The Hamiltonian-based generator-coordinate calculations of $0\nu\beta\beta$ decay NMEs

Changfeng Jiao
Department of Physics
San Diego State University

Feb 3rd @ University of Carolina at Chapel Hill

Generator Coordinate Method (GCM)

Generator Coordinate Method: an approach that treats large-amplitude fluctuations, which is essential for nuclei that cannot be approximated by a single mean field.

How it works:

① Step1: Construct basis states by constrained HFB calculation.
correlations along important coordinates (e.g., deformation).

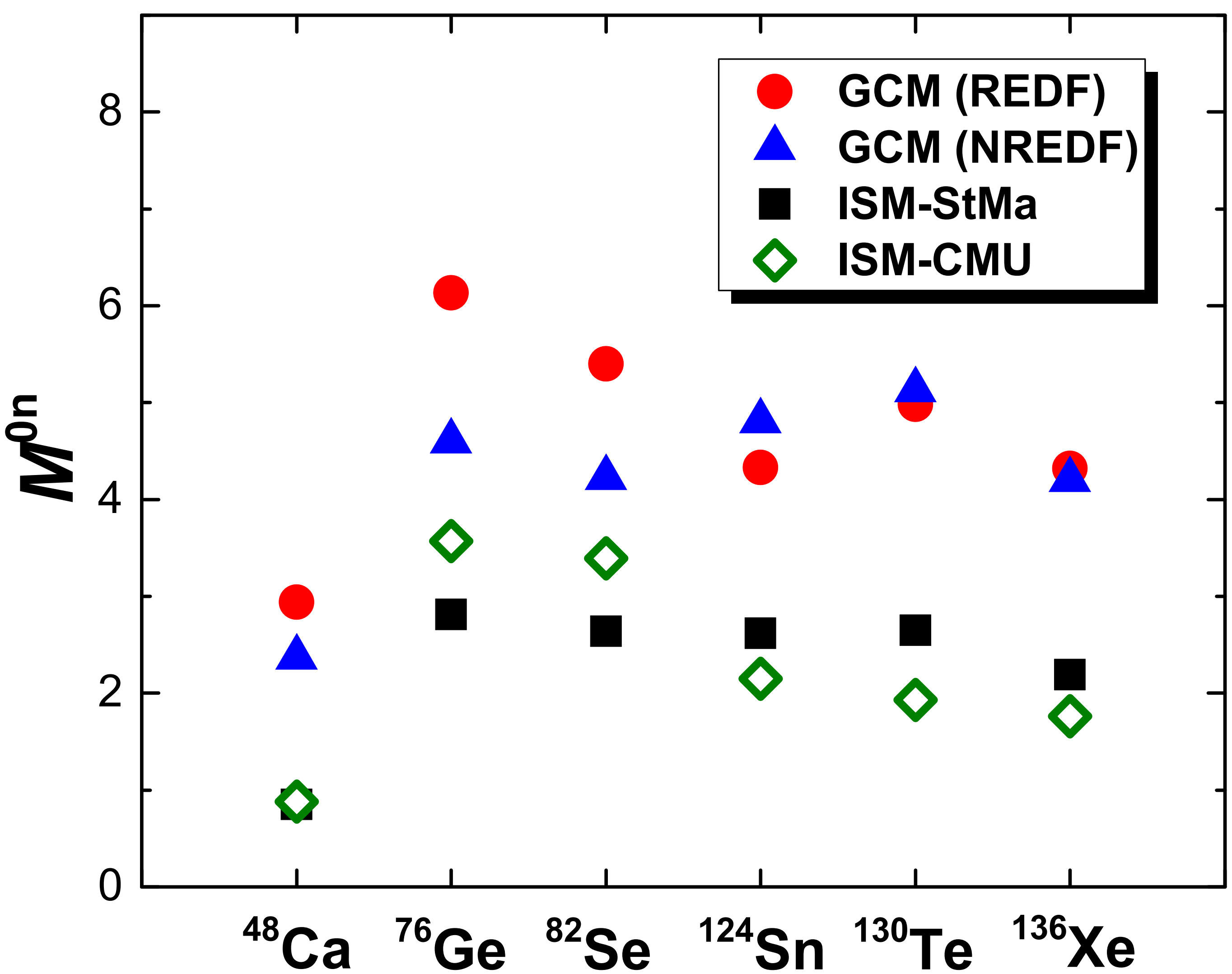
② Step2: Restore the symmetry of mean-field states. ***Projections.***

③ Step3: Diagonalize Hamiltonian in space of symmetry-restored nonorthogonal vacua.

GCM based on EDF has been applied to double-beta decay, however…

Comparison between GCM and SM

Current results with EDF-based GCM



Both the shell model and the EDF-based GCM could be missing important physics.

The discrepancy may be because:

- The GCM omits correlations.
- The shell model omits many single-particle levels.

Does the discrepancy come from methods themselves, or the interactions they use?

Ultimate goal and :

Ultimate goal:

The perfect many-body method will include all possible correlations in an infinitely large space.

To get closer to the ultimate goal:

We can use SM Hamiltonian in the GCM.

- more correlations.
- larger model space.

Our short-term goal is more modest:

a shell-model Hamiltonian-based GCM in one and two (and possibly more) shells.

Our Current Procedure

- Using a shell-model Hamiltonian.
- HFB states $|\Phi(q)\rangle$ with multipole constraints $q_i = \langle \mathcal{O}_i \rangle$.

  We are trying to include all possible collective correlations.

$$\mathcal{O}_1 = Q_{20}, \quad \mathcal{O}_2 = Q_{22},$$

$$\mathcal{O}_3 = \frac{1}{2}(P_0 + P_0^\dagger), \quad \mathcal{O}_4 = \frac{1}{2}(S_0 + S_0^\dagger),$$

- Angular momentum and particle number projection

$$|JMK;NZ;q\rangle = \hat{P}^J_{MK} \hat{P^N} \hat{P^Z} |\Phi(q)\rangle$$

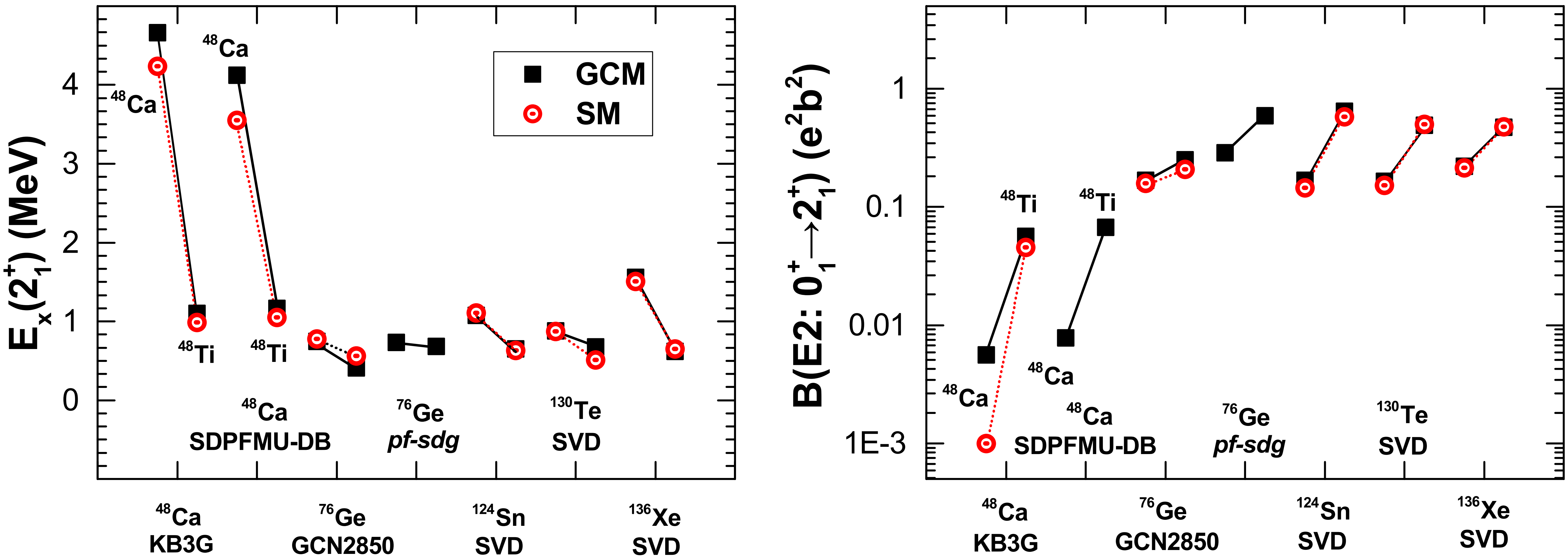
- Configuration mixing within GCM:

$$|\Psi^J_{NZ\sigma}\rangle = \sum_{K,q} f^{JK}_\sigma(q) |JMK;NZ;q\rangle$$

$$\sum_{K',q'} \{\mathcal{H}^J_{KK'}(q;q') - E^J_\sigma \mathcal{N}^J_{KK'}(q;q')\} f^{JK'}_\sigma(q') = 0 \quad \rightarrow \quad f^{JK}_\sigma(q)$$

$$M^{0\nu\beta\beta}_\xi = \langle \Psi^{J=0}_{N_f Z_f} | \hat{O}^{0\nu\beta\beta}_\xi | \Psi^{J=0}_{N_i Z_i} \rangle$$

Validation of GCM

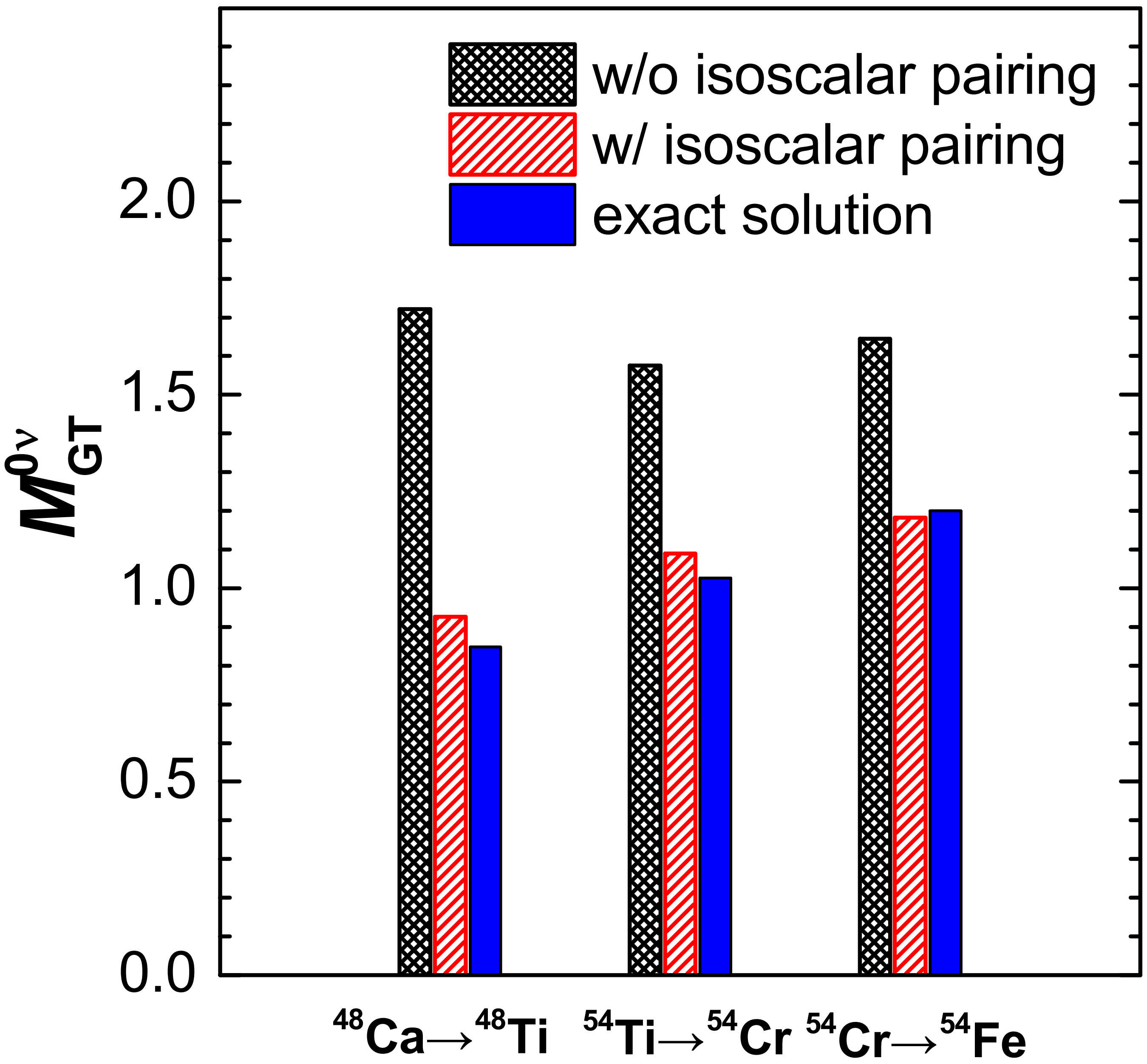


The first 2+-state energies and *B*(E2) given by Hamiltonian-based GCM are in great agreement with SM results.

Level 1 GCM: Axial shape and pn pairing fluctuation

$$H' = H - \lambda_Z N_Z - \lambda_N N_N - \lambda_0 Q_{20} - \frac{\lambda_P}{2}(P_0 + P_0^\dagger)$$

pn pairing constrained



$$P_0^\dagger = \frac{1}{\sqrt{2}} \sum_l \hat{l} [c_l^\dagger c_l^\dagger]^{L=0,S=1,T=0}_{M_S=0}$$

We use the KB3G interaction for two GCM calculations:

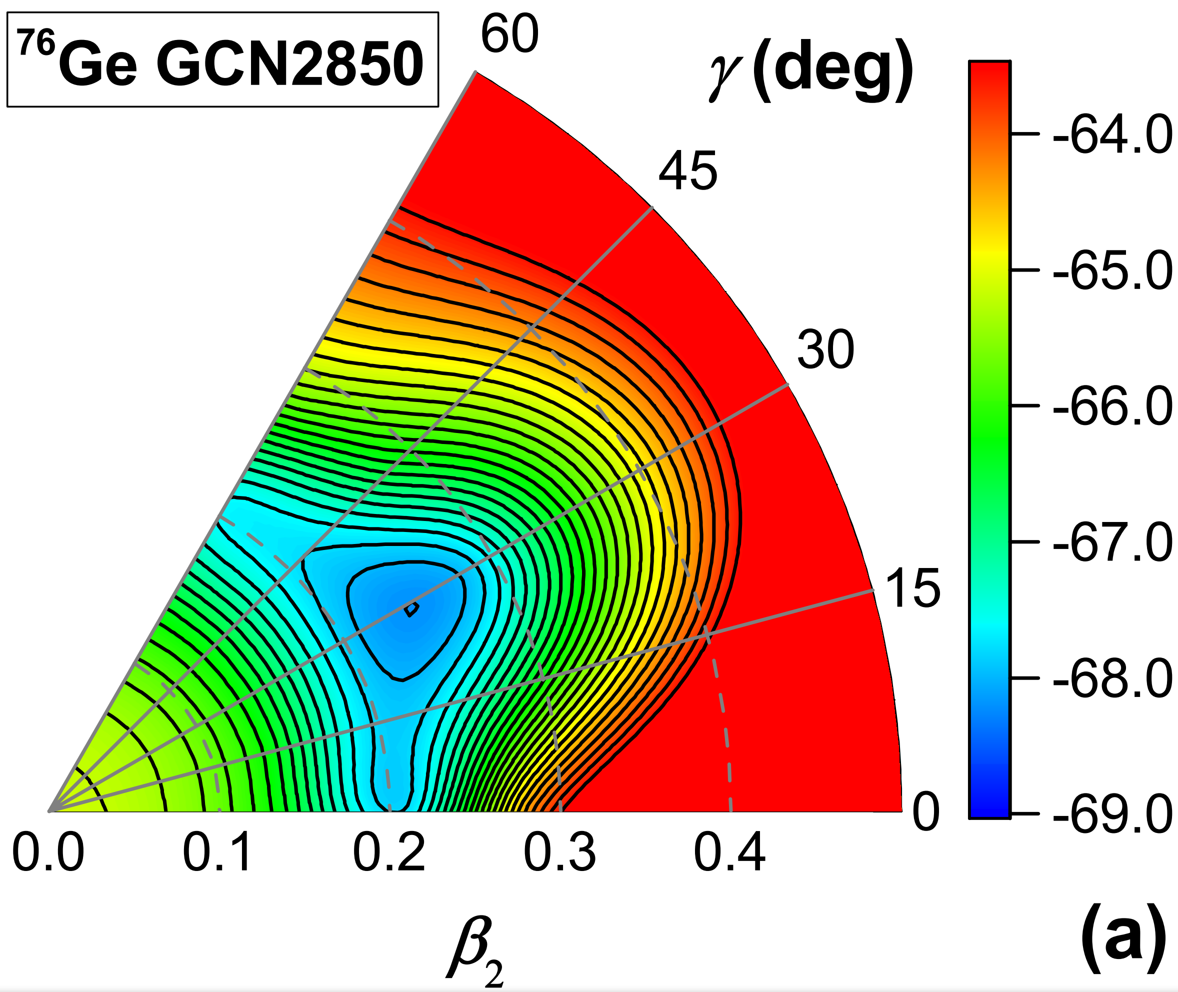
- **Black column**: we set all the two-body matrix elements of the Hamiltonian with $J = 1$ and $T = 0$ to zero.
  M_{GT} is overestimated.
- **Red column:** we use the full KB3G Hamiltonian:
  M_{GT} is suppressed, close to SM.

CJ and J. Engel, PRC 96, 054310 (2017)

Level 2 GCM: Triaxial deformation

$$H' = H - \lambda_Z N_Z - \lambda_N N_N - \lambda_0 Q_{20} - \frac{\lambda_P}{2}(P_0 + P_0^\dagger) - \lambda_2 Q_{22}$$

triaxial deformation constrained



With GCN2850 or JUN45 interaction, projected potential energy surfaces for ^{76}Ge and ^{76}Se give minima with triaxial deformation.

Level 2 GCM: triaxial deformation

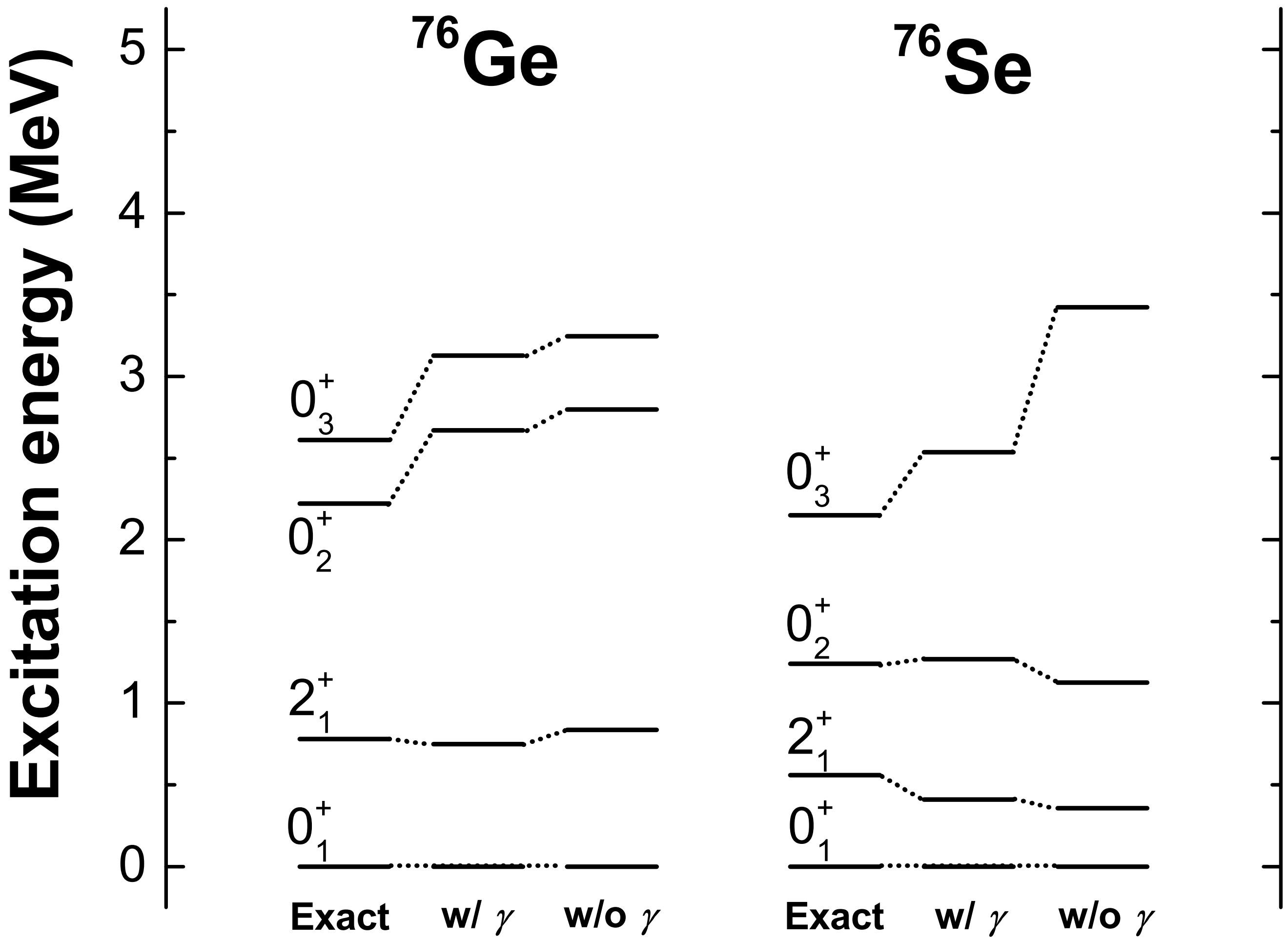


TABLE I. Matrix elements $M^{0\nu}$ produced in the GCM by GCN2850 and JUN45 for the decay of ^{76}Ge, with and without triaxial deformation as a generator coordinate, and by those same interactions with exact diagonalization.

	GCN2850	JUN45
Axial GCM	2.93	3.51
Triaxial GCM	2.56	3.16
Exact	2.81	3.37

CJ and J. Engel, PRC 96, 054310 (2017)

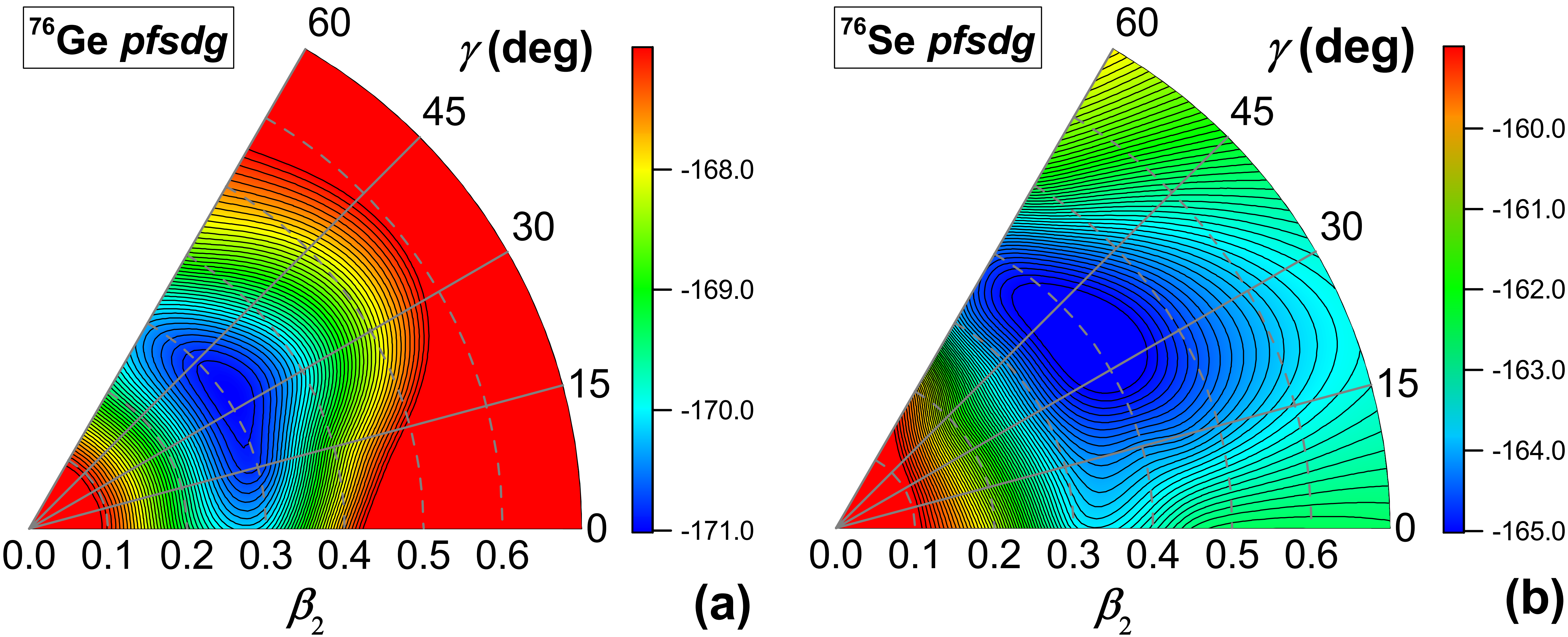
10~15% reduction of NME if triaxial shape fluctuation is included.

Two-shell GCM for ^{76}Ge

- Effective *pfsdg*-shell interaction based on chiral EFT can be calculated by many-body perturbation theory (MBPT), similarity renormalization group (SRG) or couple cluster (CC).
- We employ an effective *pfsdg*-shell interaction calculated by extended Krenciglowa-Kuo perturbative method, which are provided by J. D. Holt.
- The monopole part of the resulting Hamiltonian is sensitive to the three-body part of the initial interaction, which one generally reduces to an effective two-body interaction by summing the third particle over a set of occupied states.

pfsdg: 3N forces normal ordered with respect to ^{56}Ni
We optimize the single-particle energies for *pfsdg*-shell interactions by fitting the measured occupancies of valence neutron and proton orbits.

Two-shell GCM for ^{76}Ge



- Larger model space: triaxially deformed as predicted.
- How does triaxial shape influence NMEs?

CJ and J. Engel, PRC 96, 054310 (2017)

Two-shell GCM for ^{76}Ge

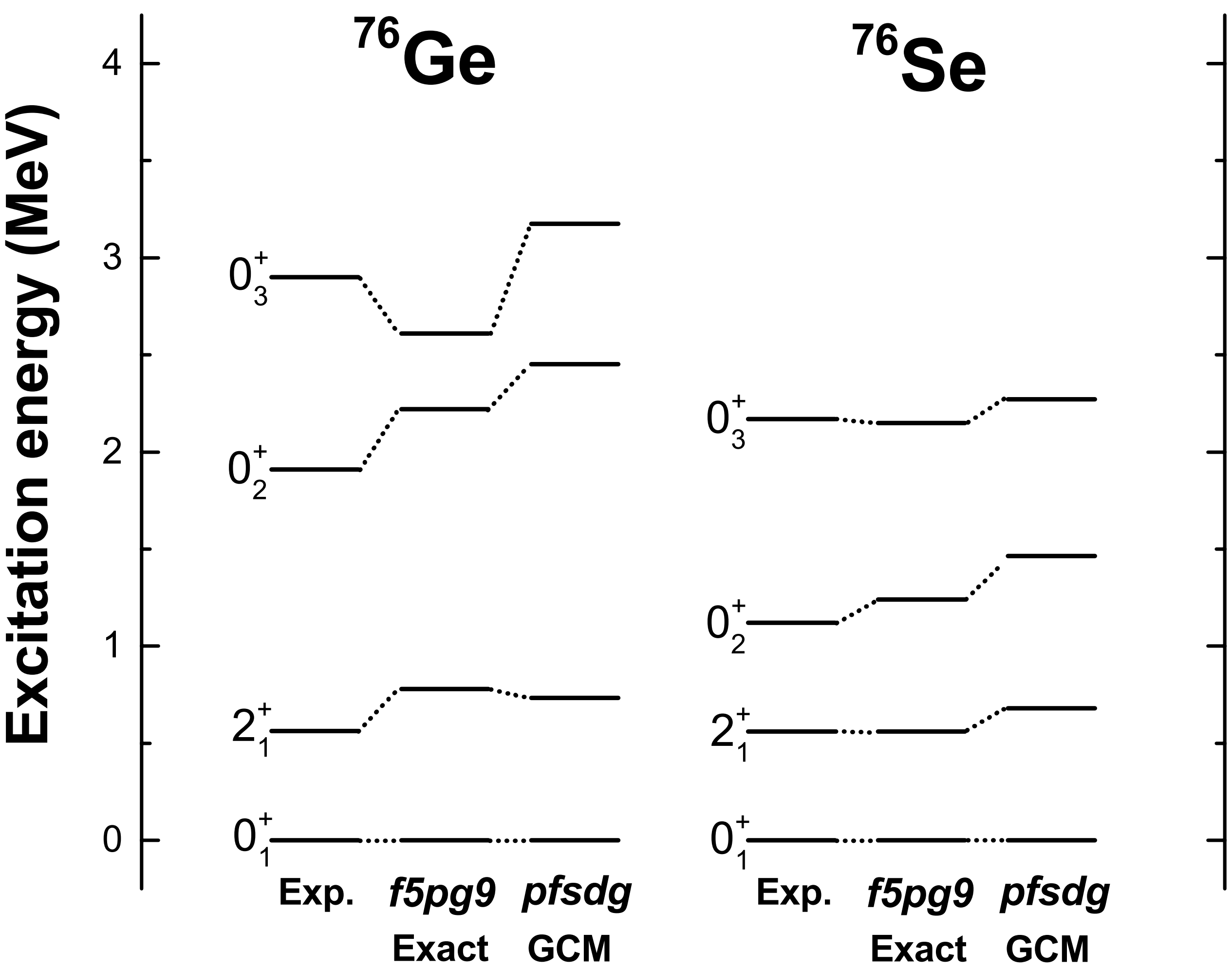


TABLE II. GCM results for the Gamow-Teller ($M^{0\nu}_{\rm GT}$), Fermi ($M^{0\nu}_{\rm F}$), and tensor ($M^{0\nu}_{\rm T}$) $0\nu\beta\beta$ matrix elements for the decay of ^{76}Ge in two shells, without and with triaxial deformation.

	Axial	Triaxial
$M^{0\nu}_{\rm GT}$	3.18	1.99
$-\frac{g_V^2}{g_A^2} M^{0\nu}_{\rm F}$	0.55	0.38
$M^{0\nu}_{\rm T}$	−0.01	−0.02
Total $M^{0\nu}$	3.72	2.35

- The low-lying spectra are well described.
- The NME is slightly smaller than the single-shell result.
- Importance of triaxial deformation in larger space.

CJ and J. Engel, PRC 96, 054310 (2017)

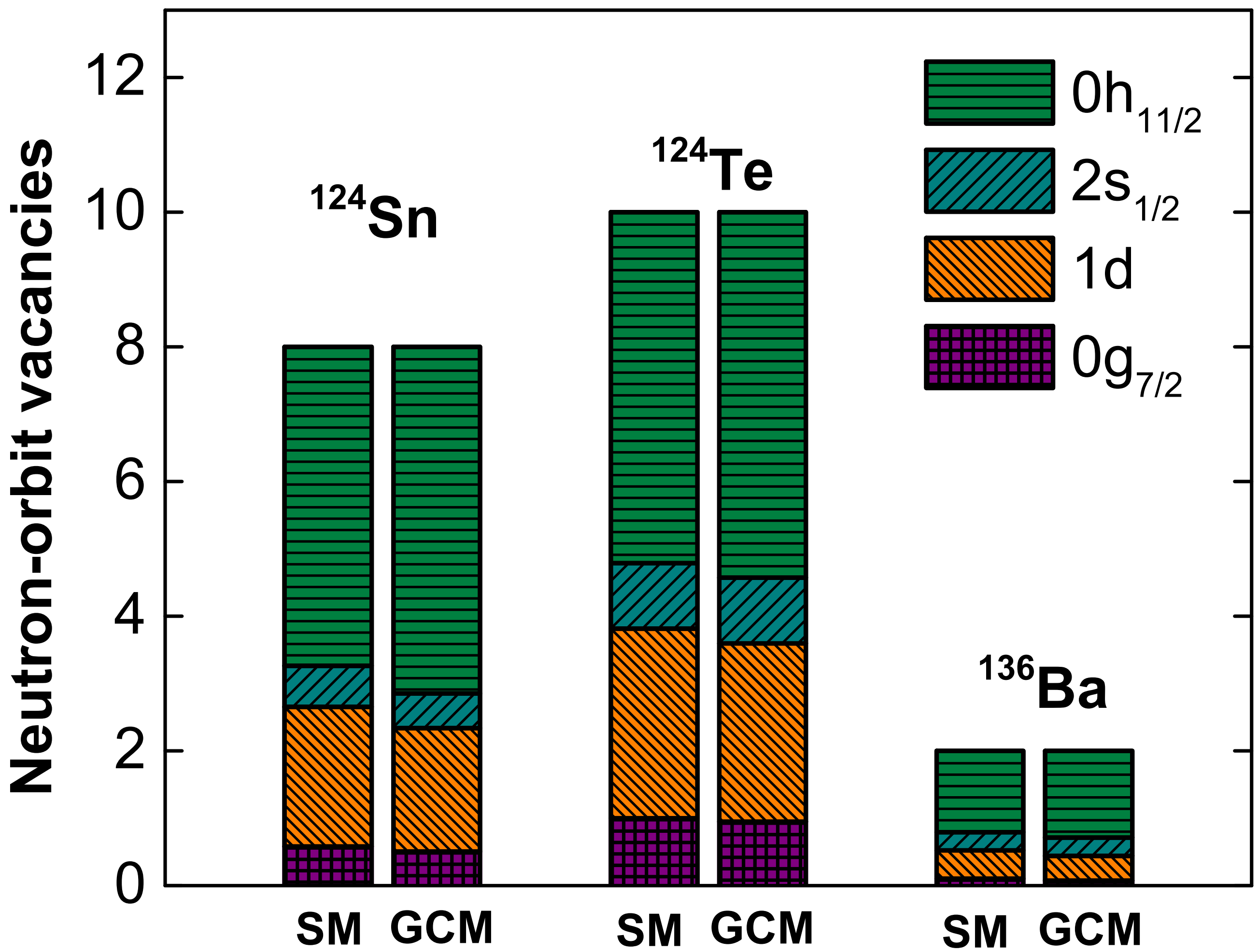
GCM with jj55 space

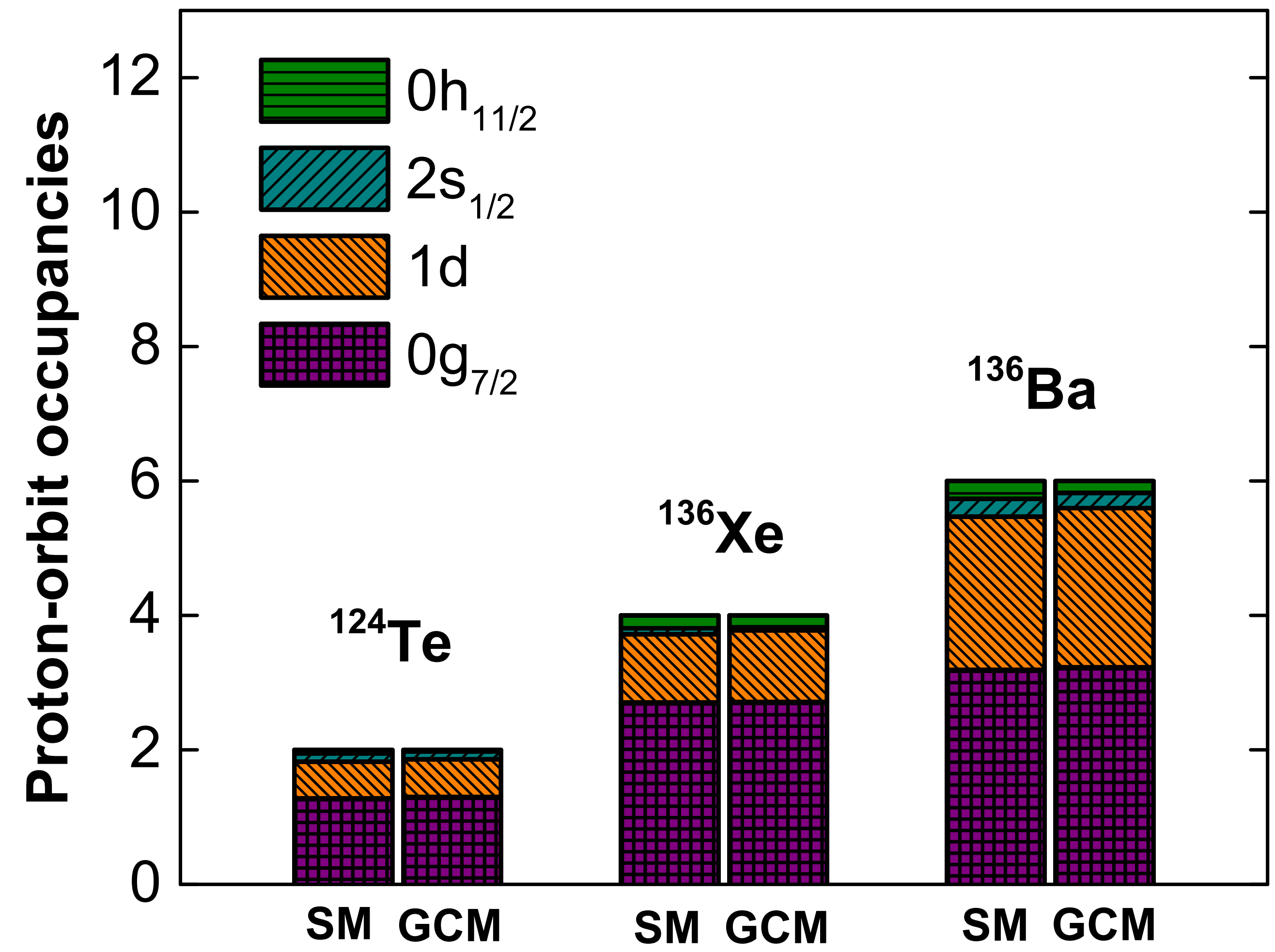
We want to extend the Hamiltonian-based GCM to larger model space and heavier $0\nu\beta\beta$-decay candidates (e.g., ^{150}Nd), for which no effective shell-model interaction exists.

STEP1: We move forwards to ^{124}Sn, ^{130}Te, and ^{136}Xe to check how GCM with shell-model Hamiltonian works for them.

- We use the SVD effective Hamiltonian within $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$. $2s_{1/2}$, $0h_{11/2}$ orbits (called *jj*55 model space here). M. Horoi's group has done a lot of shell-model calculation with this interaction, providing a great testing ground.
- Because these nuclei are considered to be nearly spherical or slightly deformed, only axial deformation, isoscalar pairing, and isovector pn pairing are treated as coordinates (but separately for latter two).

GCM with jj55 space





The theoretical neutron shell vacancies and proton shell occupancies given by GCM are very close to the exact diagonalization from SM.

GCM with jj55 space

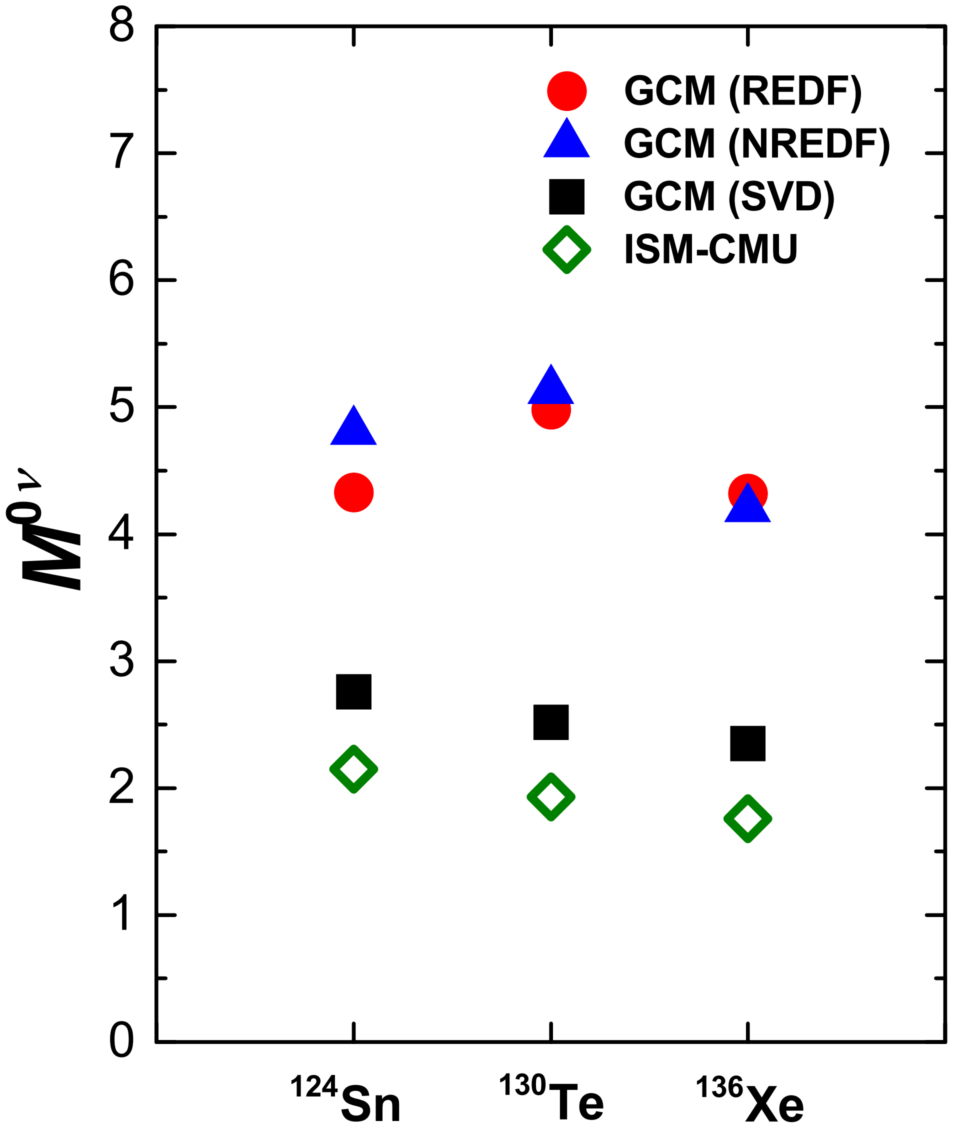
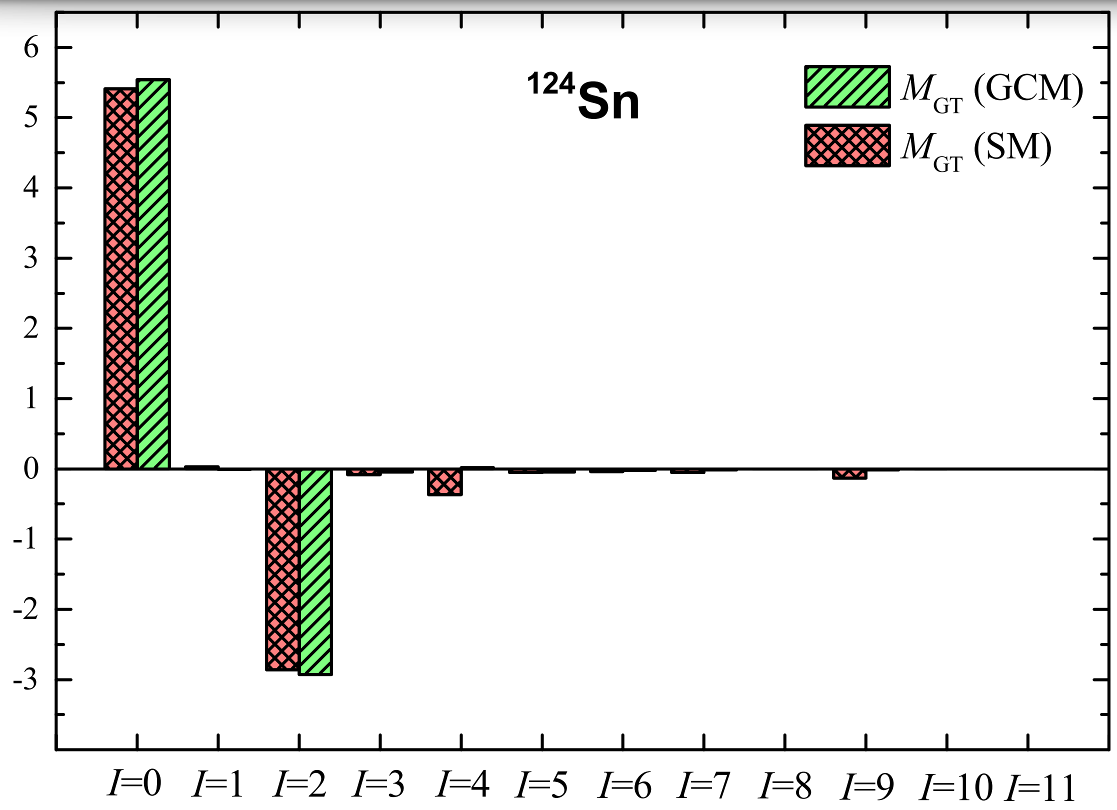


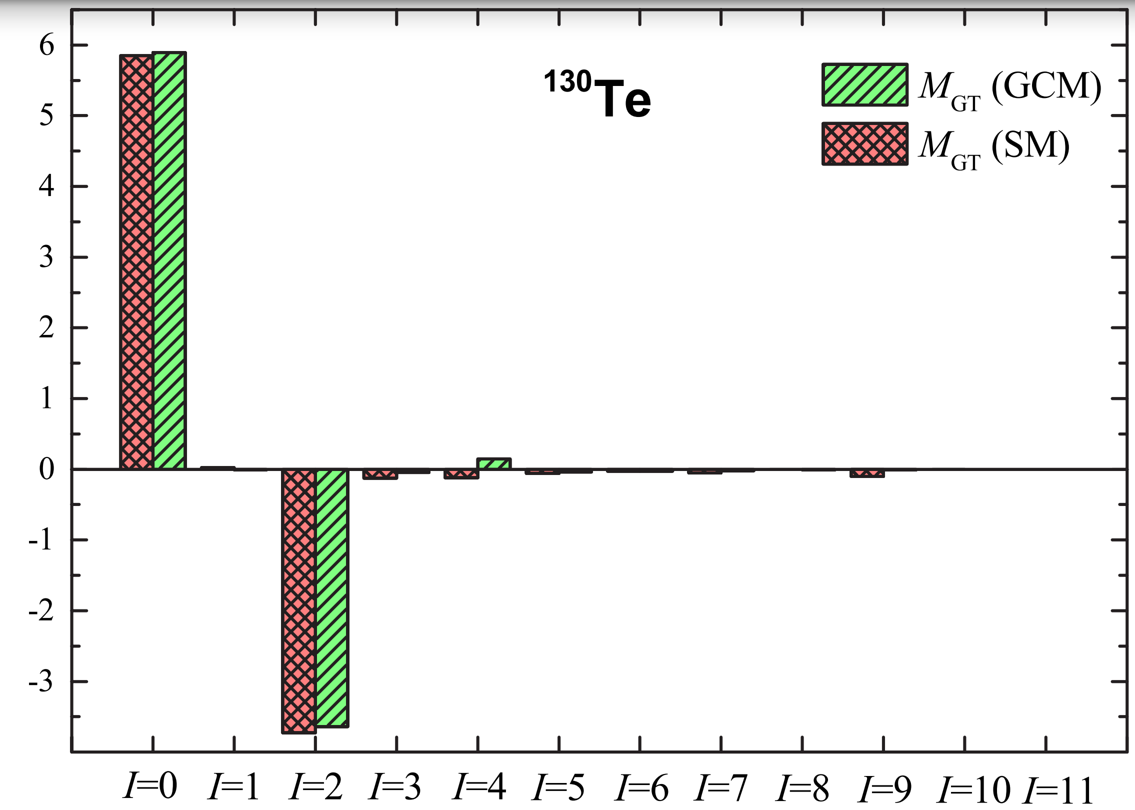
TABLE II: The NMEs obtained with SVD Hamiltonian by using GCM and SM for ^{124}Sn, ^{130}Te, and ^{136}Xe. CD-Bonn SRC parametrization was used.

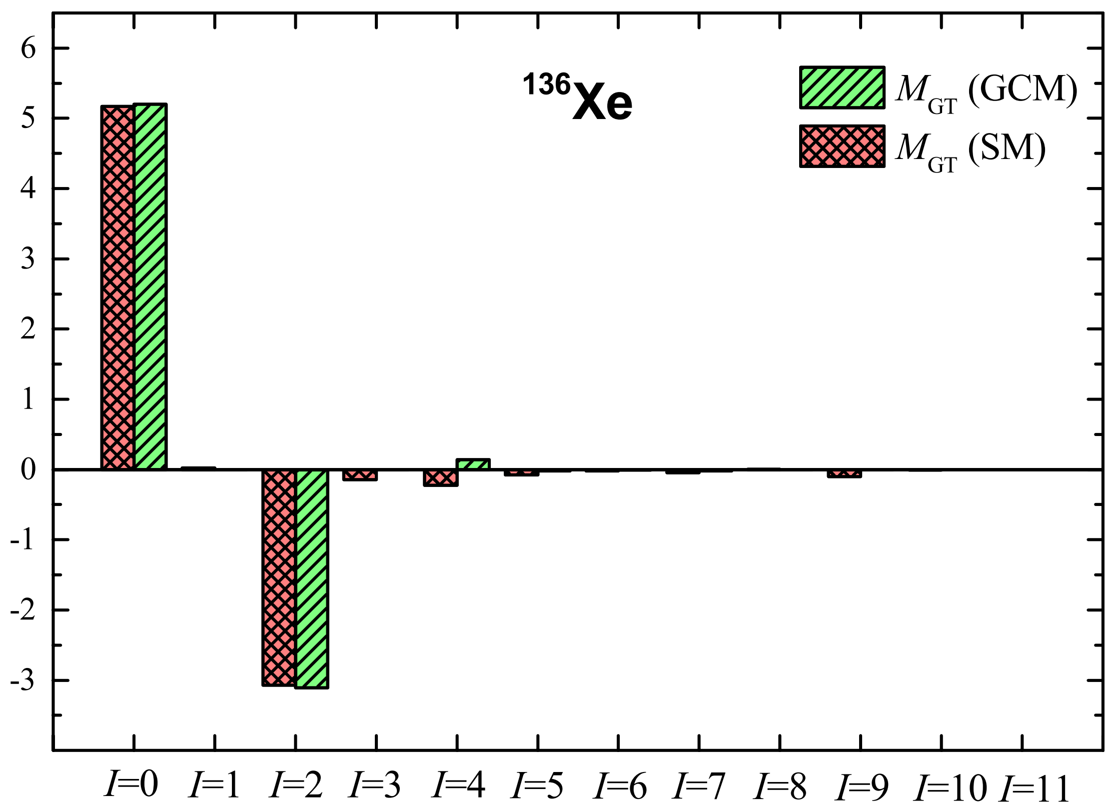
		$M^{0\nu}_{\mathrm{GT}}$	$M^{0\nu}_{\mathrm{F}}$	$M^{0\nu}_{\mathrm{T}}$	$M^{0\nu}$
^{124}Sn	GCM	2.48	−0.51	−0.03	2.76
	SM	1.85	−0.47	−0.01	2.15
^{130}Te	GCM	2.25	−0.47	−0.02	2.52
	SM	1.66	−0.44	−0.01	1.94
^{136}Xe	GCM	2.17	−0.32	−0.02	2.35
	SM	1.50	−0.40	−0.01	1.76

The NMEs given by our SVD-based GCM are closer to the exact result, ~30% larger than SM results, most of them come from GT part.

GCM with jj55 space

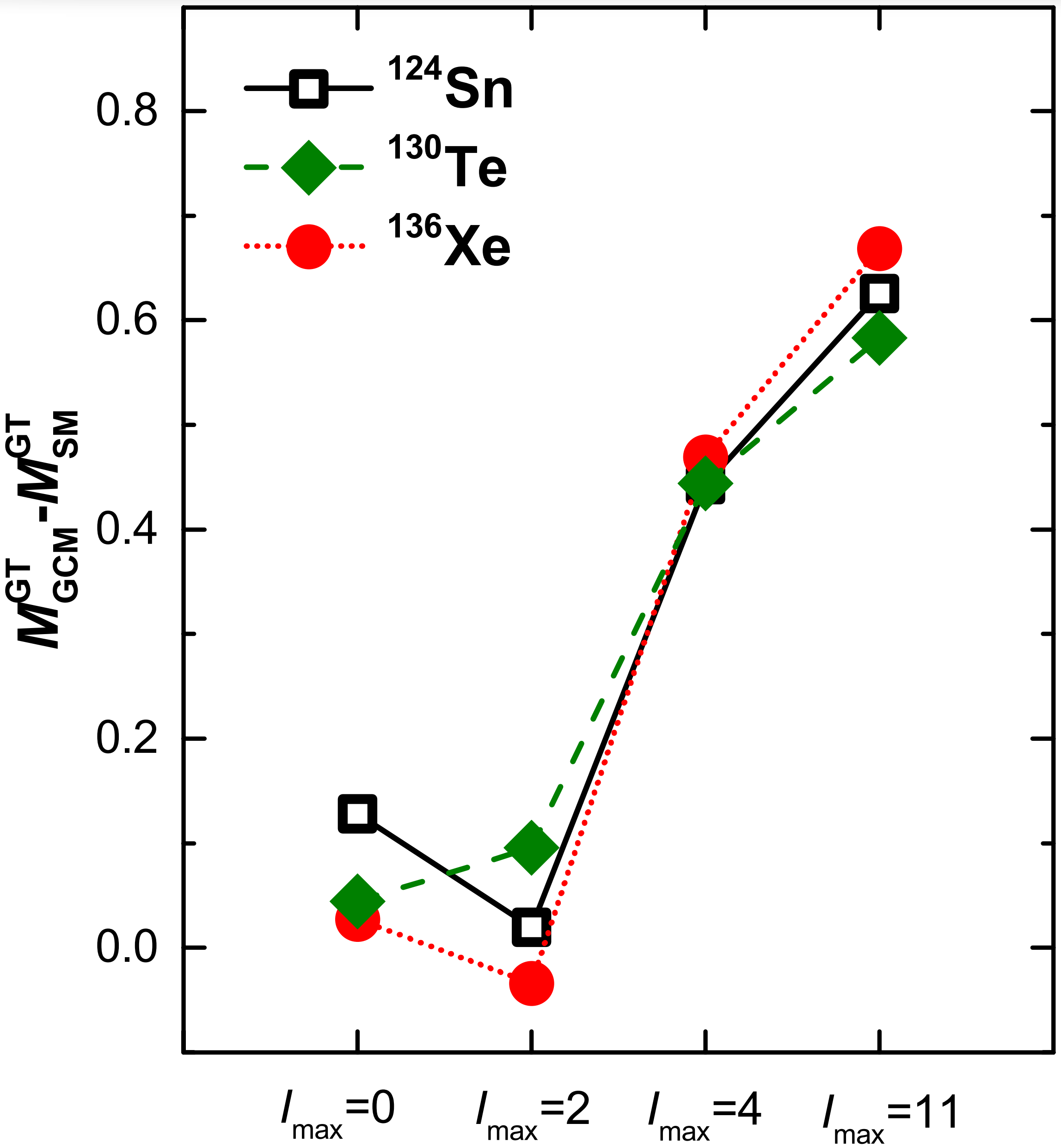






- The dramatic cancellation between I = 0 and I = 2 is well described in GCM calculations.
- GCM results barely capture I>3 contributions.

GCM with jj55 space

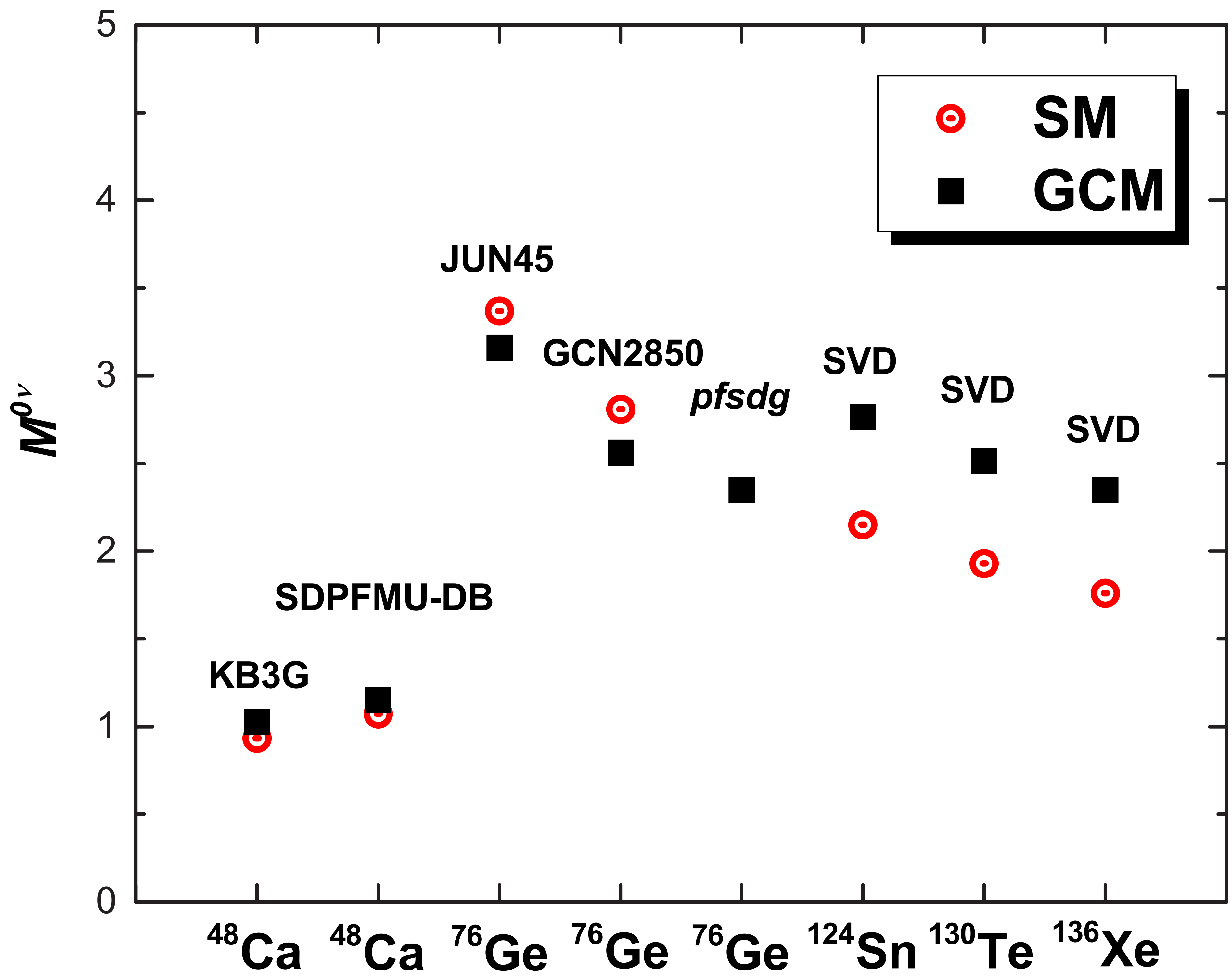


more than 80% overestimation is from *l*>3 contributions: Non-collective correlations?

Some potential improvement:

- Treat deformation, isovector pairing, isoscalar pairing as coordinates at the same time.
- Non-collective correlations should be considered. (e.g., quasiparticle excitation?)

Summary of 0vββ NME given by GCM



Summary

- We are trying to combine the virtues of the shell model and EDF calculations by the Hamiltonian-based GCM.
- Tests against exact solutions in one shell indicate that we indeed have captured important valence-space correlations.
- Calculation has been extended to two major shell (e.g., *pfsdg* shell) model space, which is out of scope of the conventional SM.
- Extending to *jj*55 model space indicates that non-collective correlations may be required.

Perspective

- We can improve the Hamiltonian-based GCM by using path integral and the auxiliary-field Monte Carlo method.
- Also, we can improve the angular-momentum and particle-number projections by using the linear algebra.

Summary

Collaborators:

- Calvin Johnson, SDSU
- Mihai Horoi, CMU
- Andrei Neascu, CMU
- Jonathan Engel, UNC
- Jiangming Yao, UNC
- Longjun Wang, UNC
- Jason Holt, TRIUMF
- Nobuo Hinohara, University of Tsukuba
- Javier Menendez, University of Tokyo

Thank you!

Supported by
DE-SC0015376 & DE-FG02-03ER41272