SRG-evolved operators for β and ββ decay

Nuclear Theory for Double-Beta Decay and Fundamental Symmetries Topical Collaboration Winter Meeting

The University of North Carolina at Chapel Hill

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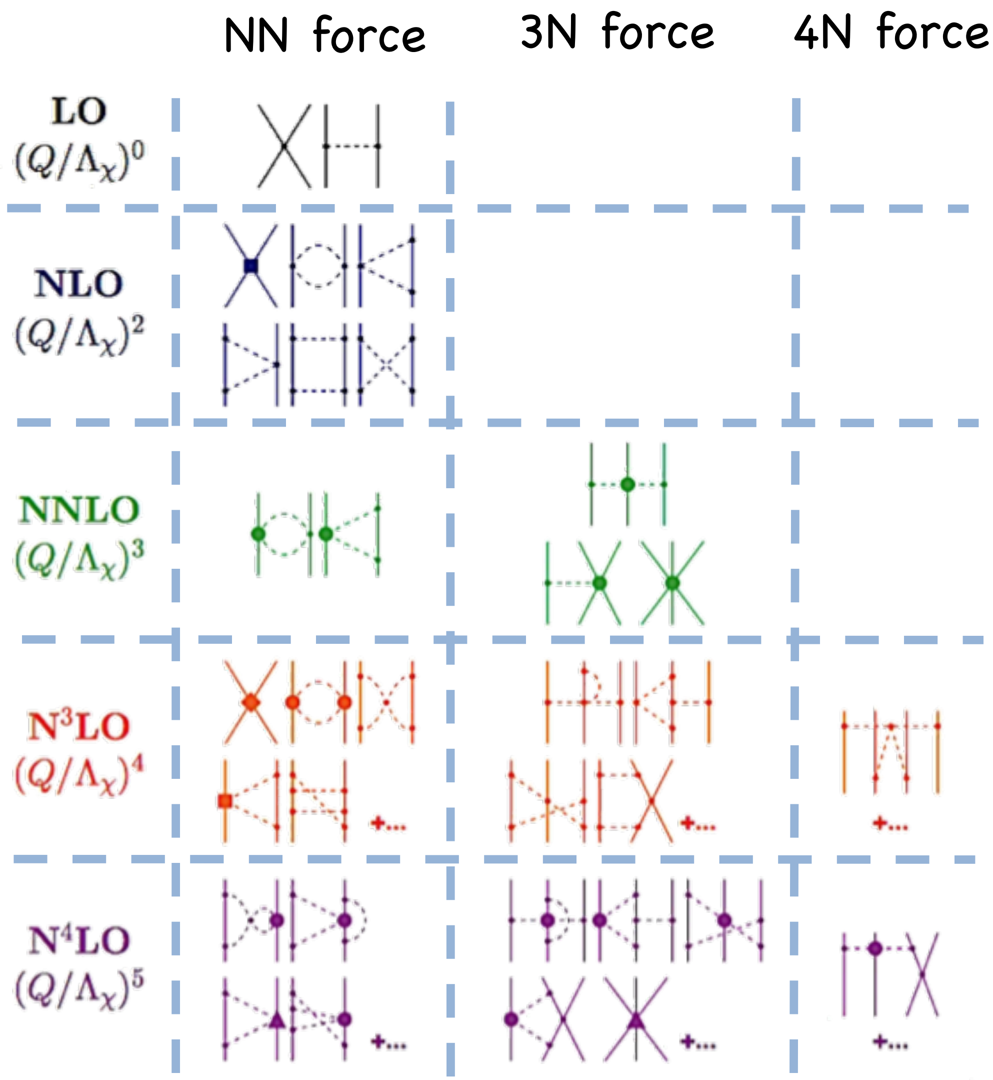
We start from the chiral effective field theory of quantum chromodynamics

Links the nuclear forces to the fundamental theory of quantum chromodynamics (QCD)

- organization in systematically improvable expansion: $(Q/\Lambda)^{\nu}$
- empirically constrained parameters capture unresolved short-distance physics

Worked out by Van Kolck, Keiser, Meissner, Epelbaum, Machleidt, ...

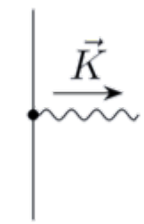


We start from the chiral effective field theory of quantum chromodynamics

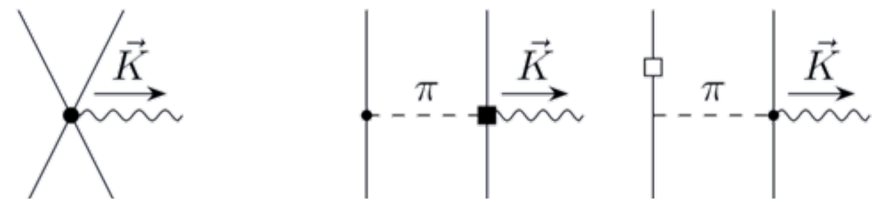
Axial-vector currents

Chiral effective field theory also provides nuclear currents

- LO: standard 1-body currents
- Sub-leading orders introduce 2-body currents (2BC or MEC)
- Application of 2BC so far largely unexplored



LO, $\sigma\tau$



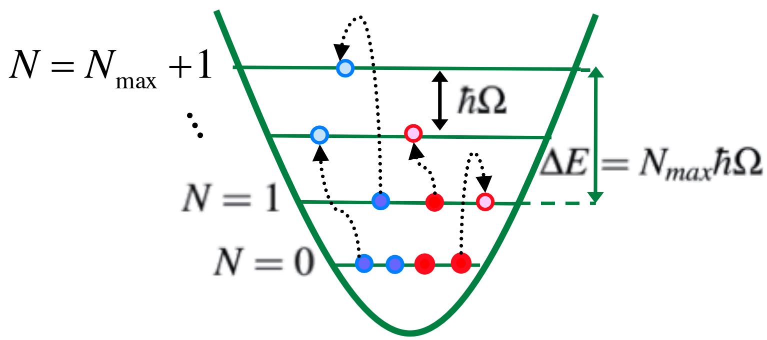
Operator for Gamow-Teller transitions

J=1 electric multiple

$$\hat{O}_{\mathrm{GT}} = g_{\mathrm{A}}^{-1}\sqrt{6\pi}\hat{E}_1^A(\vec{K}=0) = \hat{O}_{\sigma\tau}^{1b} + \hat{O}_{2\mathrm{BC}}^{2b}$$

To constrain the chiral EFT parameters we need efficient few- and many-body methods

- Ab initio no-core shell model (NCSM)
  - Superposition of harmonic oscillator (HO) wave functions
  - 'Diagonalize' Hamiltonian matrix
  - Short- and medium-range correlations
  - **Bound states, narrow resonances**
  - Relative coordinates and/or slater-determinant (SD) basis



$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^{HO}(\vec{\eta}_1, \vec{\eta}_2, \ldots, \vec{\eta}_{A-1})$$ relative coordinates

$$\Psi_{\mathrm{SD}}^A = \sum_{N=0}^{N_{\max}} \sum_j c_{Nj}^{\mathrm{SD}} \Phi_{\mathrm{SD}\,Nj}^{HO}(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A) = \Psi^A \varphi_{000}(\vec{R}_{CM})$$ SD basis

To constrain the chiral EFT parameters we need efficient few- and many-body methods

- Ab initio NCSM with continuum (NCSMC)
  - Generalized cluster expansion
  - Discrete NCSM eigenstates: short- and medium-range correlations
  - Continuous microscopic-cluster states: cluster dynamics, long range
  - Bound states, narrow and broad **resonances, scattering states**

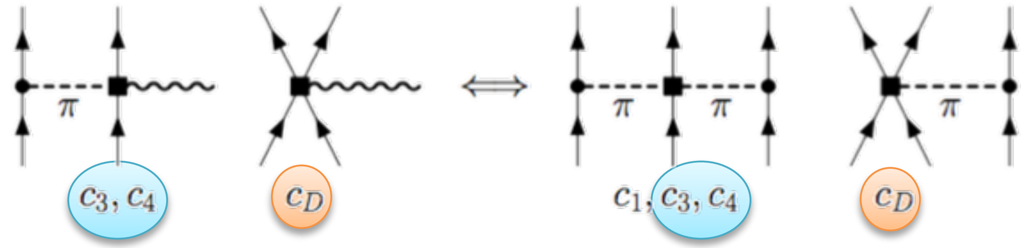
NCSM eigenstates | **NCSM/RGM continuous states**

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| ^{(A)}, \lambda \right\rangle + \sum_{\nu} \int d\vec{r}\, u_{\nu}(\vec{r})\, \hat{A}_{\nu} \left| \vec{r} \;\; _{(A-a)} \;\; _{(a)}, \nu \right\rangle$$

Unknowns: c_{λ}, $u_{\nu}(\vec{r})$

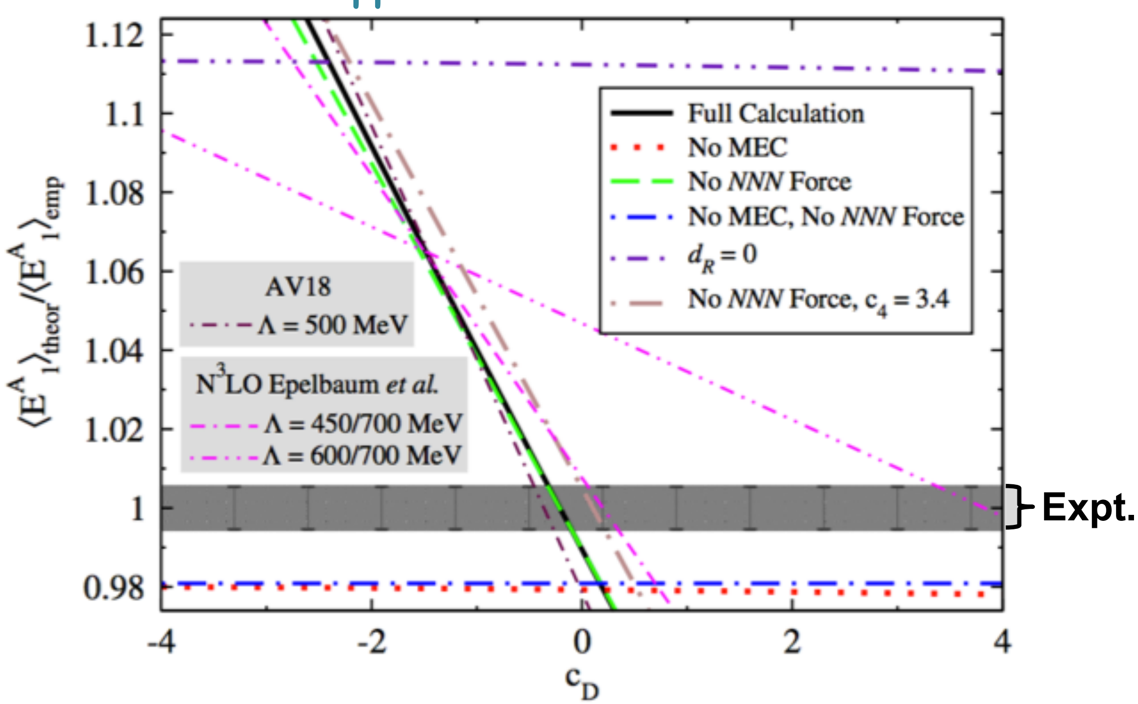
The two-body axial-vector share their parameters with the NN and 3N forces

Park et al., Gardestig & Phillips, ...



- ***c_D*:** shared with one-pion exchange plus two-nucleon contact 3N force
- ***c_3,c_4*:** shared with two-pion-exchange NN and 3N forces
- ^{3}H β-decay half-life can be used to constrain c_D
- **Note:** results all but insensitive to 3N force!

First applied to N^{3}LO NN + N^{2}LO 3N



D. Gazit, SQ, P. Navratil, PRL103, 102502

To decouple low- and high-momentum components, promote convergence, use unitary transformations

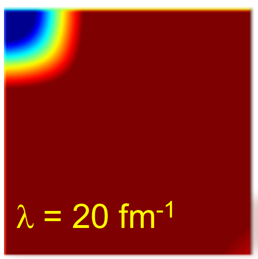
- Similarity Renormalization Group (SRG) method

E.D. Jurgenson et al., Phys. Rev. Lett. **103**, 082501 (2009)

$$H_\lambda = U_\lambda H_{\lambda=\infty} U_\lambda^+ \Rightarrow \frac{dH_\lambda}{d\lambda} = -\frac{4}{\lambda^5}\left[\eta(\lambda), H_\lambda\right]$$

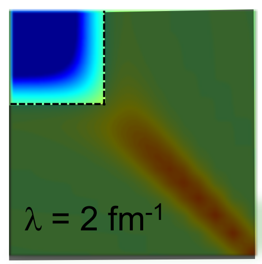
Unitary transformations

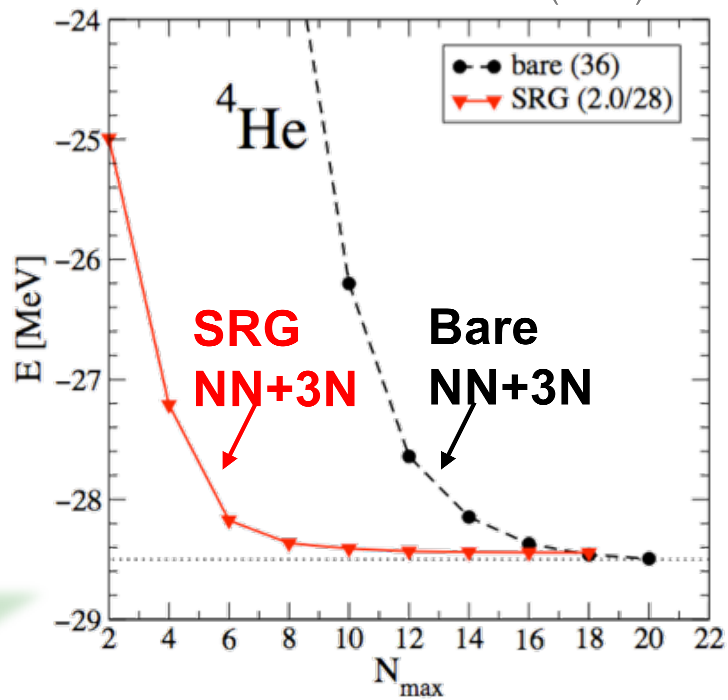
Flow parameter



Decouples low and high momenta

Induces 3-body (& higher-body) forces





For the lightest nuclei SRG-evolved NN+3N forces allow to obtain unitarily equivalent results in much smaller model spaces

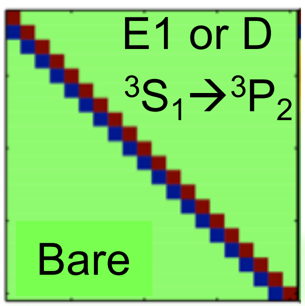
SRG evolution of transition operators

Schuster, S.Q., Johnson, Jurgenson, Navratil, PRC 90, 011301(R); PRC 92, 014320

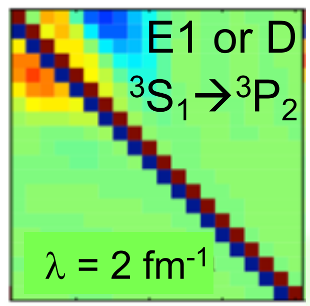
$$\hat{O}_\lambda = \hat{U}_\lambda \hat{O}_{\lambda=\infty} \hat{U}^*_\lambda ; \quad \hat{U}_\lambda = \sum_\alpha |\psi_\alpha(\lambda)\rangle\langle\psi_\alpha(\lambda=\infty)|$$

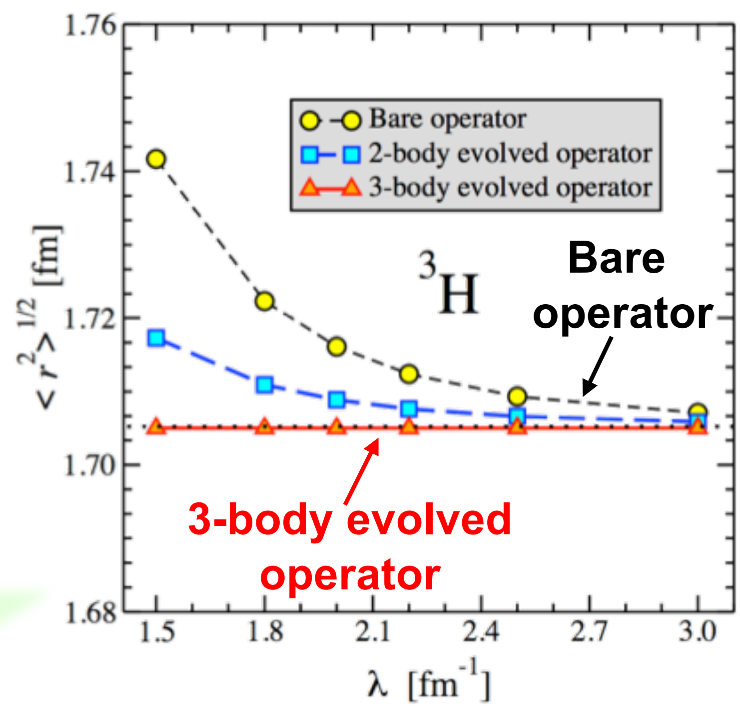
Final/initial unitary transformations

Eigenstates after & before evolution



Induces 2-body (& higher-body) operators





- In practice $U_\lambda^{J^\pi T T_z}$ is block-diagonal. For a general transition operator one has to compute and store all relevant blocks.

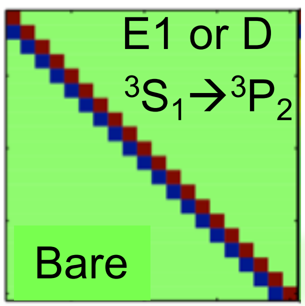
SRG evolution of transition operators

Schuster, S.Q., Johnson, Jurgenson, Navratil, PRC 90, 011301(R); PRC 92, 014320

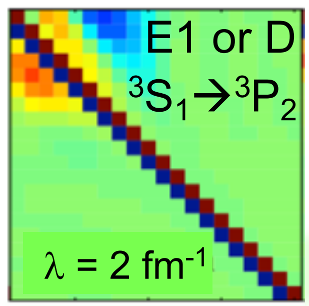
$$\hat{O}_\lambda = \hat{U}_\lambda \hat{O}_{\lambda=\infty} \hat{U}^*_\lambda; \quad \hat{U}_\lambda = \sum_\alpha |\psi_\alpha(\lambda)\rangle\langle\psi_\alpha(\lambda=\infty)|$$

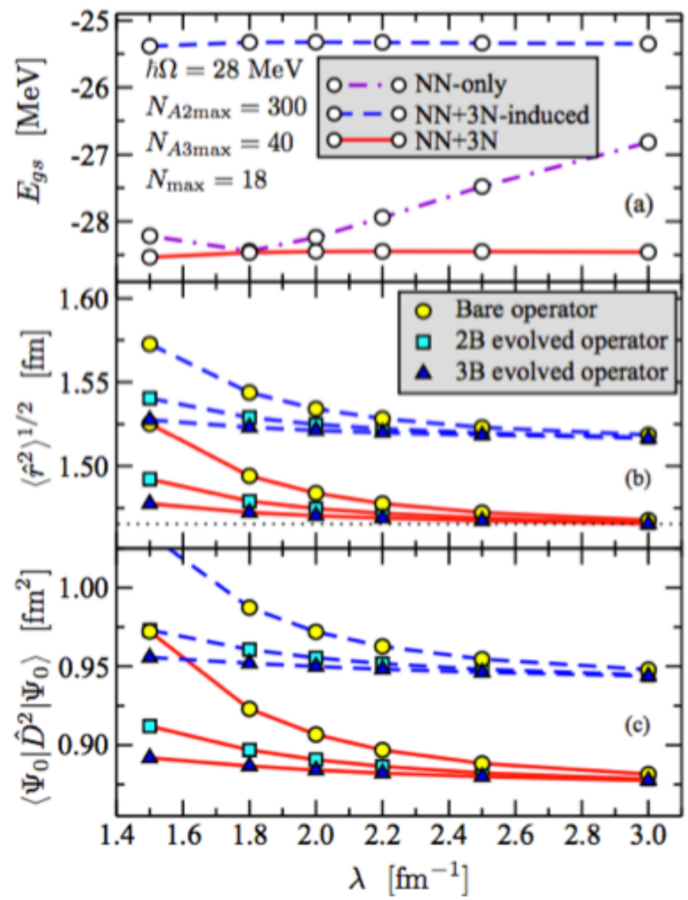
Final/initial unitary transformations

Eigenstates after & before evolution



Induces 2-body (& higher-body) operators





In ^{4}He, the inclusion of up to three-body induced terms all but completely restores the invariance of transitions under SRG

SRG 2-body evolution of transition operators

Schuster, S.Q., Johnson, Jurgenson, Navratil, PRC 90, 011301(R); PRC 92, 014320

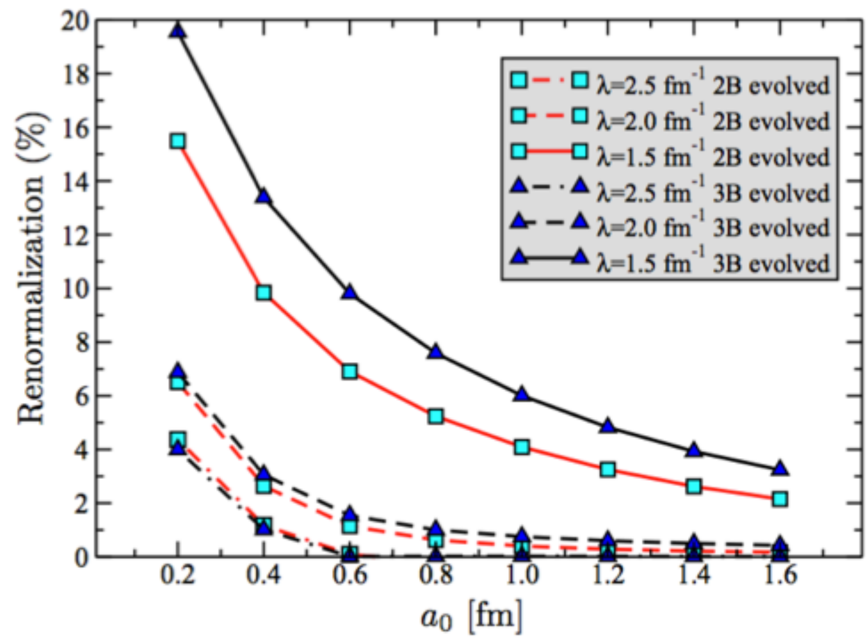
$$\hat{O}_\lambda = \hat{U}_\lambda \hat{O}_{\lambda=\infty} \hat{U}^*_\lambda; \quad \hat{U}_\lambda = \sum_\alpha |\psi_\alpha(\lambda)\rangle\langle\psi_\alpha(\lambda=\infty)|$$

Final/initial unitary transformations

Eigenstates after & before evolution

$$\hat{O}(\vec{r}_1, \vec{r}_2) = \mathcal{A} \exp\left(-\frac{(\vec{r}_1 - \vec{r}_2)^2}{a_0^2}\right)$$

$$\mathcal{A} \int \exp\left(-\frac{r^2}{a_0^2}\right) d\vec{r} = 1$$

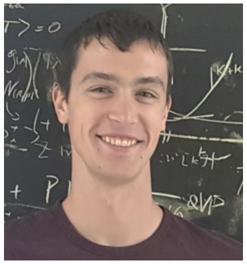


1) The shorter the range, the more renormalization
2) The 3B contribution relatively more important for the longer range

SRG 2-body evolution of general operators

Gysbers, Calci, Navratil, S.Q., Gazit, Wendt

- Evolution up to 2-body level determined in A=2 space
  - Carried out in the relative-coordinate 2-nucleon basis using the code NCSMV2B
  - Implemented for general operators of any rank
  - Matrix elements of evolved operator are also converted to the single-particle SD basis
  - Application to chiral 'Gamow-Teller' operator, 0νββ operators



P. Gysbers

$$\hat{O}_{GT} = \hat{O}^{(1)}_{\sigma\tau} + \hat{O}^{(2)}_{2BC}$$

bare operator

$$\hat{O}_{GT;\lambda} = \hat{O}^{(1)}_{\sigma\tau} + \hat{O}^{(2)}_{\sigma\tau;\lambda} + \hat{O}^{(2)}_{2BC;\lambda} + \cdots$$

evolved operator

Gamow-Teller $^3H \to {}^3He$ transition matrix element

Gysbers, Calci, Navratil, S.Q., Gazit, Wendt

$$\hat{O} = \hat{O}^{(1)}_{\sigma\tau} \to \hat{O}_{\lambda} = \hat{O}^{(1)}_{\sigma\tau} + \hat{O}^{(2)}_{\sigma\tau;\lambda} + \cdots$$

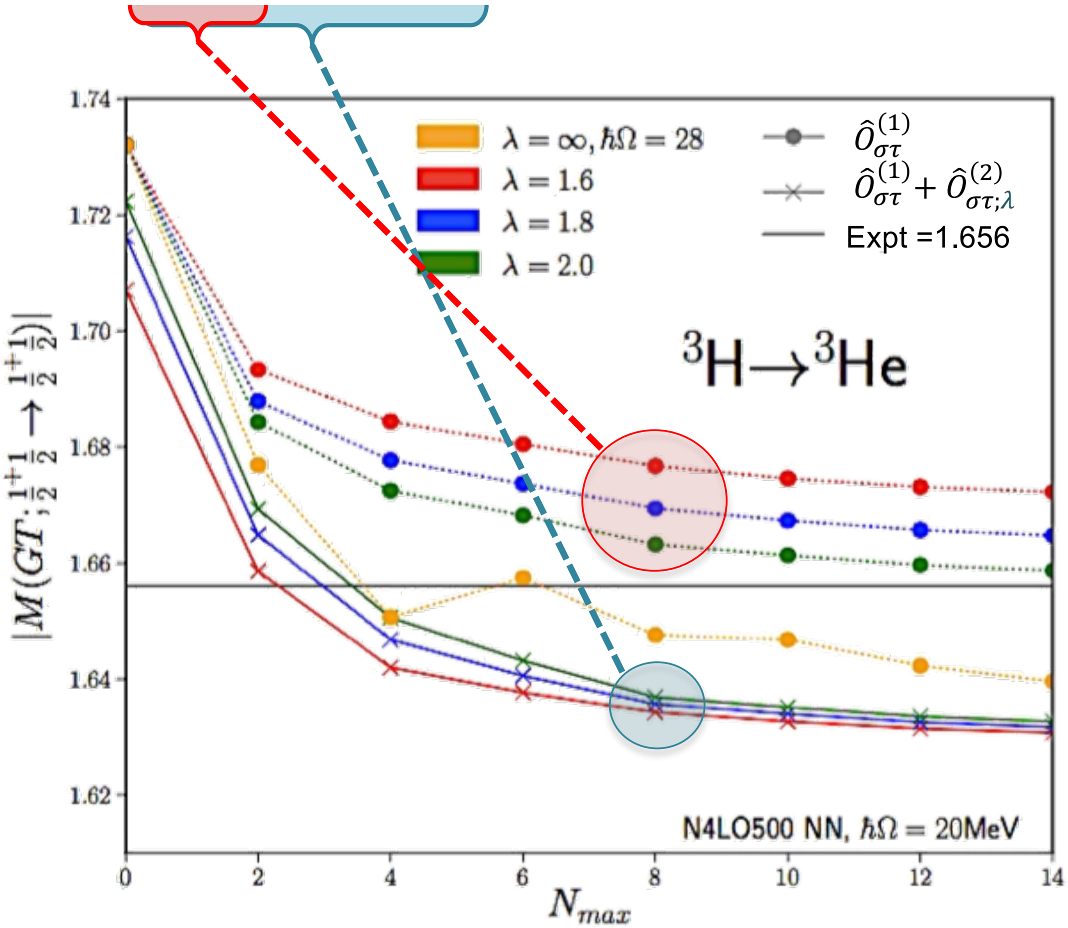
Operator:

Standard $\sigma\tau$ (1-body)

$$\left\langle \hat{O}^{(2)}_{\sigma\tau;\lambda} \right\rangle_{A=2} = \left\langle \left(\hat{O}^{(1)}_{\sigma\tau} \right)_{\lambda} \right\rangle_{A=2} - \left\langle \hat{O}^{(1)}_{\sigma\tau} \right\rangle_{A=2}$$

Interaction: N^4LO NN

- Chiral NN @ N^4LO from Entem, Machleidt & Nosyk, Λ_χ = 500 MeV



Gamow-Teller $^3H \rightarrow ^3He$ transition matrix element

Gysbers, Calci, Navratil, S.Q., Gazit, Wendt

$$\hat{O} = \hat{O}_{\sigma\tau}^{(1)} + \hat{O}_{2BC}^{(2)} \rightarrow \hat{O}_\lambda = \hat{O}_{\sigma\tau}^{(1)} + \hat{O}_{\sigma\tau;\lambda}^{(2)} + \hat{O}_{2BC;\lambda}^{(2)} + \cdots$$

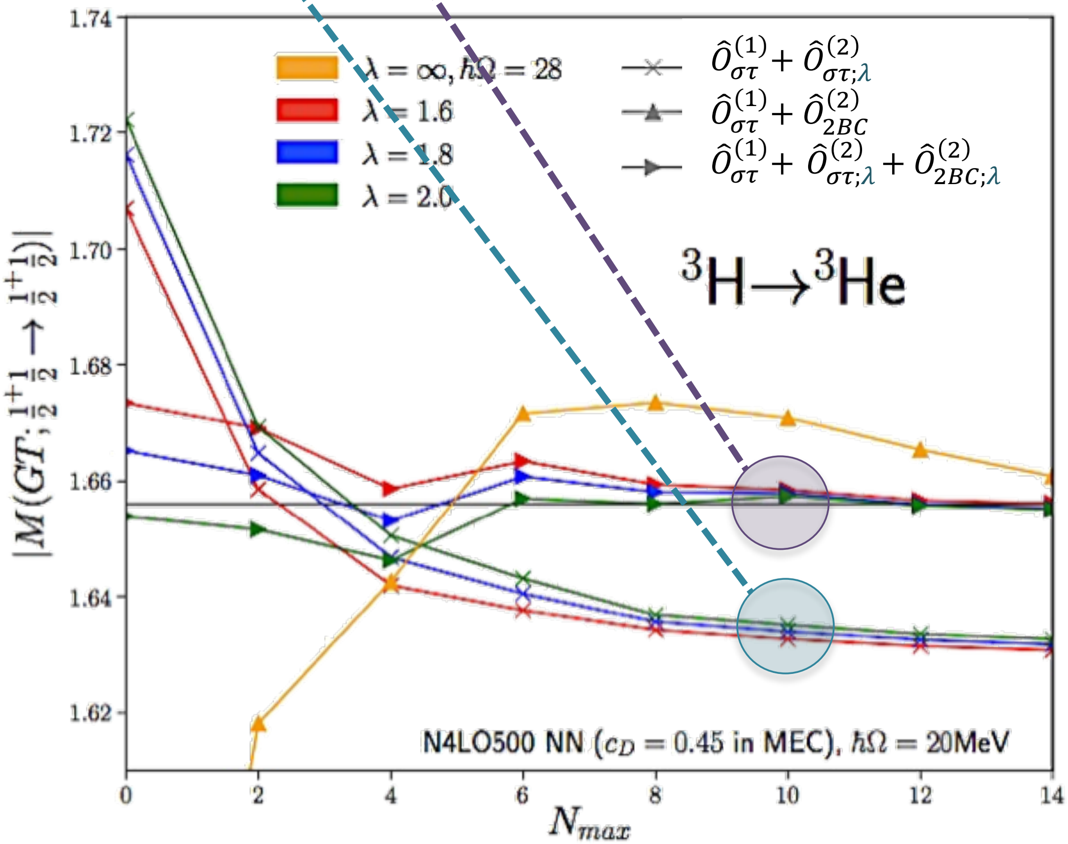
Operator:

$\sigma\tau$ (1-body) + chiral 2BC (2-body)

Interaction: N^4LO NN

- Chiral NN @ N^4LO from Entem, Machleidt & Nosyk, Λ_χ= 500 MeV
- Determined: $c_D = 0.45$

3N force repulsive!



Gamow-Teller ^{6}He→^{6}Li transition matrix element

Gysbers, Calci, Navratil, S.Q., Gazit, Wendt

$$\hat{O} = \hat{O}^{(1)}_{\sigma\tau} + \hat{O}^{(2)}_{2BC} \rightarrow \hat{O}_{\lambda} = \hat{O}^{(1)}_{\sigma\tau} + \hat{O}^{(2)}_{\sigma\tau;\lambda} + \hat{O}^{(2)}_{2BC;\lambda} + \cdots$$

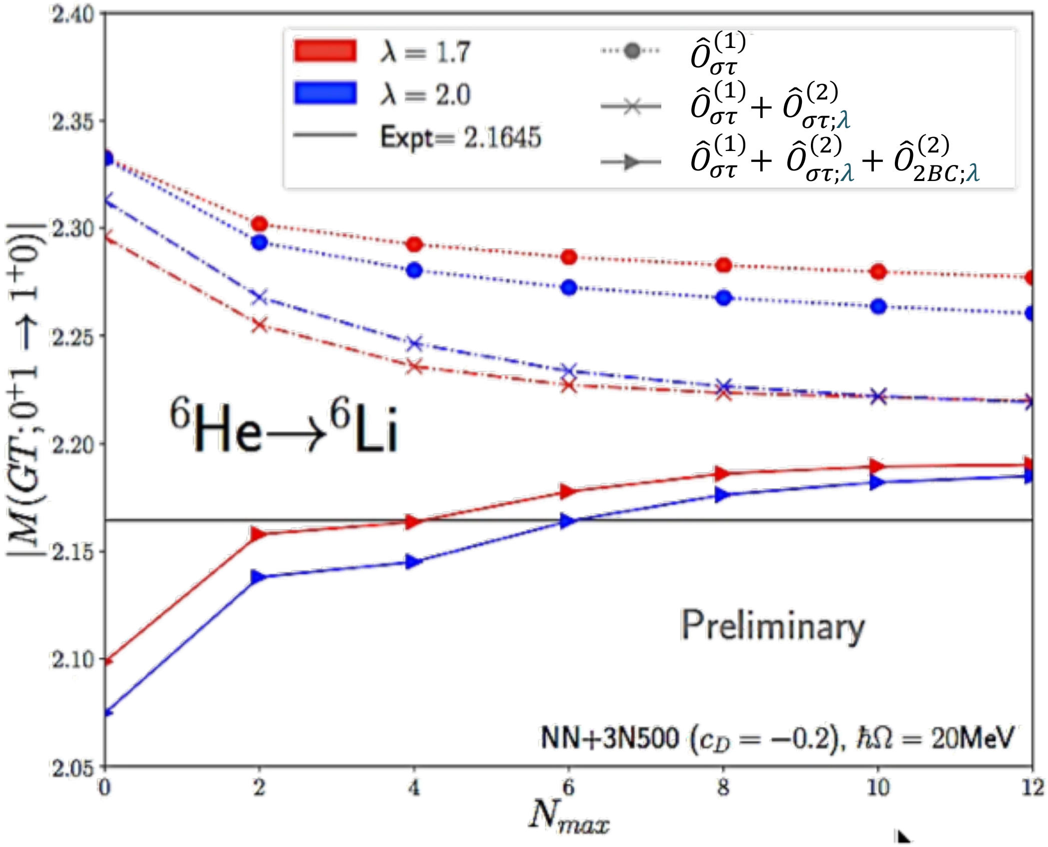
Operator:

$\sigma\tau$ (1-body) + chiral 2BC (2-body)

Interaction: NN+3N(500)

- Chiral NN @ N^{3}LO from Entem & Machleidt, Λ_χ= 500 MeV
- Local chiral 3N @ N^{2}LO from Navratil, Λ_χ= 500 MeV

Still missing: clustering, continuum effects



New set of chiral forces

Gysbers, Navratil, S.Q.

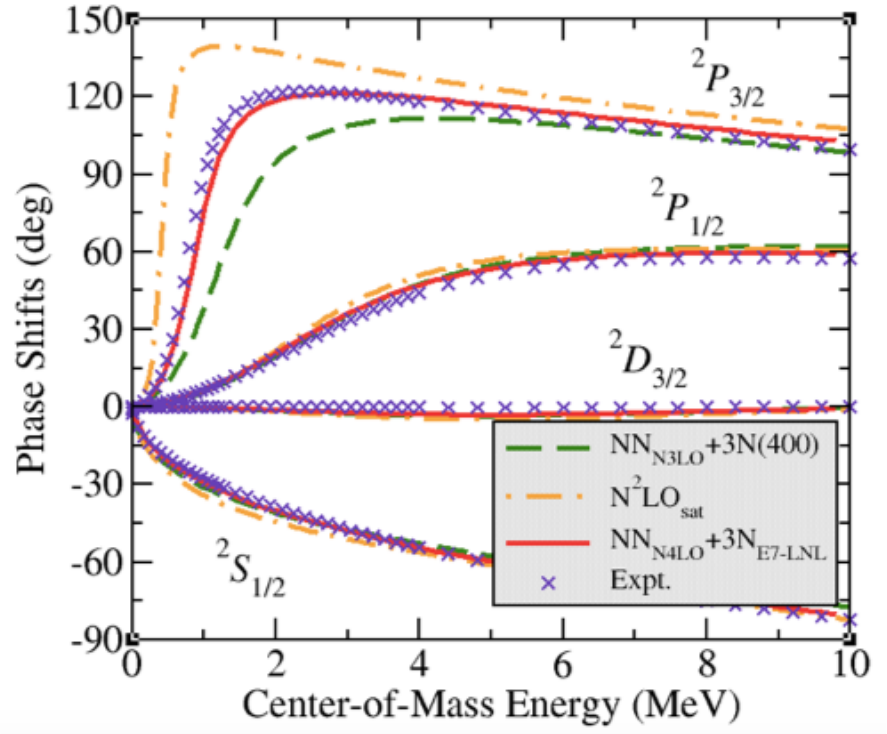
$NN_{N4LO}+3N_{LNL}$

- Chiral NN @ N^4LO from Entem & Machleidt & Nosyk, Λ_χ= 500 MeV
- Chiral 3N @ N^2LO from Navratil with local (650 MeV cutoff) and non-local (500 MeV cutoff) regulators
  - Constrained on ^3H β-decay half-life and binding energy

$NN_{N4LO}+3N_{LNL+E7}$

- $NN_{N4LO}+3N_{LNL}$ plus E7 spin-orbit term of the 3N force at N^4LO by Girlanda, Kyevsky & Viviani
  - Constrained on n-^4He phase shifts (SQ)

n+^{4}He scattering



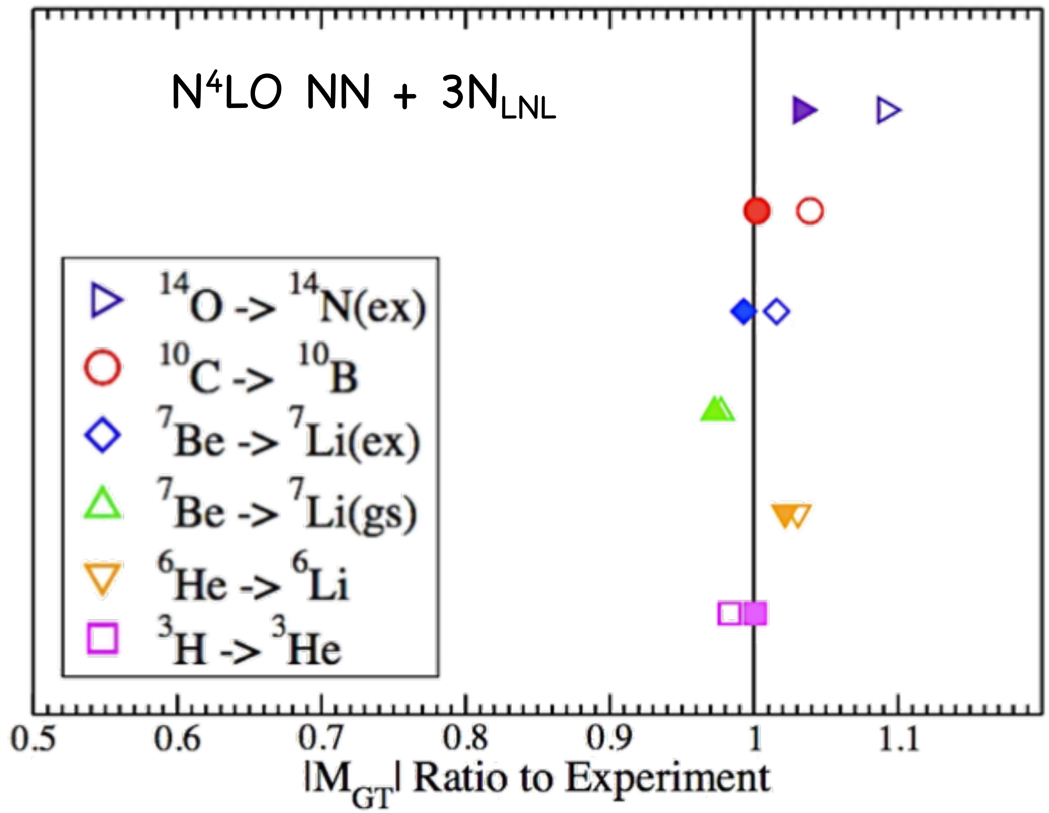
Large Gamow-Teller transition in light nuclei

Gysbers, Navratil, S.Q.

- ○ $\sigma\tau \rightarrow \hat{O}^{(1)}_{\sigma\tau} + \hat{O}^{(2)}_{\sigma\tau;\lambda}$
- ● $\sigma\tau + 2BC \rightarrow \hat{O}^{(1)}_{\sigma\tau} + \hat{O}^{(2)}_{\sigma\tau;\lambda} + \hat{O}^{(2)}_{2BC;\lambda}$

Interaction: N^4LO NN + $3N_{LNL+E7}$

- Chiral NN @ N^4LO from Entem & Machleidt & Nosyk Λ_χ= 500 MeV
- Chiral 3N @ N^2LO from Navratil, with local (650 MeV) & non-local (500 MeV) regulators



Large Gamow-Teller transition in light nuclei

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○ $\sigma\tau \rightarrow \hat{O}^{(1)}_{\sigma\tau} + \hat{O}^{(2)}_{\sigma\tau;\lambda}$

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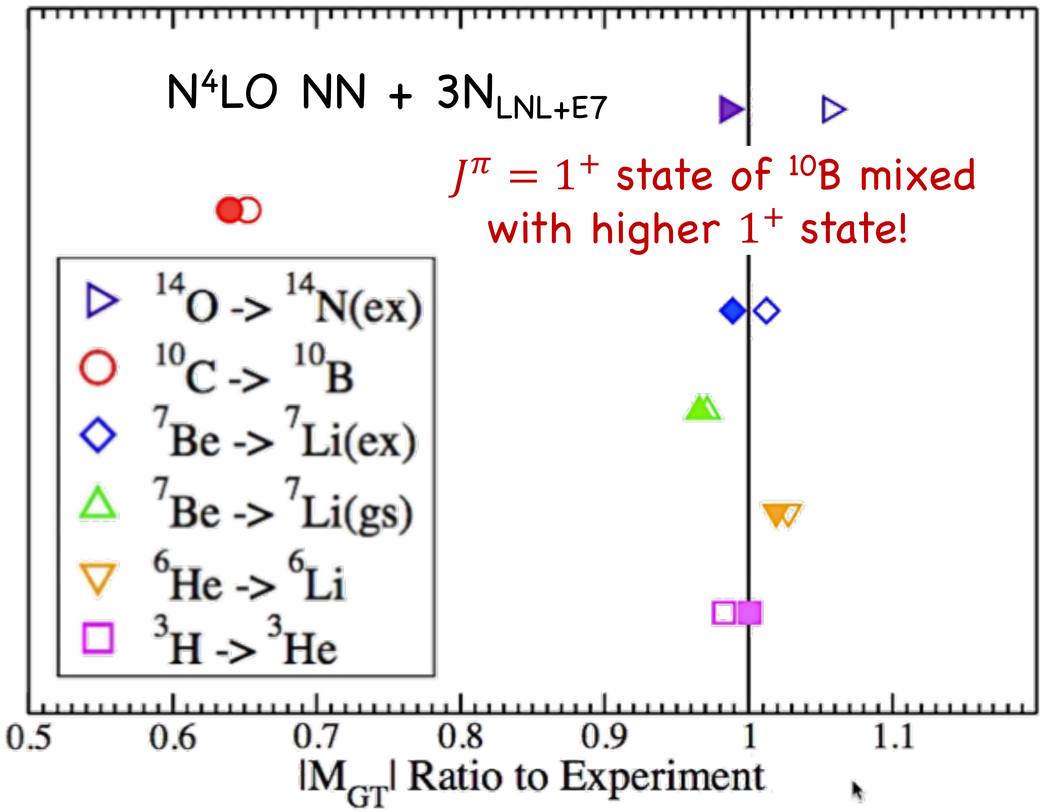
Interaction: N^4LO NN + $3N_{LNL+E7}$

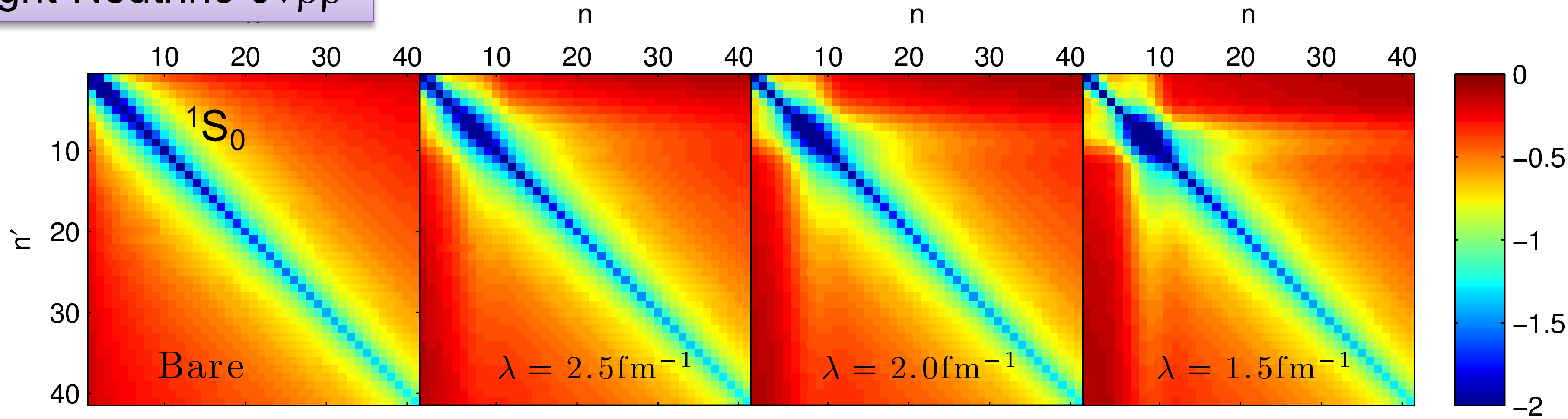
- Chiral NN @ N^4LO from Entem & Machleidt & Nosyk Λ_χ = 500 MeV
- Chiral 3N @ N^2LO from Navratil, with local (650 MeV) & non-local (500 MeV) regulators
- Chiral E7 term of 3N @ N^4LO from Girlanda, Viviani & Kievsky



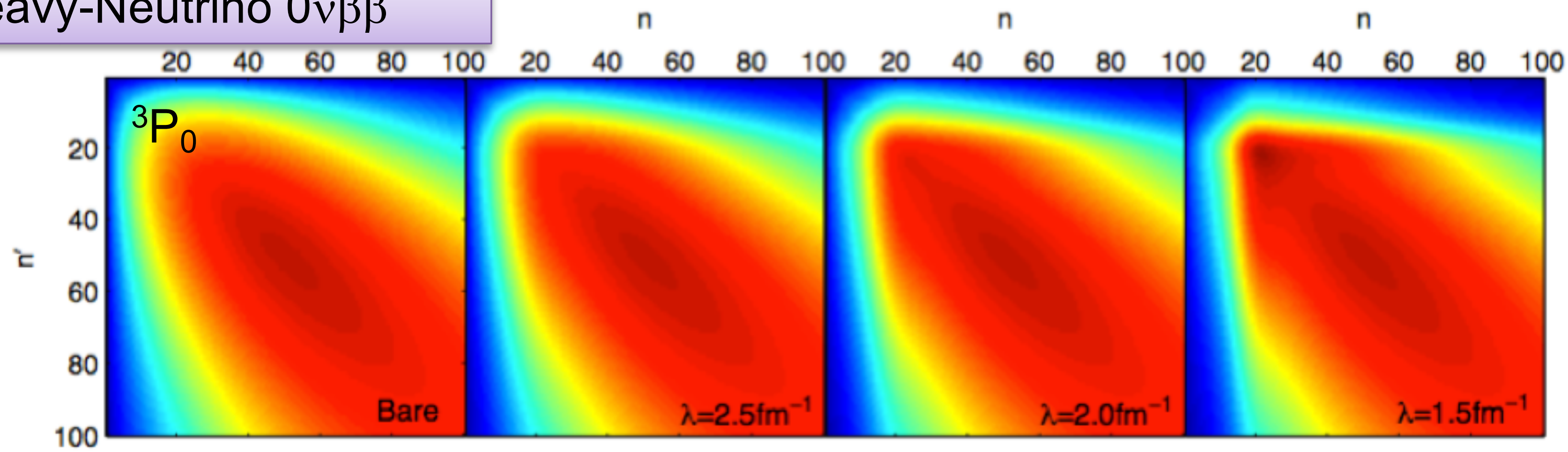
SRG 2-body evolution of the 0νββ operator

In collaboration with Schuster, Horoi, Engel, Holt, Navratil

Light-Neutrino 0νββ



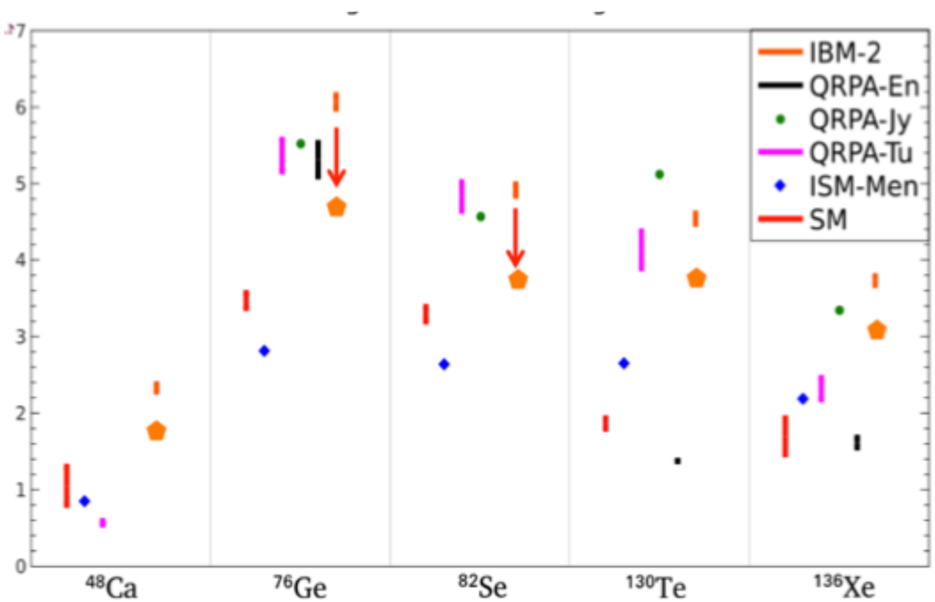
Heavy-Neutrino 0νββ

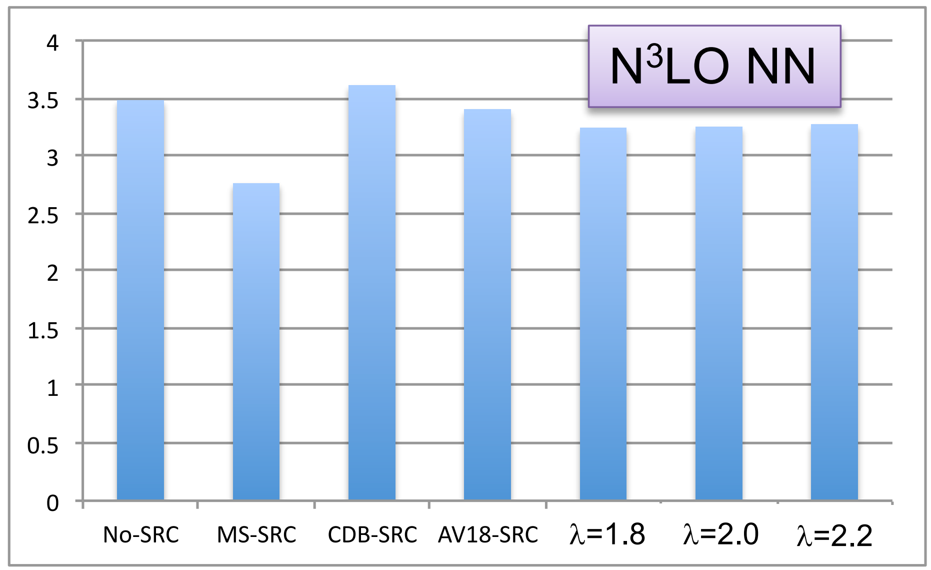


Application to ^{76}Ge $0\nu\beta\beta$ matrix elements

In collaboration with Schuster, Horoi, Engel, Holt, Navratil

- Matrix elements for light-neutrino exchange mechanism





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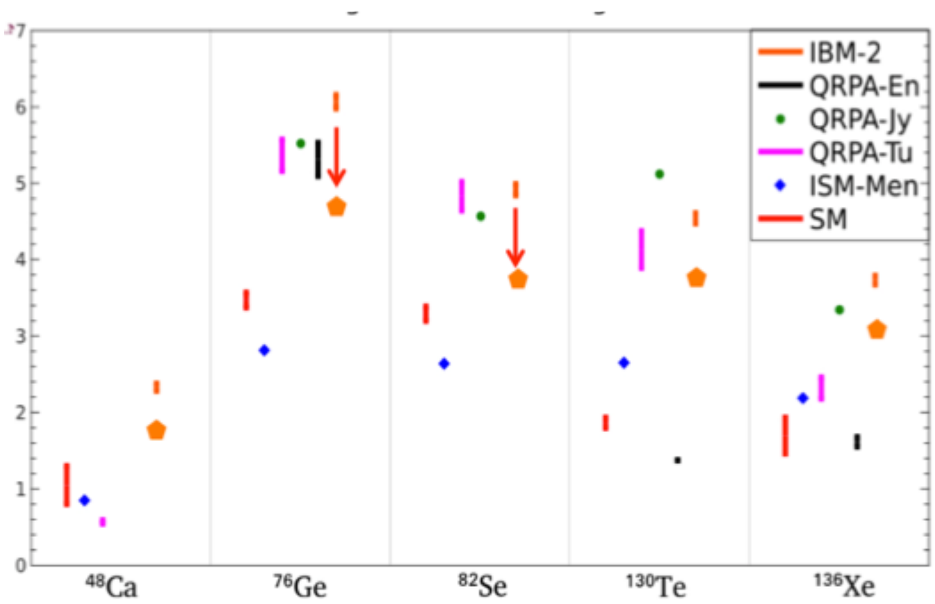
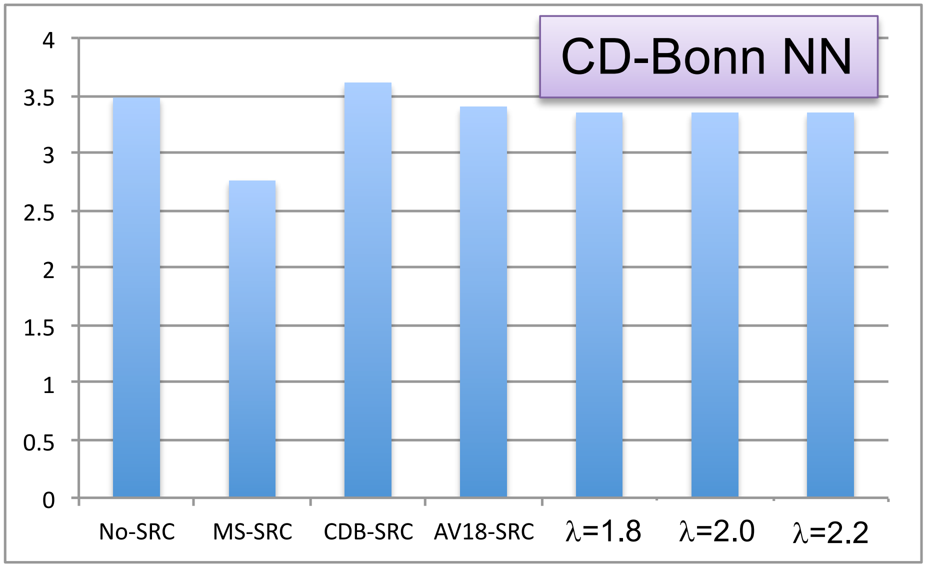
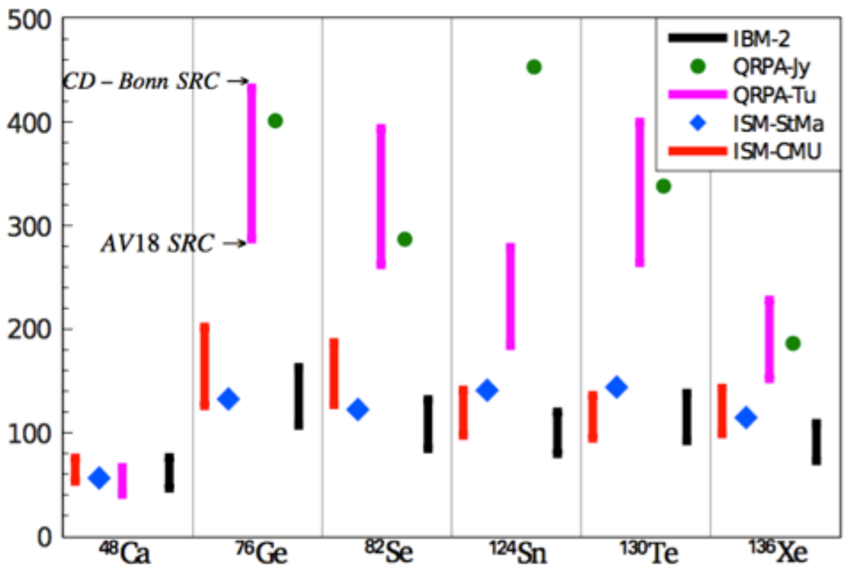


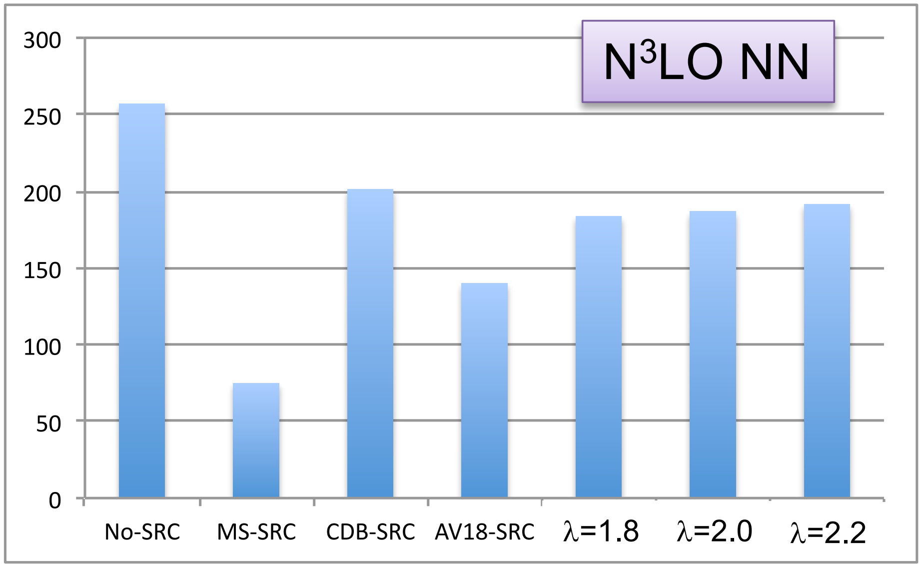


Application to ^{76}Ge $0\nu\beta\beta$ matrix elements

In collaboration with Schuster, Horoi, Engel, Holt, Navratil

- Matrix elements for heavy-neutrino exchange mechanism





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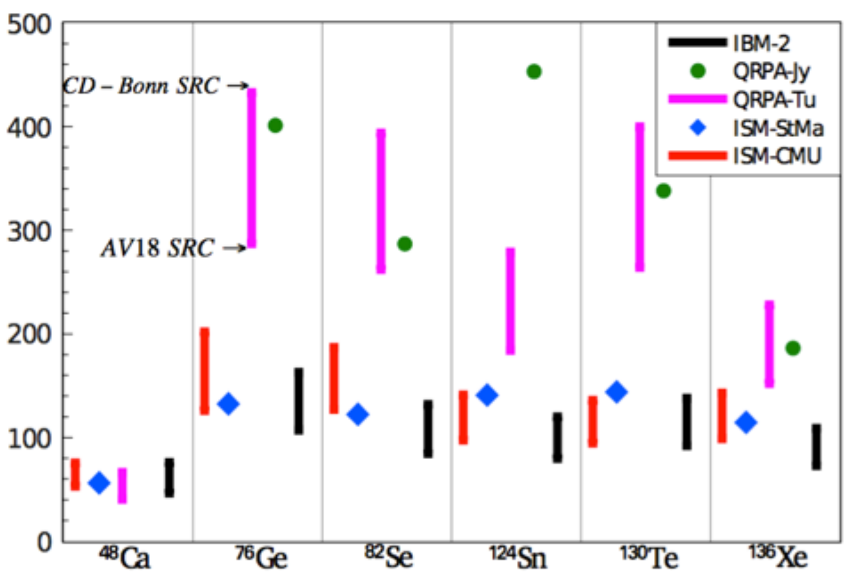
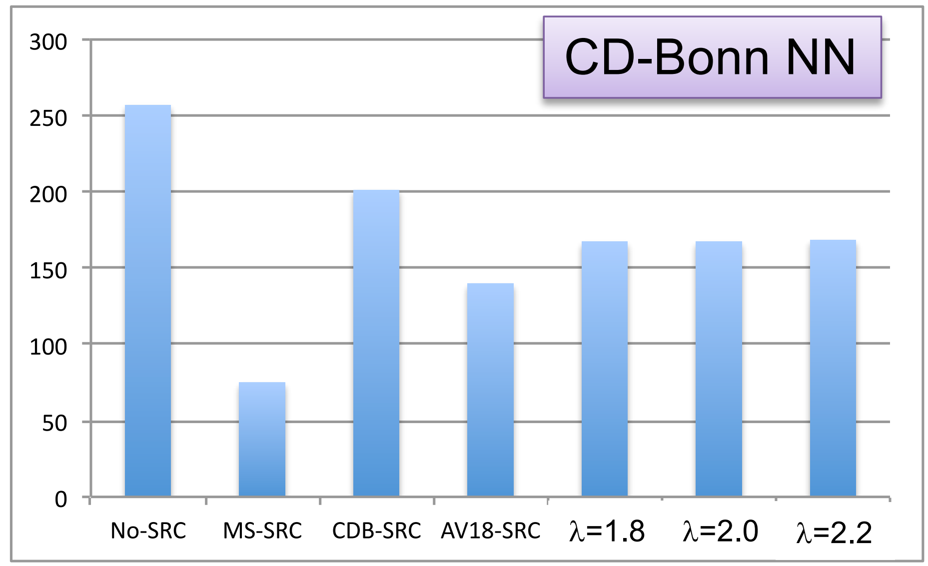


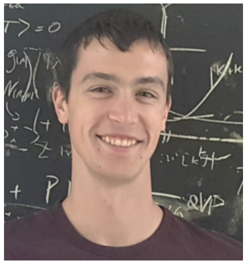


This work needs to be completed by carrying out calculations in many-body perturbation theory (MBPT)

SRG 3-body evolution of general operators

Gysbers (10 weeks internship at LLNL, funded by DBD TC), S.Q., Navratil

- Evolution at the 3-body level determined in A=3 space
  - Carried out in the relative-coordinate 3-nucleon basis using the code MANYEFF
  - Implemented for general operators of any rank
  - Conversion of matrix elements of evolved operator to the single-particle SD basis non trivial, only partially implemented
  - Applied to 0νββ operators



P. Gysbers

$$\hat{O}^{(2)}_{0\nu\beta\beta} = \hat{O}^{(2)}_{GT}, \hat{O}^{(2)}_{F}, \hat{O}^{(2)}_{T} \longrightarrow \hat{O}^{(2)}_{0\nu\beta\beta;\lambda} + \hat{O}^{(3)}_{0\nu\beta\beta;\lambda} + \cdots$$

$$\left\langle \hat{O}^{(3)}_{0\nu\beta\beta;\lambda} \right\rangle_{A=3} = \left\langle \left(\hat{O}^{(2)}_{0\nu\beta\beta} \right)_{\lambda} \right\rangle_{A=3} - \left\langle \hat{O}^{(2)}_{0\nu\beta\beta;\lambda} \right\rangle_{A=3}$$

available on NERSC

Conclusions

- SRG 2-body evolution of general operators
  - Applied to: $\sigma\tau$ operator and chiral 2BC, $0\nu\beta\beta$ operators
  - 2-body evolved matrix elements **available in relative coordinates and translated to single-particle coordinates**
- SRG 3-body evolution of general operators
  - Conversion from relative to single-particle coordinates challenging
  - Applied to $0\nu\beta\beta$ operators; 3-body evolved matrix elements **available in relative coordinates and translated to single-particle coordinates**
- Important to describe weak transitions
  - Gamow-Teller transitions of light nuclei
  - ^{76}Ge $0\nu\beta\beta$ transitions, especially with heavy neutrino (**to be completed with calculations in many-body perturbation theory**)

Collaborators contributing to the present results

- K. Wendt (LLNL)
- **P. Gysbers** (UBC/TRIUMF)
- P. Navratil, J. Holt (TRIUMF)
- M. Horoi (CMU), J. Engel (UNC)
- E. Jurgenson (LLNL)
- C. Johnson (SDSU)
- M. Schuster (WIT)
- Doron Gazit (Hebrew U)

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National Laboratory