Automobile Flywheel

**Flywheel:**
- M = 250 kg
- r = 0.60 m
- \( \frac{5000}{g} = 4.9 \times 10^5 \text{m/s}^2 \)

**Car:**
- \( M_c = 850 \text{ kg} \)
- F = 150 N at 60 mi/h = 27 mg \( g \)

Assumptions:
- negligible loss of energy in power transfer mechanisms (realistically we should assume 10% to 50% efficiency)
- negligible weight of cargo for determining max. range.

Estimations:
- Cost of electricity \( \approx 84 \text{¢/kWh} = 84 / 3.6 \text{MJ} \)
- Cost of gasoline \( \approx 3.10 / \text{gal} \); typical fuel economy = 30 MPG

**Find:** \( W_{\text{max}} \), max. range, orientation of wheel, feasibility, + cost to operate

**Solution:**

1. Conservation of energy problem: 
   \( \text{Rot. KE} \to \text{trans. KE} + \text{work to oppose friction} \)

2. KE = KE + W_{nc}
   \[ KE = \frac{1}{2}I\omega^2 \]
   \[ W_{nc} = KE - KE \]
   \[ F\cdotd = \frac{1}{2}I\omega^2 - \frac{1}{2}MV^2 \]

3. \( d = \frac{1.84 \text{MJ} - 0.40 \text{MJ}}{150 \text{N}} \)
   \[ d = 9.6 \text{ km} = 6.0 \text{ mi} \]

4. \( KE = \frac{1}{2}I\omega^2 = \frac{(280 \text{ lb}) (0.6 \text{ m})}{2} = 45 \text{ kg} \cdot \text{m}^2 \)
   \( I = 8.17 \times 10^4 \text{ rad}^2/\text{s}^2 \)
   \( \omega_{\text{max}} = \frac{W_{\text{max}}}{I} \)
   \[ \omega_{\text{max}} = 8.17 \times 10^4 \text{ rad}^2/\text{s} = 2.730 \text{ rev/min} \]
   \[ KE = \frac{1}{2}I\omega^2 = \frac{1}{2} (850 \text{ lb}) (250 \text{ lb}) (27 \text{ mg}) \]
   \[ KE = 0.40 \text{ MJ} \]

5. Time to "charge": 
   \[ t = \frac{KE}{P} = \frac{1.84 \text{ MJ}}{3730 \text{ W}} = 4.93 \text{ s} = 81.2 \text{ mm} \]

6. Cost for electricity:
   \[ 1.84 \text{ MJ} \times (0.08 / 3.6 \text{ MJ}) = 0.04 \text{¢} \]
   cheaper

7. Cost for gasoline:
   \[ \frac{6 \text{ mi}}{30 \text{ mi/gal}} \times (3.10 / \text{gal}) = 0.26 \text{¢} \text{ per mile} \]
   gas is more expensive

8. It appears that this flywheel could only propel the car a maximum of 6 miles before needing to be re-charged. (Range of ~3 miles is more realistic at 50% efficiency)

   Although this car does not meet the 50 mi minimum range for marketability, the flywheel design with an electric motor looks like a cost-effective and environmentally-friendly transportation solution. This is probably why some city buses in Europe use flywheels similar to this.

One important design consideration is the orientation of the flywheel inside the vehicle. Since the flywheel has considerable angular momentum, it must be positioned horizontally (as shown above) so that the vehicle can turn left or right without exerting a torque on the flywheel. This then means that the car will tend to veer left or right whenever the car tilts up or down!