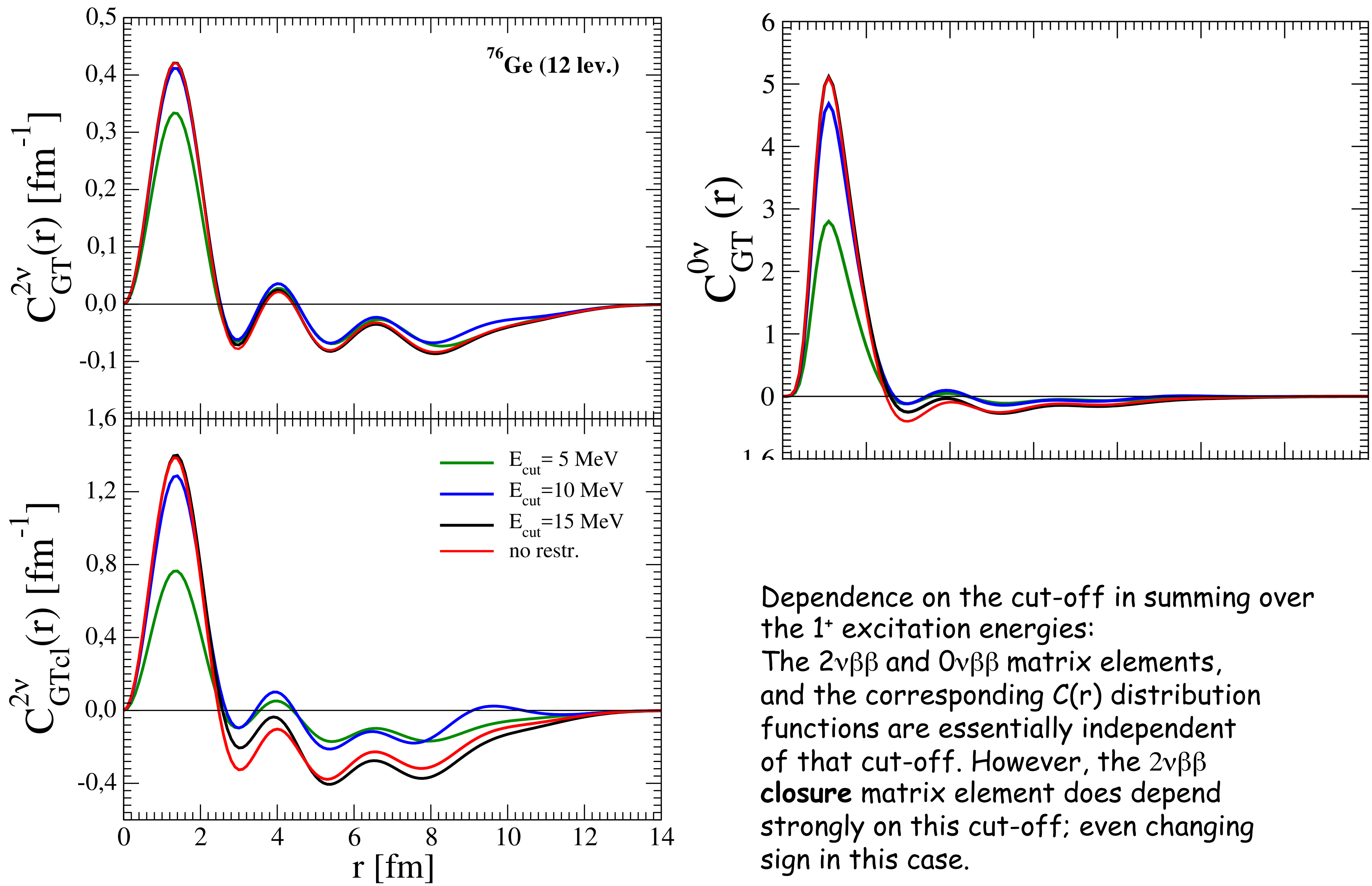




Again, this feature appears to be present in other nuclear models as well. Here are the shell model results for $M^{2\nu}$ in ^{48}Ca (upper panel) and in the model case of ^{36}Ar. (From Kortelainen and Suhonen, J. Phys. *G* **30**, 2003 (2004)).

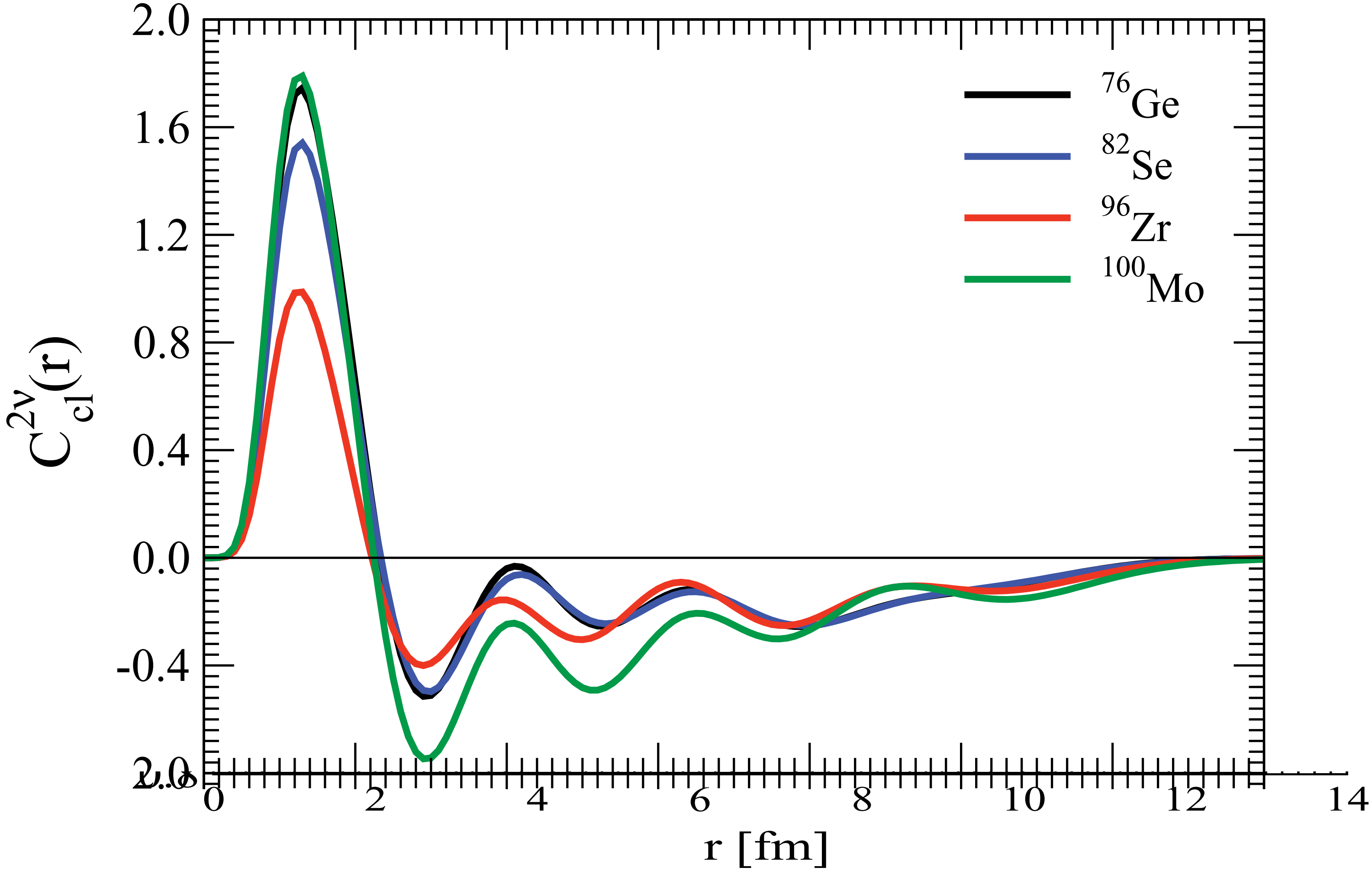
The drop at ~ 10 MeV is again visible, perhaps it is less apparent that in the heavier nuclei treated by QRPA.

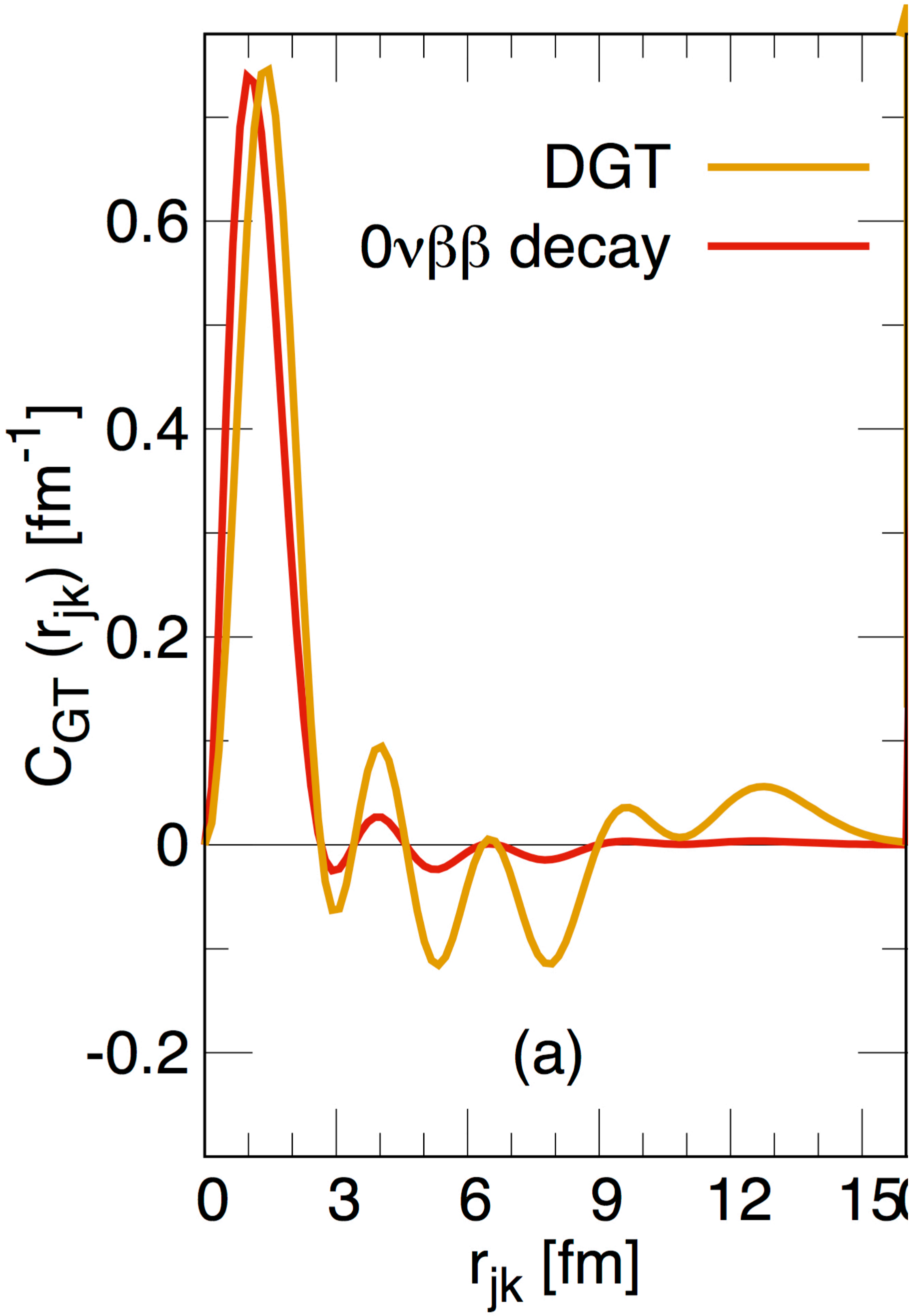
Nevertheless, the inherent uncertainty in $M^{2\nu}_{cl}$ is substantial.



Dependence on the cut-off in summing over the 1^+ excitation energies:
The $2\nu\beta\beta$ and $0\nu\beta\beta$ matrix elements, and the corresponding *C*(r) distribution functions are essentially independent of that cut-off. However, the $2\nu\beta\beta$ **closure** matrix element does depend strongly on this cut-off; even changing sign in this case.

Here are the functions $C^{2\nu}_{cl}(r)$ evaluated with QRPA for several nuclei. The peak at small r is essentially compensated by the substantial tail at larger r because $M^{2\nu}_{GTcl}$ is very small. Besides, the $C^{2\nu}_{cl}(r)$ depends on the nuclear parameters used, thus it is uncertain, particularly its tail at r > 2fm.





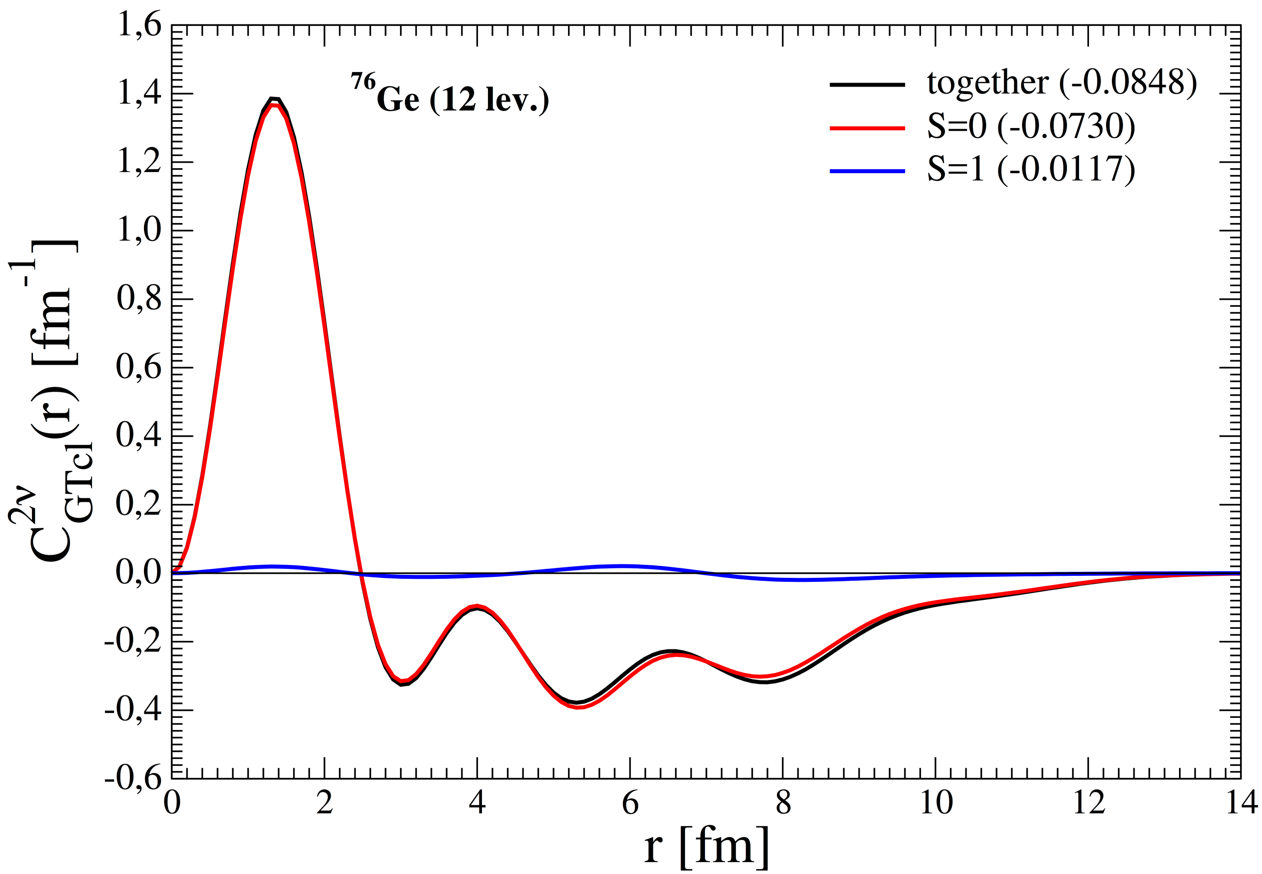
For comparison, the C(r) function for ^{136}Xe evaluated in the NSM by Shimizu et al. The yellow line corresponds to the $C^{2\nu}{}_{cl}(r)$. It is qualitatively Similar to the corresponding QRPA curve. However, differences are be expected due to the absence of the giant GT state in the NSM in this case.

Further insight can be gained by the transformation into the LS scheme. Since T=1, the S=1 requires odd parity, suppressed at small r. Hence, we expect that most contribution comes from S=0, as shown.

Further, by using the spin projection operators one can show that $M^{2\nu}_{GTcl}$ = [-3 $M^{2\nu}$(S=0) + $M^{2\nu}$(S=1)]/4 and $M^{2\nu}_{Fcl}$ = [$M^{2\nu}$(S=0) + $M^{2\nu}$(S=1)]/4 .

The same is true for the corresponding C(r) functions. Neglecting the S=1 contribution, we conclude that $C^{2\nu}_{GTcl}(r) = -3\ C^{2\nu}_{Fcl}(r)$ and naturally also

$$M^{2\nu}_{GTcl} = -3\ M^{2\nu}_{Fcl}$$



However, $M^{2\nu}_{Fcl}$ = 0 if the nuclear states have a definite isospin. This is automatic in NSM. In QRPA based on the BCS functions this requirement based on the isospin symmetry can be fulfilled without isospin projection simply by requiring that the isovector (T=1) nucleon-nucleon interaction has the same strength in the pp, nn, and np channel. That is true even if the pp and nn pairing are treated by BCS and the pn by QRPA.

In previous slide I showed that if the S=1 part is negligible, then $M^{2\nu}_{GTcl} = -3\, M^{2\nu}_{Fcl}$. Hence $M^{2\nu}_{Fcl}$ = 0 implies that also **$M^{2\nu}_{GTcl} = 0$**.
That is in agreement (essentially) with our numerical results, and in disagreement with the assumption of Shimizu et al.
It also suggests a ``restoration" of the Wigner SU(4) symmetry.

We know that $M^{2\nu}$ is small but non-vanishing. Could it be compatible with **$M^{2\nu}_{GTcl}$ = 0 ?**

Since $M^{2\nu} \times \Delta E = M^{2\nu}_{GTcl}$, where ΔE is the average energy denominator, the above is possible provided a contraintuitive $\Delta E = 0$.

But that is perfectly possible with the numerators in $M^{2\nu}$, $M^{\beta+}_{GT}(m)\ M^{\beta-}_{GT}(m)$, having positive and negative signs, despite The fact that all excitation energies are positive.

Summary

1) Only small distances, r < 2 fm, contribute to the $M^{0\nu}$.
That seems to be an universal conclusion, common to all methods where it was tested.
2) That explains, or justifies at least, why the calculated $M^{0\nu}$ change little with A or Z, unlike $M^{2\nu}$.
3) There is a close relation between $M^{0\nu}$ and the $2\nu\beta\beta$ closure matrix element $M^{2\nu}_{cl}$.
4) If $M^{2\nu}_{cl}$ or, better yet, its radial dependence $C^{2\nu}_{cl}(r)$ could be experimentally determined, it would make the determination of the $M^{0\nu}$ easier for all possible $0\nu\beta\beta$ mechanisms.
5) I argued that the assumption of Shimizu et al. that there is a proportionality between $M^{0\nu}$ and $M^{2\nu}_{cl}$ is unlikely correct, since $M^{2\nu}_{cl} \sim 0$.
6) Reliable determination of $M^{2\nu}_{cl}$ and $C^{2\nu}_{cl}(r)$ is not easy.
But more work, in theory and experiment, is needed to see how realistic this is.

spares

Calculated $M^{0\nu}$ by different methods (color coded)
The spread of the M^{0n} values for each nucleus is ~ 3. On the other hand, there is relatively little variation from one nucleus to the next.

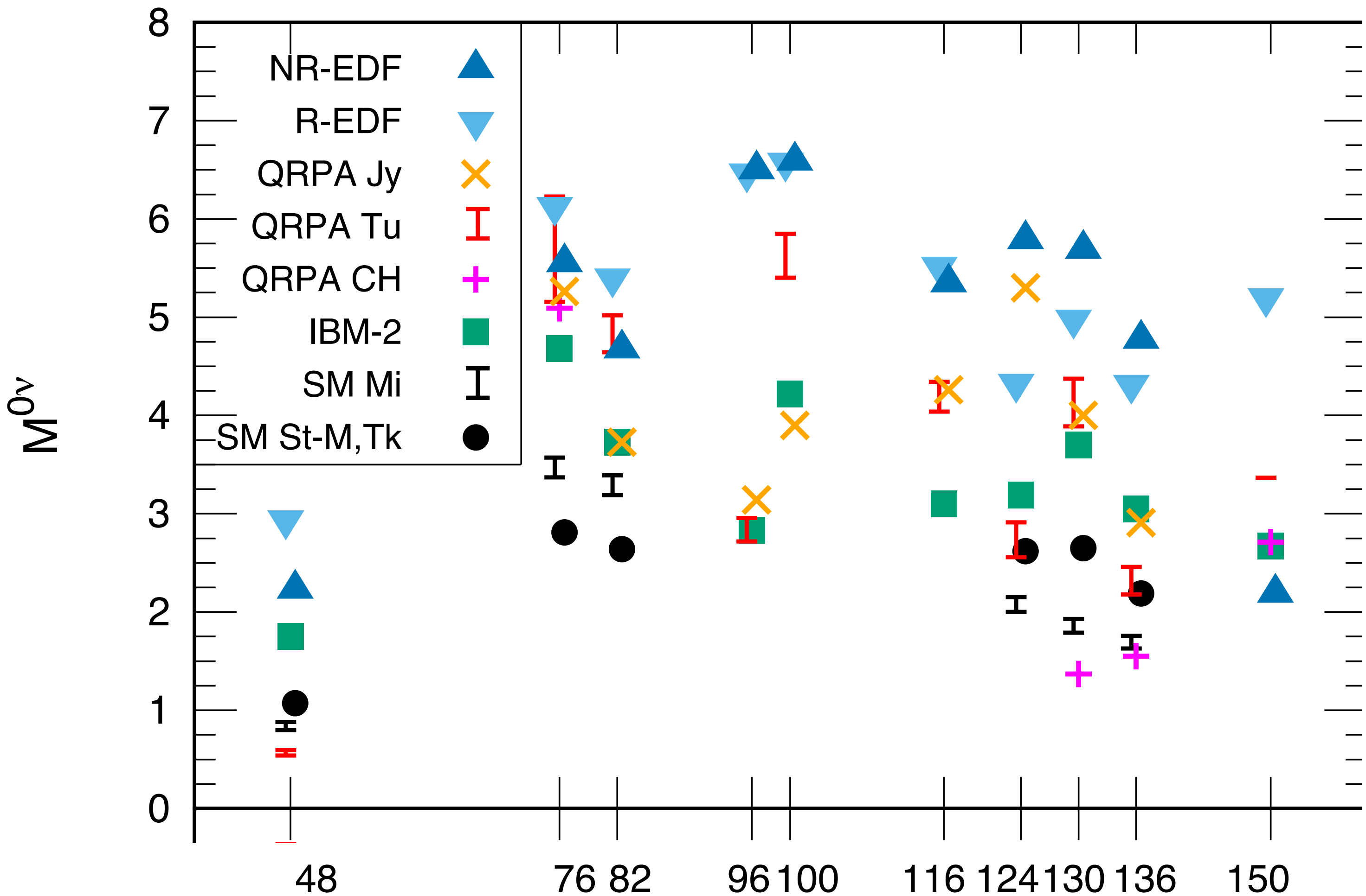
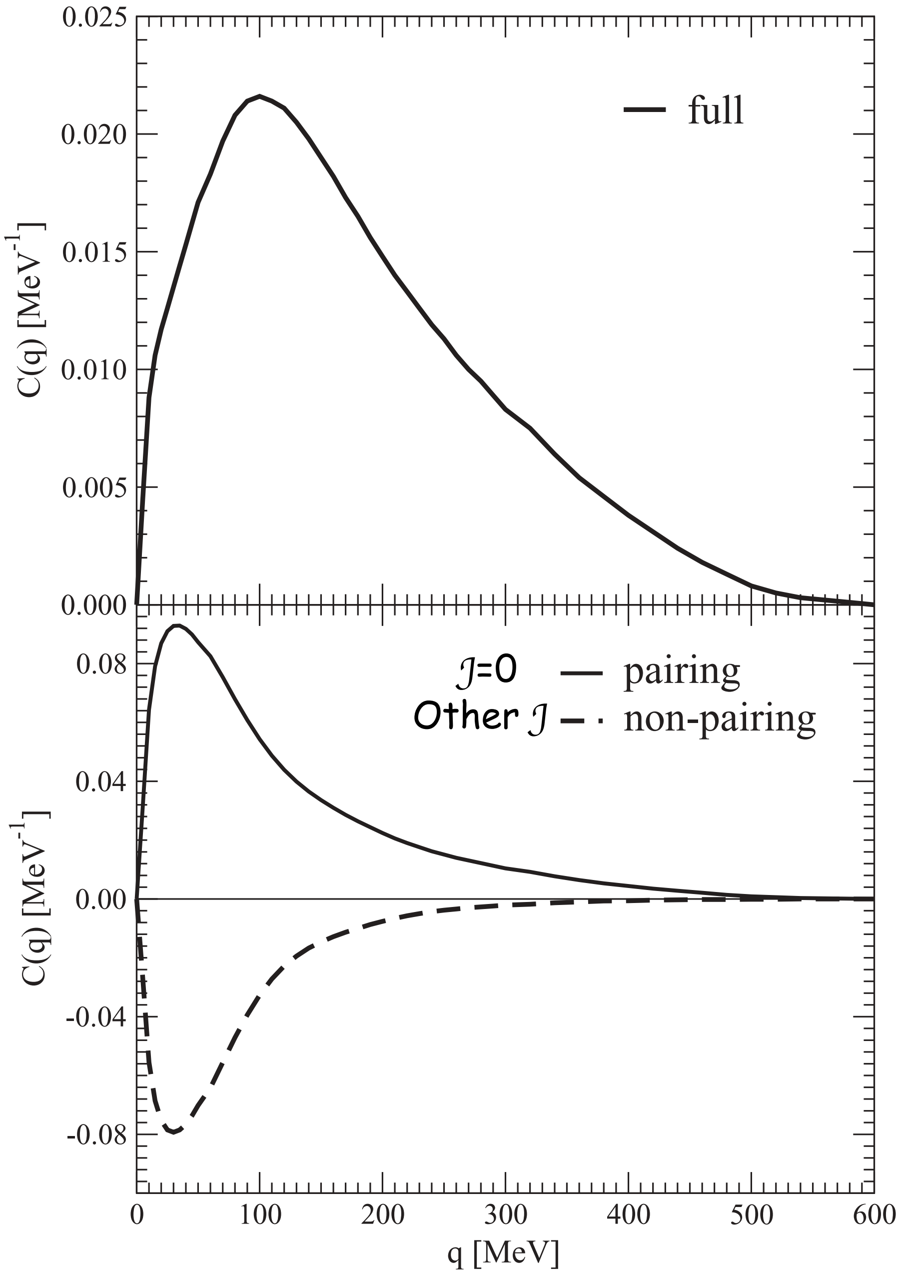
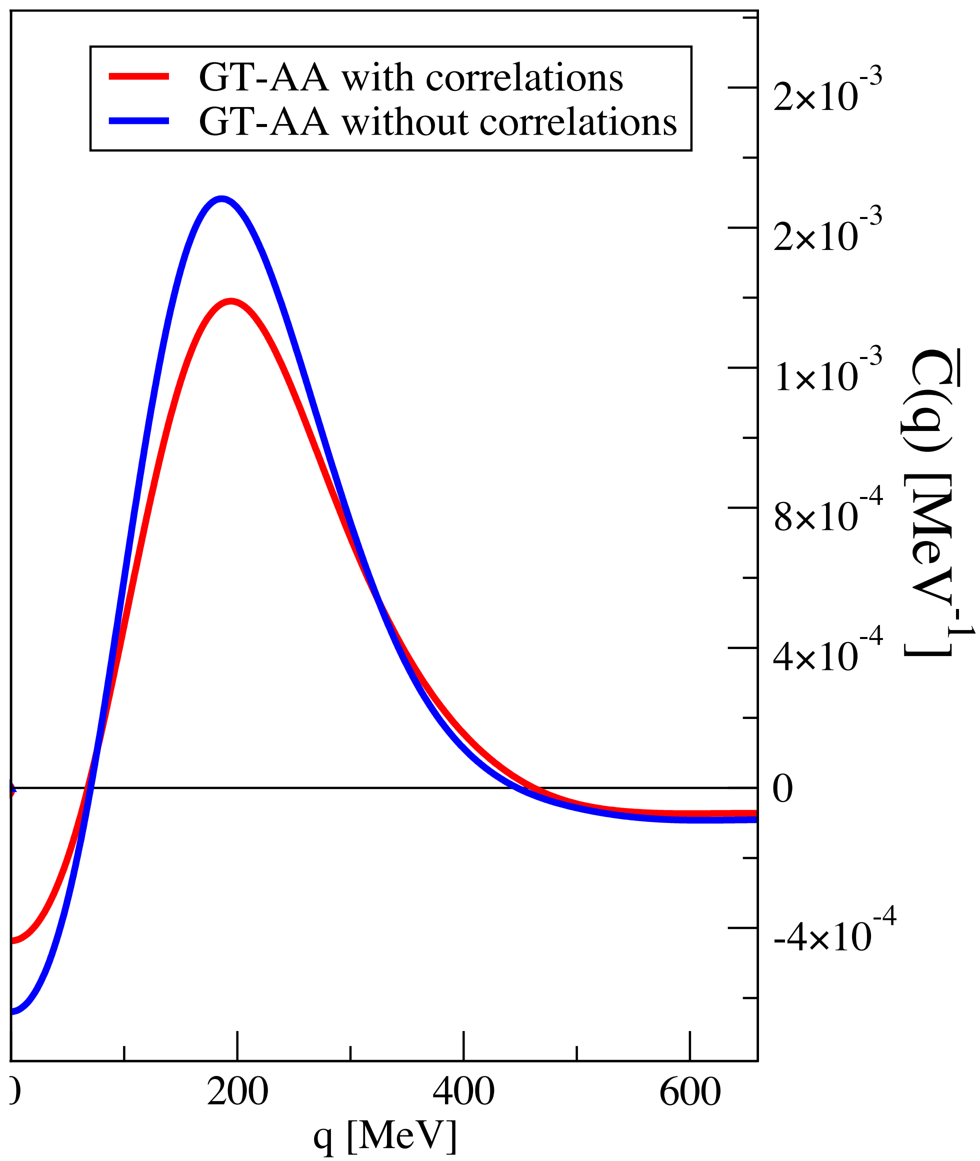


Figure from review by Engel and Menendez



From C(r) we know that the $0\nu\beta\beta$ operator has a short range character. That is also visible in the momentum analog C(q). The characteristic momentum is not $\hbar c/R$ but $\hbar c/r_0 \sim 200$ MeV.

This is again the result of cancellation between the $\mathcal{J}$=0 (pairing) part and the other $\mathcal{J}$ (broken pairs) parts. Note that the lower panel has ~ 3 times larger y scale.



Again, $C(q)$ for the hypothetical ^{10}He $0\nu\beta\beta$ decay, evaluated using the variational Monte Carlo method, with no approximation. The behavior at large values of q (q > 400 MeV) is a bit different. This has to do with the different treatment of the nucleon finite size.